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SPECIFICATIONS AND CLARIFICATIONS REGARDING THE ADAPT VPA ASSESSMENT/PROJECTION COMPUTATIONS CARRIED OUT DURING THE SEPTEMBER 2000 ICCAT WEST ATLANTIC BLUEFIN TUNA STOCK ASSESSMENT SESSION

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SUMMARY

Detailed algebraic specifications are given for the ADAPT VPA assessment and projection computations carried out at the 2000 assessment session. Some minor errors made at that time, related to plus-group mass and how this is taken into account in MSY computations, are corrected. These adjustments have been incorporated in revised FORTRAN code, which has also been extended to compute all the assessment diagnostics reported in Table 7 of the report of the 2000 session. Replication of some of these has, however, proved problematic and their specification requires further clarification during the 2002 session.

RÉSUMÉ

Le présent document fournit des spécifications algébriques détaillées pour l'évaluation ADAPT VPA ainsi que des calculs de projections réalisés à la session d'évaluation de 2000. Certaines erreurs mineures commises à l'époque, relatives à la masse du groupe-plus et à la façon dont ceci est pris en compte dans les calculs de la PME, ont été corrigées. Ces ajustements ont été incorporés au code révisé FORTRAN, qui a également été étendu pour calculer tous les diagnostics d'évaluation consignés au Tableau 7 du rapport de la session de 2000. La réplique de certains d'entre eux s'est toutefois avérée problématique et leur spécification nécessite davantage de clarification au cours de la session de 2002.

RESUMEN

Se presentan especificaciones algebraicas detalladas para la evaluación del VPA ADAPT y los cálculos de proyección realizados en la sesión de evaluación de 2000. Se corrigen los errores menores en los que se incurrió en ese momento, relacionados con la masa del grupo plus y el modo en que se consideró en los cálculos del RMS. Estos ajustes se han incorporado en el código FORTRAN revisado, que también ha sido ampliado para calcular todos los diagnósticos de evaluación de la Tabla 7 del informe de la sesión de 2000. Sin embargo, la réplica de algunos de éstos ha demostrado ser problemática y su especificación requiere una nueva aclaración durante la sesión de 2002.

KEYWORDS

Bluefin tuna; Mathematical models; Population numbers

1. INTRODUCTION

The ADAPT-VPA assessments and projections which have provided the baseline for scientific advice for West Atlantic bluefin tuna over recent years have become very complicated, so that it is

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important to maintain an accurate record of exactly what the technical specifications agreed at a particular meeting have been.

For the computations carried out at the September 2000 ICCAT SCRS West Atlantic bluefin tuna stock assessment session (ICCAT, 2001), much of this detail is provided in Appendix B of Geromont and Butterworth (2001). An updated version of that Appendix is provided as the Appendix to this paper. Its purpose is to correct some earlier minor errors, and to cover aspects of the assessment / projection computations reported in ICCAT (2001) that have not been fully defined previously. The associated process of extending code to output some of the diagnostic and related statistics reported in ICCAT (2001) has brought to light both some errors and some matters requiring clarification, and these are detailed below.

2. ERRORS IDENTIFIED

1) A minor error was identified in the code pertaining to the computation of future projections of the plus-group masses, as detailed in the final paragraph of section A.8.5 of the Appendix. This has hardly any quantitative impact on key projection results. The error has been corrected in the FORTRAN code.

2) p-values reported for model deviance (see Equation A.21) in Table 7a of ICCAT (2001) were erroneously calculated for n instead of n-p[3] degrees of freedom.

3. CALCULATIONS PROVIDED

Section A.8.9 has been substantially extended to clarify exactly how the FORTRAN code computes MSY, B^{sp}_{MSY} and K^{sp}. This incorporates a minor adjustment to previous practice, in that the mass-at-age for the plus-group is no longer taken to equal its value for the most recent year ($y_{\max}$); instead account is taken of the fact that the equilibrium plus-group mass will change with the magnitude of fishing mortality (see Equations A.31 and A.34).

4. CLARIFICATIONS NEEDED

1) Historical practice (see section A.8.9) arguably lacks self-consistency in that computation of K^{sp} uses masses-at-age based upon the growth curve (Equation A.26), whereas that of MSY uses masses-at-age for the most recent year (except now for the plus-group, as per the preceding paragraph). Some further "reconciliation" might be desirable here.

2) We have been unable to reproduce the "average normalized weights by series" for ADAPT runs as reported in Table 7b of ICCAT (2001). We suggest the following formulation:

$$W^i_N = n^I [W^i / \sum_j W^j] \qquad\qquad W^i = (1/n^i)\sum_y 1/(\tilde{\sigma}^i_y)^2 \tag{1}$$

where W^i_N is the normalized weight for index i,
n^I is the number of indices considered,
n^i is the number of years for which a value for index i is available, and
$\tilde{\sigma}^i_y$ is the residual standard deviation associated with index i in year y, as elaborated in the text following Equation A.9.

Note that series "weight" in this context has factored out the aspect that series with more data points carry a greater weight in the fit for that reason.

[3] p is the number of estimable parameters.

2) The numbers of estimated parameters listed in Table 7a of ICCAT (2001) appear to be wrong in some cases (Runs 2a and 2b), conceivably because of failure to count the number of *q* parameters estimated.

   The AIC_c values listed in Table 7a also do not seem to correspond to the formula used at the time (Equation A.20a). Further, however, consideration needs to be given as to whether Equation A.20b would be the more appropriate to use in this case, following the rationale provided in Appendix C, section C.1.1 of Geromont and Butterworth (2001).

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APPENDIX

SPECIFICATIONS FOR THE ADAPT VPA CODE AS USED FOR THE 2000 WESTERN NABFT ASSESSMENT

This Appendix details options in the existing ADAPT VPA code for western North Atlantic bluefin tuna assessments and includes information on the selections made for the base case ADAPT assessments at the 2000 SCRS bluefin assessment session in Madrid - this information is given in italics.

Note that at times, in the interests of economy of symbols, no distinction is made between a model parameter / variable x, and its maximum likelihood estimate $\hat{x}$, the latter symbol being used in both instances. The meaning intended will be clear from the context.

A.1 Model

The fundamental age-structured equations describing the dynamics are:

$$N_{y+1,a+1} = N_{y,a}\ e^{-(F_{y,a}+M_{y,a})} = N_{y,a}\ e^{-Z_{y,a}} \tag{A.1}$$

and:

$$C_{y,a} = \frac{N_{y,a}\ F_{y,a}\ (1-e^{-Z_{y,a}})}{Z_{y,a}} \tag{A.2}$$

or, substituting Equation A.1 into Equation A.2:

$$C_{y,a} = \frac{N_{y+1,a+1}\ \ F_{y,a}\ \ (e^{Z_{y,a}}-1)}{Z_{y,a}} \tag{A.3}$$

where $N_{y,a}$ is the number of fish of age a at the start (taken as 1 January in this instance) of year y, estimated by the model,

$F_{y,a}$ is the instantaneous fishing mortality rate on fish of age a during year y, estimated by the model,

$M_{y,a}$ is the instantaneous natural mortality rate on fish of age a during year y, which is assumed time-invariant (i.e. independent of y) and is input, and

$C_{y,a}$ is the observed number of fish of age a caught during calendar year y, and is input.

Given initial guesses for the terminal population numbers ($N_{y_{\max}+1,a}$ - see Sections A.2.1 and A.2.2 below), the VPA involves calculating the remaining numbers-at-age ($N_{y,a}$) and fishing mortalities-at-age ($F_{y,a}$) by proceeding backwards up each cohort by solving Equations A.2 and A.3 successively. The plus-group population numbers are calculated as follows:

$$N_{y+1,m} = N_{y,m-1}\ e^{-Z_{y,m-1}} + N_{y,m}\ e^{-Z_{y,m}} \tag{A.4}$$

where m is the oldest age, taken to be a plus-group,

$N_{y,m-1}$ and $N_{y,m}$ are given by Equation A.1, and

$F_{y,m}$ is defined by Equation A.5 below.

A.2 Parameters

A.2.1 Estimating terminal population numbers

The population numbers ($N_{y_{\max}+1,a}$) in the year following the last year ($y = y_{\max}$) for which catch-at-age data are available are estimated in the model fitting procedure.

[Note: The 2000 assessments ($y_{\max}$ =1999) involved estimating population numbers at the start of 2000 for ages a=3, 5, 7 and 9 for the western North Atlantic.]

A.2.2 Selectivity of terminal year ($y = y_{\max}$)

Given values for the estimable parameters specified above ($N_{y_{\max}+1,a}$), fishing mortalities ($F_{y_{\max},a-1}$) can then be computed. For those ages a for which population numbers are not estimated, fishing mortalities cannot be calculated directly. For such ages, the F's need to be linked to those fishing mortalities that are directly estimated using a vector specifying which ages are grouped, as well as the relative selectivities within each group.

[Note: The 2000 assessments assumed for the western North Atlantic that ages 1-3, 4-5, 6-7 and 8-9 are grouped and that: $F_{1999,1} = 0.318\, F_{1999,2} = 0.318\, F_{1999,3}$, $F_{1999,4} = F_{1999,5}$, $F_{1999,6} = F_{1999,7}$, *and* $F_{1999,8} = F_{1999,9}$ *.]*

A.2.3 Estimating the F-ratio for the plus group

Fishing mortalities F for the oldest age m (taken to be a plus-group) and any year y are given by:

$$F_{y,m} = r_y\, F_{y,m-1} \tag{A.5}$$

where r_y is the F-ratio for year y (which can either be input or estimated for any year y, or estimated for blocks of years).

[Note: The 2000 base case assessments corresponded to m=10 and to the F-ratio being pre-specified ($r_y = 1.0$) for the period 1970-73, a single value estimated for the period 1974-81, and a different value estimated for the most recent period (1982-99), subject to a penalty term included in the likelihood function (see section A.6.3).]

A.2.4. Natural mortality rates

The natural mortality rate M_a input can either be constant or age-dependent.

[Note: M_a was assumed age-independent (=0.14yr^{-1}) for the 2000 assessments.]

A.3 Data

Aggregated, as well as fleet disaggregated, catches-at-age are input for a period (*1970-99 for the 2000 base case assessments*), in addition to mid-year masses-at-age for the same period.

Values of indices of abundance used to fit the model are input. They can either be indices in terms of population numbers or biomass, and relate to the beginning or middle of the year as well as to any pre-specified range of ages. The (sampling) standard errors (i.e. square roots of variances) associated with each value (σ_y^i) may also be input. When computing the expected value of each index ($\hat{I}_y^i$) from the model estimates of biomass-at-age or numbers-at-age, "selectivities" are specified to give a relative weight (W_a) to each age within the range specified. There are three possible choices:

- "uniform": equal weighting by age.
  [Note: For 2000 assessments, this applied to the JLL GOM, larval, and tagging indices.]
- "partial catches": first partial fishing mortalities are calculated for the associated fleet (*f*):

$$F^f_{y,a} = F_{y,a}\frac{C^f_{y,a}}{C_{y,a}} \tag{A.6}$$

where $C^f_{y,a}$ is the observed number of fish of age *a* caught by fleet *f* during calendar year *y*,

and then the weights for any year are given by:

$$W^f_a = \sum_y F^f_{y,a} \Big/ \max_a \left[\sum_{y'} F^f_{y',a}\right]$$

Thus, for example, if the index applied to mid-year biomass over the age range a_1 to a_2:

$$\hat{I}^f_y = \sum_{a=a_1}^{a_2} W^f_a\, w_{y,a+1/2}\, N_{y,a}\, e^{-Z_{y,a}/2}$$

where $w_{y,a+1/2}$ is the mid-year mass of a fish of age *a* during year *y*.

[Note: For 2000 assessments, this applied to the Can SWNS, JLL NWAtl, USLL GOM, and USRR (with size range) indices.]

- "pre-specified": weights for each year and the age range under consideration are input on the basis of consideration of the catch-at-age matrix ($C^f_{y,a}$) for the fleet concerned.

  [Note: For the 2000 assessments, this applied to the Can GSL index, and is intended to make allowance for this index applying to fish aged 13+ rather than 10+.]

A.4 Model fitting

A.4.1 Maximum likelihood for weighting indices

The abundance indices are assumed either to be normally or log-normally distributed about the model predictions:

$$I^i_y = \hat{q}^i\,\hat{I}^i_y + \varepsilon^i_y \qquad \ell\mathrm{n} I^i_y = \ell\mathrm{n}(\hat{q}^i\,\hat{I}^i_y) + \varepsilon^i_y \qquad \varepsilon^i_y \sim N(0;(\tilde{\sigma}^i_y)^2) \tag{A.7}$$

where I^i_y is the abundance index for year *y* and abundance series *i*,

$\hat{I}^i_y$ is the corresponding model estimate,

$\tilde{\sigma}^i_y$ is the residual standard deviation for year *y* and abundance index *i*, and

q^i is the catchability coefficient for abundance series *i*, estimated by maximum likelihood:

$$\hat{q}^i = \frac{\sum_y I^i_y\,\hat{I}^i_y\,(\tilde{\sigma}^i_y)^{-2}}{\sum_y \hat{I}^i_y\,\hat{I}^i_y\,(\tilde{\sigma}^i_y)^{-2}} \qquad \ell\mathrm{n}\,\hat{q}^i = \frac{\sum_y(\ell\mathrm{n}\,I^i_y - \ell\mathrm{n}\,\hat{I}^i_y)(\tilde{\sigma}^i_y)^{-2}}{\sum_y(\tilde{\sigma}^i_y)^{-2}} \tag{A.8}$$

[Note: The 2000 assessments assumed log-normal errors.]

Ignoring constants independent of the model parameters, the contribution by the abundance index data to the objective function is given by:

$$\sum_i\sum_y\left\{\ell\mathrm{n}\tilde{\sigma}^i_y + \tfrac{1}{2}(\varepsilon^i_y/\tilde{\sigma}^i_y)^2\right\} \tag{A.9}$$

The values for the residual standard deviations, $\tilde{\sigma}_y^i$, depend on whether individual data point weights are pre-specified and how "additional variance" is handled.

1. Data point weights pre-specified / no "additional variance". For this case, $\tilde{\sigma}_y^i$, is estimated externally to the resource assessment model (from, for example, random effects GLM-based models).
2. Data point weights pre-specified / series-specific "additional variance". For this case, $\tilde{\sigma}_y^i$, is defined as:

$$\tilde{\sigma}_y^i = \sqrt{(\tau^i)^2 + (\sigma_y^i)^2} \tag{A.10a}$$

where τ^i is the (estimated) extent of "additional variance" for abundance series *i*.

3. Data point weights pre-specified / series-independent "additional variance". For this case, $\tilde{\sigma}_y^i$, is defined as:

$$\tilde{\sigma}_y^i = \sqrt{\tau^2 + (\sigma_y^i)^2} \tag{A.10b}$$

where τ is the (estimated) extent of "additional variance", assumed common to all abundance series.

4. Data points equally weighted within each abundance series / series-specific "additional variance". For this case, τ^i only is estimated, and is effectively equivalent to $\tilde{\sigma}_y^i$.
5. Data points equally weighted within each abundance series / series-independent "additional variance". For this case, $\tilde{\sigma}_y^i$, equals τ. Only τ is estimated and effectively all $\tilde{\sigma}_y^i$'s are taken equal to τ.

The values for the parameters that determine the extent of additional variance (τ^i and τ) can be determined by treating them as estimable parameters in the non-linear optimization, or using the formulae:

$$\sum_y \frac{1}{(\tau^i)^2 + (\sigma_y^i)^2} = \sum_y \frac{(\varepsilon_y^i)^2}{((\tau^i)^2 + (\sigma_y^i)^2)^2} \tag{A.11a}$$

$$\sum_i \sum_y \frac{1}{\tau^2 + (\sigma_y^i)^2} = \sum_i \sum_y \frac{(\varepsilon_y^i)^2}{(\tau^2 + (\sigma_y^i)^2)^2} \tag{A.11b}$$

The maximum likelihood estimates for τ^i and τ have closed-form solutions if the indices are equally weighted (so that σ_y^i can be set to zero in Equation (A.11)).

[Note: The 2000 base case assessment was based on weighting option 5.]

A.5 Variance estimation

A.5.1 Bootstrapping of the index data

[Note: This was the option for the 2000 base case assessment, which assumed log-normally distributed errors.]

Multiple pseudo-data sets are generated from each abundance index by adding error using the assessment model-estimated standard deviation for each index:

$$I_y^{i,U} = \hat{I}_y^i + \varepsilon_y^{i,U}; \qquad I_y^{i,U} = \hat{I}_y^i e^{\varepsilon_y^{i,U}} \qquad \varepsilon_y^{i,U} \sim N(0, (\tilde{\sigma}_y^i)^2) \tag{A.12}$$

where $I_y^{i,U}$ is the abundance index for year *y* and abundance series *i* in bootstrap data set *U*.

Note that a parametric bootstrap procedure (assuming distribution normality or log-normality) is being used.

A.5.2 Bootstrapping of the F-ratios

For any year, or block of years, for which the *F*-ratio (*r*) is estimated in the fitting process, this is simply re-estimated given the corresponding pseudo data for each replicate. For years for which a fixed value is input for the fit itself, to allow for the *F*-ratios in any year *y* to fluctuate about the constant assumed for that year, bootstrap replicates are generated as follows:

$$F^U_{y,m} = r_y\, \hat{F}_{y,m-1}\, e^{\eta^U_y} \qquad \eta^{i,U}_y \sim N(0,\sigma^2_y) \tag{A.13}$$

where σ_y is the standard deviation of the variation about the *F*-ratio for year *y* which is taken to be:

$$\sigma_y = \begin{cases} 0.4 & \text{if } y < 1975 \\ 0.25 & \text{otherwise} \end{cases}$$

r_y is the *F*-ratio input by the user.

A.6 Additional options

[Note: The following options were not implemented for the 2000 base case assessment, except as specifically indicated hereafter.]

A.6.1 Beverton-Holt stock-recruit penalty function

The number of recruits during year *y* ($N_{y,a_{\min}}$) is assumed to be related to the spawner stock size by a Beverton-Holt stock-recruit relationship with auto-correlated stochastic deviations:

$$N_{y,a_{\min}} = \frac{\alpha\, B^{sp}_{y-a_{\min}}}{\beta + B^{sp}_{y-a_{\min}}} e^{\zeta_y} \qquad \zeta_y = \rho\zeta_{y-1} + \sqrt{1-\rho^2}\,\varepsilon_y \qquad \varepsilon_y \sim N(0,\sigma^2_R) \tag{A.14}$$

where ρ is the auto-correlation coefficient, which is input,

$a_{\min}$ is the smallest age of animals present in the catch,

B^{sp}_y is the spawner stock size in the middle of year *y* (as annual spawning peaks in about July):

$$B^{sp}_y = \sum_{a=a_{\min}}^{m} f_a\, w_{y,a+1/2}\, N_{y,a}\, e^{-Z_{y,a}/2} \tag{A.15}$$

α and β are the stock-recruit relationship parameters estimated in the fitting procedure, and

f_a is the proportion of fish of age *a* that are mature.

[Note: For the 2000 assessments $f_a = 0$ for $a < 8$ and $f_a = 1$ for $a \geq 8$, i.e. knife-edge maturity at age 8 was assumed and the smallest age was taken to be $a_{\min}$ =1.]

The contribution of the penalty function (if included) to the quantity minimized (the negative of the log-likelihood function) is therefore:

$$\sum_{y=y_1}^{y_2} \left\{ \ell\text{n}\sigma_R + \tfrac{1}{2}\Big[\frac{\zeta_y - \rho\zeta_{y-1}}{\sigma_R\sqrt{1-\rho^2}}\Big]^2 \right\} \qquad \zeta_y = \ell\text{n}\, N_{y,a_{\min}} - \ell\text{n}\Big[\frac{\alpha\, B^{sp}_{y-a_{\min}}}{\beta + B^{sp}_{y-a_{\min}}}\Big] \tag{A.16}$$

where y_1 and y_2 are the first and last years considered in the penalty function, which are input, and σ_R is the associated standard deviation, which is input.

In the interests of simplicity, expression A.16 omits a term involving ζ_{y_1}, which generally is of little quantitative consequence to values estimated.

[Note: For the 2000 "high recruitment" base case assessment option, this penalty function was not applied for the assessment itself. However, estimation of the parameters of the Beverton-Holt stock-recruitment function used for projections was effected by inclusion of this penalty, given a very low weight, for $y_1 = 1970$ to $y_2 = 1996$ with $\rho = 0.5$ and $\sigma_R = 0.4$. The value for α was constrained to be less than average recruitment over 1970-74.]

A.6.2 2-line stock-recruit penalty function

The number of recruits is assumed to be related to the spawner stock size by a simple 2-line stock-recruit relationship:

$$N_{y,a_{\min}} = \begin{cases} \alpha\,\varepsilon^{\zeta_y} & \text{if } B^{sp}_{y-a_{\min}} \geq B^{sp}_{crit} \\ \alpha\,(\dfrac{B^{sp}_{y-a_{\min}}}{B^{sp}_{crit}})\,\varepsilon^{\zeta_y} & \text{otherwise} \end{cases} \tag{A.17}$$

where α is the 2-line stock-recruitment parameter estimated in the fitting procedure, and

B^{sp}_{crit} is the critical spawner stock size above which recruitment is constant, given by:

$$B^{sp}_{crit} = \tfrac{1}{5}\sum_{y=1991}^{1995} B^{sp}_y$$

The contribution of the penalty function to the quantity minimised (the negative of the log-likelihood function) is given by Expression A.16 above, but with ζ_y defined now by Equation A.17.

[Note: For the 2000 "low recruitment" base case assessment option, this penalty function was not applied in the assessment itself. However, estimation of the parameters of the 2-line stock-recruitment function used for projections was effected by inclusion of this penalty, given a very low weight, for $y_1 = 1976$ to $y_2 = 1996$ with $\rho = 0$ and $\sigma_R = 0.4$.]

A.6.3 Penalty function for the "last-block" F-ratio

The "prior" distribution for the *F*-ratio (r_y) for the period 1982-99 is assumed to be log-normal and centred on an expected value. In likelihood maximisation terms, this corresponds to a penalty function added to the negative log-likelihood of the form:

$$-\ell\mathrm{n}\,L = \frac{(\ell\mathrm{n}\,\tilde{r}_y - \ell\mathrm{n}\,\hat{r}_y)^2}{2(\sigma_r)^2} \tag{A.18}$$

where $\tilde{r}_y$ is the expected *F*-ratio for the most recent period (taken to be the value assumed for the 1996 base case assessment, 1.14),

$\hat{r}_y$ is the corresponding model estimate, and

σ_r is the standard deviation of the "prior" distribution (assumed to be 0.25).

A.6.4 Penalty function for the F-ratios

It is possible to place a penalty on the inter-annual change in the *F*-ratio if *F*-ratios are estimated for each historical year:

$$-\ell nL = (y_{\max} - 1)\ell\mathrm{n}\sigma_F + \tfrac{1}{2\sigma_F^2}\sum_{y=1970}^{y_{\max}-1} \ell\mathrm{n}(r_y / r_{y+1})^2 \tag{A.19}$$

A.7 Diagnostic statistics

The value for the AIC statistic, corrected for the impact of small sample size (i.e. the AIC_c statistic – see Burnham and Anderson (1998) section 2.4.1) is provided for each fit to allow comparisons among models to be made. The value of AIC_c is computed using the formulae:

$$AIC_c = \begin{cases} -2\ell nL + 2p + \dfrac{2p(p+1)}{n-p-1} & \text{(A.20a)} \\ -2\ell nL + 2p + \sum\limits_i \dfrac{2p_i(p_i+1)}{n_i-p_i-1} & \text{(A.20b)} \end{cases}$$

where p is the number of parameters (the number of terminal *N*s, *F*-ratios, "additional variances", and catchability coefficients) that are estimated,

p_i is the number of parameters estimated from data-subset i (from each of which a variance is estimated) for which there are n_i data points (see discussion in Geromont and Butterworth, 2001, Appendix C), and

n is total number of data points.

[Note: The 2000 assessment session used Equation A.20a.]

The overall model deviance (as defined in ICCAT, 2001) is computed to provide a measure of goodness of fit. The value of this statistic can be compared with a χ^2-distribution with n-p degrees of freedom:

$$\text{Deviance} = \sum_i \sum_y \frac{(I_y^i - \hat{q}^i \hat{I}_y^i)^2}{(e^{(\hat{\sigma}_y^i)^2} - 1)(\hat{q}^i \hat{I}_y^i)^2} \qquad \text{(A.21)}$$

where n is the total number of terms in this double summation.

A.8 Technical specifications of projections

The specifications below are those adopted for the ADAPT projections conducted during the 2000 bluefin SCRS meeting in Madrid.

A.8.1 2-line stock-recruitment relationship [Note: 2000 "low recruitment" option.]

The future number of recruits is assumed to be related to the spawning stock biomass by a "2-line" stock-recruitment relationship.

$$N_{y,a_{\min}} = \begin{cases} \alpha\, \varepsilon^{\zeta_y} & \text{if } B^{sp}_{y-a_{\min}} \geq B^{sp}_{crit} \\ \alpha\, (\dfrac{B^{sp}_{y-a_{\min}}}{B^{sp}_{crit}})\, \varepsilon^{\zeta_y} & \text{otherwise} \end{cases} \qquad \zeta_y = \rho \zeta_{y-1} + \sqrt{1-\rho^2}\, \varepsilon_y \qquad \text{(A.22)}$$

where $\varepsilon_y \sim N(0, \sigma_R^2)$.

[Note also that for the 2000 (as in 1998) assessments, zero auto-correlation (ρ=0) was assumed and the value of σ_R was fixed to 0.4; $a_{\min}$ was set to 1.]

A.8.2 Beverton-Holt stock recruitment relationship [2000 "high recruitment" option.]

The future number of recruits is assumed to be related to the spawning stock biomass by a Beverton-Holt stock-recruitment relationship with auto-correlated stochastic deviations:

$$N_{y,a_{\min}} = \frac{\alpha \; B^{sp}_{y-a_{\min}}}{\beta + B^{sp}_{y-a_{\min}}} \varepsilon^{\zeta_y} \qquad\qquad \zeta_y = \rho\zeta_{y-1} + \sqrt{1-\rho^2}\,\varepsilon_y \qquad (A.23)$$

where $\varepsilon_y \sim N(0,\sigma_R^2)$.

[Note that for the 2000 (as in 1998) assessment, the values of ρ and σ_R were fixed to 0.5 and 0.4 respectively, and $a_{\min}$ to 1 .]

A.8.3 Past recruitments

The VPA is unable to estimate $N_{y_{\max}-2,a_{\min}}$, $N_{y_{\max}-1,a_{\min}}$, or $N_{y_{\max},a_{\min}}$ with reliability. Thus, projections are based on values for recruitment in these years given by Equations A.22 (for the "low recruitment" scenario), or A.23 (for the "high recruitment" scenario) with $\zeta_{y_{\max}-3}$ as estimated in the fitting procedure. Known catches-at-age from the associated year-classes for these years are taken into account in projecting forward to give numbers at ages 2, 3 and 4 at the start of year $y_{\max}+1$.

The random components of Equations A.22 and A.23 can sometimes lead to situations in which the recruitment generated is insufficient to allow the catches already made from one of these year-classes to be realized. In such cases, the recruitment in question is regenerated from the distribution specified in Equations A.22 or A.23.

A.8.4 Selectivity-at-age for future catches

The geometric mean over the years $y_{\max}-4$ to $y_{\max}-2$ is taken of the fishing mortality at age for each age. The values obtained are scaled by dividing by their maximum over all the ages to provide selectivities-at-age, S_a, for future catches.

A.8.5 Weight-at-age in the future

Fish of ages 1 to *m*-1 are assumed to have the same average weight for all future years as estimated for year $y = y_{\max}$. The average weight of age *m*+ fish in the future varies because of the change in the age composition of the older fish over time. This average is calculated from the average age of animals aged *m* and older by means of the equations:

$$w(t) = 1.520\times10^{-5}[l(t)]^{3.05305} \qquad (A.26a)$$

$$l(t^*) = 382.0\;\;[1-e^{-0.079(t^*+0.707)}] \qquad (A.26b)$$

$$l(t) = 382.0\;\;[1-e^{-0.079(t+0.374)}] \qquad (A.26c)$$

where age t^* measures time from 1 May (the original basis for the estimation of the growth curve parameters) and $t = t^* - 0.333$ measures time from the "start of the year" of 1 January used for these assessments. Weight w is in kg, and length l in cm.

If $\bar{a}_y$ is the average age at the start of the year of *m*+ fish in year *y*, and $N_{y,a}$ the number of fish at that time of age *a*, then:

$$\bar{a}_{y+1} = \frac{(\bar{a}_y+1)\,N_{y,m}\,e^{-Z_{y,m}} + m\,N_{y,m-1}\,e^{-Z_{y,m-1}}}{N_{y,m}\,e^{-Z_{y,m}} + N_{y,m-1}\,e^{-Z_{y,m-1}}} \qquad (A.27)$$

The value of $\bar{a}$ for year $y = y_{\max}$ is calculated from the growth curve (Equation A.26) using the equation:

$$\overline{a}_{y_{\max}} = -0.374 - \tfrac{1}{0.079}\ell\text{n}[1-(\tfrac{w_{y_{\max},m}}{1.520x10^{-5}})^{1/3.05305}/382.0] - 0.5 \tag{A.28}$$

Equation A.27 is applied recursively to calculate the mean age of animals aged m and older for all future years. Equation A.26c with $t = \overline{a}_y + 1/2$ used to evaluate the corresponding average weight (this approach assumes approximate linearity of weight with age for ages of m and above). The reason for adding 1/2 is that age t in Equation A.26c is measured from 1 January, so that the middle of the calendar year for which catches are reported corresponds roughly to age $t+1/2$. These calculations assume that there is uniform selectivity on fish of age m and above, so that the average weight mass of m+ animals caught is the same as that in the population.

[Note: For 2000, as in 1998, the plus-group masses were computed using Equation A.27 with a value of –0.707 rather than –0.374 for the t_0 *parameter of the von Bertalanffy growth curve (in terms of t) of Equation A.26. However, this incorrect value has hardly any quantitative impact on key projection results.]*

A.8.6 Future catches

These are specified on input and taken exactly, except that if fishing mortality on the fully-selected age-class would exceed 2 to achieve this, such F is set to 2 and the corresponding lesser catch assumed to be taken. [In 1996, this limit was set at 1.4.]

A.8.7 Deterministic projections

These are based on the point estimates of numbers-at-age at the start of year $y = y_{\max}+1$ from the assessment, together with the specifications above except that recruitments from year $y = y_{\max}-2$ onwards are given by Equation A.22 or A.23 with $\varepsilon_y = 0$ (i.e., no variation about the assumed stock-recruitment relationship).

A.8.8 Stochastic projections

Realizations of distributions of quantities of interest are provided by a large number of bootstrap replicates of the process described above. First, the point estimates of numbers-at-age at the start of year $y = y_{\max}+1$ are replaced by their bootstrap replicates evaluated in terms of the prescription set out in Section A.5 above. The parameters σ_R, α and β or B^{sp}_{crit} (depending on the stock-recruitment relationship assumed) required to calculate the time series of recruitments from year $y = y_{\max}-2$ onwards by application of Equation A.22 or A.23 are then calculated from the past numbers-at-age matrix for that bootstrap replicate VPA fit. The value for σ_R is set to 0.4 if the bootstrap estimate is less than 0.4. Note that the median abundance trend for the stochastic projections will lie above the corresponding trajectory for deterministic projections. The reason is that the stochastic median essentially reflects average recruitment, which is a factor $e^{\sigma_R^2/2}$ greater for the stochastic than the deterministic projections as a result of the mean-median difference for the log-normal distribution assumed for recruitment variability. Selectivity-at-age and m+ weight projections are re-evaluated similarly for each bootstrap replicate.

A.8.9 Estimation of MSY, B^{sp}_{MSY}*, and* K^{sp}.

The Maximum Sustainable Yield, *MSY*, is calculated by finding the value of F such that:

$$\frac{dC(F)}{dF} = 0 \tag{A.29}$$

where $C(F)$ is the equilibrium catch when the fully-selected fishing mortality is fixed equal to F:

$$C(F) = \tilde{C}(F).R(F) \tag{A.30}$$

$\tilde{C}(F)$ is the yield-per-recruit when the fully-selected fishing mortality is fixed equal to F:

$$\tilde{C}(F)=\sum_a w_{a+1/2}(F)\frac{S_a F}{M_a+S_a F}\tilde{N}_a(F)(1-e^{-(M_a+S_a F)}) \qquad \text{(A.31)}$$

$\tilde{R}(F)$ is the expected recruitment when the fully-selected fishing mortality is fixed equal to F:

$$\tilde{R}(F)=\begin{cases}\alpha & \text{2-line}\\ \alpha-\beta/\tilde{S}(F) & \text{Beverton-Holt}\end{cases} \qquad \text{(A.32)}$$

$\tilde{S}(F)$ is the spawner biomass-per-recruit when the fully-selected fishing mortality is fixed equal to F:

$$\tilde{S}(F)=\sum_a f_a\, w_{a+1/2}(F)\,\tilde{N}_a(F)e^{-(M_a+S_a F)/2} \qquad \text{(A.33)}$$

$\tilde{N}_a(F)$ is the number of animals of age a when the fully-selected fishing mortality is fixed equal to F and the number of $a_{\min}=1$-year-olds equals 1:

$$\tilde{N}_a(F)=\begin{cases}1 & \text{if } a=1\\ \tilde{N}_{a-1}(F)e^{-(M_a+S_{a-1}F)} & \text{if } 2\le a<m\\ \tilde{N}_{m-1}(F)e^{-(M_{m-1}+S_{m-1}F)}/(1-e^{-(M_m+S_m F)}) & \text{if } a=m\end{cases} \qquad \text{(A.34)}$$

$w_{a+1/2}(F)$ is the mass-at-age for an animal of age a in the middle of the year when the fully-selected fishing mortality is fixed equal to F. The masses-at-age for ages $a < m$ are set equal to the masses-at-age for these ages for the most-recent year, $y_{\max}$, while the mass-at-age of the plus-group is calculated using Equations (A.26a) and (A.26c) where the average age of the plus-group at the start of the year is taken to be $m+1/(M_m+S_m F)$.

The spawner biomass at which *MSY* is achieved, B^{sp}_{MSY}, is calculated as $\tilde{S}(F_{MSY}).R(F_{MSY})$ while the pre-exploitation equilibrium biomass, K^{sp}, is calculated as $\tilde{S}(0).R(0)$. When calculating K^{sp}, the masses-at-age for ages $a < m$ are based on Equations (A.26a) and (A.26c) rather than being assumed equal to the masses-at-age for the most-recent-year, $y_{\max}$.