

# Further Updates of an Assessment of the Namibian Hake Resource, Including Both Catch-at-Age and Catch-at-Length Information

R.A. Rademeyer and D.S. Butterworth

MARAM (Marine Resource Assessment and Management Group)  
 Department of Mathematics and Applied Mathematics  
 University of Cape Town, Rondebosch 7701, South Africa

November 2004

## 1. Introduction

An earlier version of this paper was presented in February as an assessment of the Namibian hake resource based on both catch-at-age and catch-at-length information. Age-length keys for the Namibian hake are available for 1993, 1999 and 2000 and for those years, the catch-at-age data are used as input. For the years where no age-length keys are available, catch-at-length data have been used to fit the model. This follows the recommendation made by the January 2004 BENEFIT-NRF-BCLME Stock Assessment Workshop.

More recent additions to the paper include examination of the effects of taking account of the January 2004 survey result, and of CPUE data from seven vessels that fished both before and after Namibian independence in 1990.

## 2. Data

Inputs to the ASPM assessment model include catch, abundance indices, catch-at-age and catch-at-length data. Annual total catch ( $C_y$ ) are listed in Table 1, together with the abundance indices used ( $I_y^i$ ), all of which are treated as relative indices in the Reference Case assessment. Further information on the sources of this information is provided in the footnotes to this Table. Since the earlier version of this paper, the 2004 survey result together with “seven-vessel” CPUE series (Bergh and Barkai, 1995) have been added to this Table.

Commercial catches-at-age ( $C_{y,a}$ ) for the pre-independence period and after that only for years for which age-length keys are available are shown in Table 2. Catches-at-age for the *Nansen* surveys ( $C_{y,a}^{surv}$ ) – including the surveys conducted by commercial vessels since 2000 – are shown in Tables 3 and 4.

Commercial length-at-age proportions, obtained from the Namibian observer program since 1997, are shown in Table 5. Tables 6 and 7 shows the length-at-age proportions from the summer and winter surveys respectively. These proportions are for *M. capensis* and *M. paradoxus* combined (obtained by adding the numbers estimated in each length class).

## 3. Methods

The model used in this analysis is an ASPM, which is fitted to abundance indices (CPUE and survey biomass estimates series) as well as to catch-at-age and catch-at-length information. The assessment methodology used is given in Appendix 1.

Model specifications for the Reference Case assessment are listed below:

- 1) Natural mortality  $M$  is taken to be independent of age and is estimated in the model fitting procedure.
- 2) Commercial selectivity-at-age takes the form of a logistic curve (equation A.34) with the slope parameter  $s$  (equation A.35) estimated in the model fitting procedure. The selectivity is taken to differ pre- and post-independence (after independence, vessels were excluded from waters shallower than 200 m so that the selectivity became lower for the younger age classes). The selectivity function is taken to be independent of time during the

post-independence period (1990 to 2003, denoted by  $S_{post,a}$ ), while for the pre-independence period, the selectivity is modelled by a form  $S_{pre1,a}$  which is constant over the 1964 to 1973 period, after which it changes in a linear fashion to reach a form ( $S_{pre2,a}$ ) in 1984, and this remains constant from then until the end of this earlier period in 1989. The selectivity slope parameter  $s$  is estimated separately for each of the three periods; the selectivity is taken to decrease with age for fish older than 5.

- 3) Survey selectivity-at-age also takes the form of a logistic curve (equation A.34) with the slope parameter  $s$  (equation A.35) estimated in the model fitting procedure. The selectivity is taken to decrease with age for fish older than 3. It is kept constant over the whole period of the assessment and is assumed to differ for summer and winter surveys.
- 4) Stock-recruitment residuals are estimated from 1964 to 2002, with  $\sigma_R = 0.25$ .
- 5) The length-at-age is estimated by the Von Bertalanffy growth equation, with the Von Bertalanffy parameter values used shown in Table 8.

Several sensitivities on the Reference Case assessment have been conducted, they are listed below:

- 1a. "Reference Case"
- 1b. "Von Bert fixed,  $q=1.0$ ": same as the Reference Case assessment but the multiplicative bias for the *Nansen* summer and winter surveys ( $q_{Nansen}$ , pre-2001) is fixed at 1.0.
- 1c. "Von Bert fixed,  $q=0.7$ ": as above but with  $q_{Nansen}$  fixed at 0.7.
- 1d. "Von Bert fixed,  $q=0.4$ ": as above but with  $q_{Nansen}$  fixed at 0.4.
- 1e. "Von Bert fixed,  $h=0.5$ ": same as the Reference Case assessment ( $q_{Nansen}$  is estimated) but the steepness parameter  $h$  is fixed to 0.5 instead of estimated in the model fitting procedure. This sensitivity run struggles to converge so that the Hessian-based CV's are not available for this case.
- 2a. "Growth curve estimated,  $q$  estimated": same as the Reference Case assessment but the length-at-age is modelled by a straight line (instead of the Von Bertalanffy equation), the parameters of which are estimated directly in the model fitting procedure. As in the Reference Case, all the survey multiplicative biases are estimated in the model.
- 2b. "Growth curve estimated,  $q=0.4$ ": as for 2a above but with  $q_{Nansen}$  fixed at 0.4.
3. "Change in  $K$  (50% decrease over 1970-75)": same as the Reference Case but the carrying capacity ( $K$ ) of the resource is assumed to have decreased by 50% over the 1970-75 period.
4. "GLM CPUE series not included": same as the Reference Case but the GLM standardized CPUE series is not included when calculating the likelihood.
- 5a. "Including extra CPUE series (all years)": same as the Reference Case but including the seven-vessel CPUE series (Bergh and Barkai, 1995) from 1972-1994.
- 5b. "Including extra CPUE series from 1985 onwards": as for 5a above but with the data for the seven-vessel CPUE series from 1985 only (there may be comparability problems in this series as the data are unstandardised, so that considering a shorter period may be more reliable).
- 5c. "Including extra CPUE series from 1985 onwards, with  $q^{extra} = p^*q^{GLM}$ ": as for 5b above but  $q^{extra} = p^*q^{GLM}$  where
 
$$p^* = \frac{\sum_{y=1992}^{1994} CPUE_y^{extra} / 3}{\sum_{y=1992}^{1994} CPUE_y^{GLM} / 3}$$
 , i.e. these two CPUE series are scaled directly using values for the years of overlap, rather than estimating their  $q$ 's within the model fit.
6. "Including 2004 survey biomass estimate": same as the Reference Case but including the 2004 survey biomass abundance estimate which has more recently become available.

#### 4. Results and Discussion

Results of the Reference Case assessment (1a) and the various sensitivity tests are shown in Table 9. Graphical output of the results obtained for the Reference Case and its fits to the available data are shown in Figs 1-12. The Figures also include results for some of the sensitivity tests: Figs 9 and 11 for case (2a) where the somatic growth curve is estimated in the model fit, Figs 10, 11 and 12 for case (4) when the GLM standardised CPUE series is omitted from the fitting criterion, Fig 13 for the cases which include the seven-vessel CPUE series (cases 5a, b and c) and Fig. 14 for case (6), which includes the 2004 survey biomass estimate.

The Reference Case assessment reflects a heavily reduced resource which has declined over the past decade (Fig. 1). There is no major indication of lack of fit to the data (Figs. 3-7), though there is correlation evident in the catch-at-age and catch-at-length residuals (Figs 5 and 7). Surprisingly, the estimated multiplicative bias ( $q$ ) for the *Nansen* summer survey in this fit (1.38) exceeds 1 (suggesting that this survey overestimates abundance in absolute terms). However if  $q$  is forced to lower values (sensitivity tests (1b) to (1d)), although the resource status ( $B_{2003}^{sp}/K$ ) is slightly improved, the estimated steepness  $h$  drops below its already (unrealistically?) low value of 0.3 for the Reference Case, so that estimates of resource productivity (e.g. MSY) drop lower.

Estimating the somatic growth curve parameters in the fit (tests (2a) and (2b)), or allowing for a 50% decrease in carrying capacity over the period the pilchard resource collapsed in the early 1970s, makes little qualitative difference to results. In contrast, forcing steepness  $h$  to be higher (test (1e)) gives a much more optimistic appraisal (abundance above MSY level); this improves the fit to many of the data series, but that is more than counterbalanced by an appreciable deterioration in the fit to the GLM standardised CPUE series over 1992-2003. If this series is omitted from the fitting criteria (test (4)), a more positive picture emerges with abundance increasing slightly since Namibian independence in 1990 rather than declining (Fig. 11). Given this difference, it is not surprising that constant catch projections show very different biomass trends for the Reference Case (1a) and test (4) in Fig. 12. The former suggests that only catches in the 50 thousand ton region would be sustainable in the medium term, whereas the latter would suggest this figure to exceed 150 000 tons.

To examine this difference further in a simpler and less model-dependent way, the average annual surplus production ( $\bar{P}$ ) evidenced by the resource since 1990 was estimated separately using the recent survey and the recent GLM CPUE data. From the simple biomass equation:

$$B_{t+1} = B_t + P_t + C_t \quad (1)$$

it follows that on averaging over time:

$$\begin{aligned} \bar{P} &= \bar{C} + \bar{B} \left[ \frac{B_{t+1} - B_t}{B_t} \right] \\ &= \bar{C} + \bar{B}r \end{aligned} \quad (2)$$

where  $r$  is the proportional annual change in abundance. This can be estimated from the GLM standardised CPUE or the *Nansen* survey data which yield (see Fig. 13 for the corresponding plots):

$$\begin{aligned} \hat{r}_{CPUE} &= -0.101 \quad (\text{s.e. } 0.018) \\ \hat{r}_{survey} &= +0.017 \quad (\text{s.e. } 0.022) \end{aligned} \quad (3)$$

Given that  $\bar{C} = 130$  thousand tons, and taking the *Nansen* summer survey estimates to provide an estimate of  $\bar{B}$  (after adjusting for trawler/*Nansen* bias) yields estimates of average productivity (with 95% confidence intervals based on the  $r$  standard errors shown in parenthesis) of:

$$\begin{aligned} \hat{P}_{CPUE} &= 46 \text{ thousand tons} & [16; 77] \\ \hat{P}_{survey} &= 143 \text{ thousand tons} & [106; 181] \end{aligned} \quad (4)$$

Arguments could be offered both to use higher or lower values for  $\bar{B}$ , which would exacerbate or diminish the differences in these two estimates. However the greater concern is that the two estimates of  $r$  in equation (3) are statistically significantly different at the 5% level. Though they are measuring slightly different components of the overall hake biomass, this difference in  $r$  remains a concern, and suggests that at least one of the two is not providing a reliable signal of medium term resource trend. As such, basing TAC recommendations on analyses combining both

these sets of data is open to statistical question, and debate as to which might constitute the more reliable index of hake abundance and its trend is warranted.

Including the seven-vessel CPUE series developed by Bergh and Barkai (1995) in the model fit (tests (5a, b and c)) gives consistently similar much more optimistic appraisals (abundance well above MSY level) (Fig. 14). Fits to the seven-vessel and GLM CPUE series for test 5a are shown in Fig. 15. This more optimistic appraisal results from the fact that the steepness parameter  $h$  is estimated to be above 0.5 in all three cases, compared to less than 0.3 in the Reference Case. While this larger estimate for  $h$  seems not unrealistic, the value of  $q$  for the *Nansen* summer surveys suggest that these measure only some 40% of abundance, an inference which might be subject to question. It is important to note that these series were based on industry records which have yet to be checked, and the series standardised.

Including the 2004 survey biomass estimates makes little qualitative difference to the Reference Case results (Fig. 16).

## References

- Baranov F.T. 1918. On the question of the dynamics of the fishing industry. *Nauch. Issled. Ikhtiol. Inst. Izv.* **1**: 81-128 (In Russian).
- Bergh M. and Barkai A. 1995. Further numerical analyses to establish a TAC for the Namibian hake resource for 1996. Unpublished report. 18pp.
- Beverton R.J.H. and Holt S.J. 1957. *On the dynamics of exploited fish populations*. Fisheries Investment Series 2, Vol. 19, U.K. Ministry of Agriculture and Fisheries, London.
- Brandão A., Butterworth D.S., Watkins B.P. and Staverees L. 2002. An updated assessment of the toothfish (*Dissostichus eleginoides*) resource in the Prince Edward Island vicinity and extensions taking commercial catch-at-length data into account. Unpublished report, CCAMLR. WG/FSA/02-76. 23pp.
- Brandão A. and Butterworth D.S. 2004. Updated GLM analyses applied to the commercial CPUE data for Namibian hake for the period 1992 to 2003. HWG/WkShop/2004/04/Doc. 2.
- Butterworth D.S., Punt A.E., Plaganyi E.E. and Rademeyer R.A. 2001. An updated assessment and projections for the Namibian hake resource. Unpublished report, NatMIRC, Namibia. HWG/WKShop/2001/04/Doc.1. 16pp.
- Geromont H.F. and Butterworth D.S. 1998. Initial evaluation of a range of possible management procedures for west coast hake. Unpublished report, MCM, South Africa. WG/02/98/D:H:18. 26pp.
- Pope J.G. 1972. An investigation of the accuracy of virtual population analysis using cohort analysis. *Res. Bull. Int. Commn. Northwest Atlant. Fish.* **9**: 65-74.
- Rademeyer R.A. and Butterworth D.S. 2003. An updated assessment for the Namibian Hake Resource. Unpublished report, NatMIRC, Namibia. HWG/WKShop/2003/03/Doc.2. 28pp.
- Rice J.A. 1995. *Mathematical statistics and data analysis* (2<sup>nd</sup> ed.). Wadsworth Publishing, Belmont, California.

**Table 1:** Total annual landings, CPUE and survey abundance data for Namibian hake (ICSEAF Divisions 1.3, 1.4 and 1.5) for the period 1964 to 2003. Catches and survey biomass estimates are in thousand tons.

Year	Catches	CPUE ICSEAF 1.3 + 1.4 (tons/h)	CPUE ICSEAF 1.5 (tons/h)	CPUE seven- vessel	CPUE GLM (kg/h)	Spanish winter survey	Spanish summer survey	<i>Nansen</i> summer survey biomass (CV)	<i>Nansen</i> winter survey biomass (CV)
1964	47.852								
1965	193.200	1.78	2.24						
1966	334.627	1.31	2.62						
1967	394.445	0.91	1.47						
1968	630.392	0.96	1.38						
1969	526.657	0.88	1.15						
1970	627.198	0.90	1.10						
1971	595.215	0.87	1.44						
1972	820.110	0.72	1.00	10.50					
1973	667.965	0.57	1.00	10.75					
1974	514.558	0.45	0.70	8.50					
1975	488.208	0.42	0.82	8.75					
1976	601.045	0.42	0.58	8.25					
1977	431.483	0.49	0.69	6.25					
1978	379.390	0.43	0.56	6.50					
1979	310.175	0.40	0.74	7.75					
1980	171.848	0.45	0.71	7.50					
1981	211.534	<i>0.55</i>	<i>0.85</i>	8.00					
1982	307.078	<i>0.53</i>	<i>0.84</i>	7.50					
1983	339.590	<i>0.58</i>	<i>0.90</i>	7.25		708.50			
1984	364.993	<i>0.64</i>	<i>0.93</i>	8.75		2128.26	2187.60		
1985	386.184	<i>0.66</i>	<i>1.03</i>	10.50		1215.84			
1986	381.189	<i>0.65</i>	<i>0.93</i>	9.50		938.29	1018.61		
1987	300.249	<i>0.61</i>	<i>0.88</i>	8.75		721.02			
1988	336.000	<i>0.63</i>	<i>0.84</i>	8.50		562.59	532.55		
1989	309.329			7.50		485.68	1737.84		
1990	132.379			6.00			1957.13	586.771 (0.154)	725.893 (0.119)
1991	56.135			9.00				545.824 (0.212)	
1992	87.497			17.00	<b>1.669</b>			817.302 (0.110)	1005.620 (0.093)
1993	108.000			16.75	<b>1.820</b>			942.584 (0.128)	798.308 (0.112)
1994	112.206			11.75	<b>1.377</b>			750.374 (0.119)	964.510 (0.090)
1995	130.362				<b>0.891</b>			584.928 (0.121)	647.135 (0.104)
1996	129.102				<b>0.777</b>			819.415 (0.139)	729.610 (0.112)
1997	116.593				<b>0.848</b>			663.349 (0.123)	
1998	150.825				<b>1.081</b>			1572.857 (0.145)	
1999	160.690				<b>1.108</b>			1071.529 (0.129)	
2000	162.821				<b>0.767</b>			<i>1357.193 (0.195)</i>	
2001	157.000				<b>0.593</b>			586.726 (0.233)	
2002	165.000				<b>0.476</b>			725.000 (0.286)	
2003	176.000				<b>0.594</b>			776.000 (0.248)	
2004								<b><i>1157.394 (0.291)</i></b>	

## Note:

1. Here and in subsequent Tables, data that are newly added to or changed from those used in the Rademeyer and Butterworth (2003) assessment are shown in bold.
2. The ICSEAF CPUE values for 1981-1988 are shown in italics as they are not used in the Reference Case assessment.
3. The 1992-2002 GLM CPUE is as revised by Brandão and Butterworth (2004).
4. Values for the “seven-vessel” CPUE series were read off the plot in Fig. 5 of Bergh and Barkai (1995).
5. The 2000-2003 “*Nansen*” summer surveys were conducted by commercial trawlers so that their results are in italic and/or bold in the column concerned. An additional variance (CV=0.153) has been added to the survey sampling variances to make allowance for uncertainty in the relative catchability of the trawlers compared to the *Nansen* (see text for more details).

**Table 2:** Commercial catches-at-age (shown as proportions) for the Namibian hake fishery for years for which age-length keys are available. Catches for the period 1968 to 1988 are from ICSEAF records for Divisions 1.3, 1.4 and 1.5, while those from 1999 to 2000 are from NatMIRC catch-at-length data for the fishery off Namibia.

Age	Proportions caught at age: <i>Merluccius capensis</i> and <i>M. paradoxus</i> combined								
	0	1	2	3	4	5	6	7	8+
1968	0.000	0.002	0.094	0.548	0.244	0.081	0.024	0.005	0.003
1969	0.000	0.006	0.126	0.368	0.346	0.098	0.034	0.015	0.007
1970	0.000	0.000	0.155	0.402	0.269	0.127	0.031	0.011	0.004
1971	0.000	0.001	0.067	0.302	0.429	0.130	0.043	0.019	0.008
1972	0.000	0.004	0.101	0.468	0.282	0.095	0.034	0.014	0.003
1973	0.000	0.022	0.099	0.465	0.324	0.055	0.020	0.008	0.007
1974	0.000	0.068	0.278	0.278	0.147	0.127	0.073	0.024	0.005
1975	0.000	0.030	0.155	0.435	0.197	0.108	0.046	0.020	0.009
1976	0.000	0.054	0.280	0.416	0.192	0.043	0.011	0.003	0.001
1977	0.000	0.112	0.120	0.379	0.279	0.086	0.012	0.008	0.005
1978	0.000	0.059	0.399	0.341	0.112	0.055	0.023	0.008	0.002
1979	0.000	0.032	0.243	0.330	0.200	0.120	0.046	0.020	0.008
1980	0.000	0.143	0.157	0.267	0.217	0.112	0.065	0.025	0.013
1981	0.000	0.096	0.249	0.259	0.190	0.117	0.061	0.019	0.008
1982	0.000	0.148	0.354	0.236	0.127	0.061	0.041	0.022	0.010
1983	0.000	0.473	0.397	0.083	0.030	0.009	0.005	0.002	0.001
1984	0.000	0.058	0.532	0.294	0.077	0.025	0.009	0.003	0.001
1985	0.000	0.098	0.245	0.391	0.198	0.051	0.012	0.003	0.001
1986	0.000	0.048	0.391	0.251	0.169	0.094	0.032	0.013	0.003
1987	0.000	0.035	0.233	0.389	0.214	0.085	0.033	0.009	0.002
1988	0.000	0.023	0.268	0.451	0.202	0.041	0.011	0.003	0.001
1999	0.003	0.007	0.099	0.148	0.158	0.265	0.168	0.118	0.034
2000	0.000	0.002	0.108	0.204	0.447	0.188	0.036	0.009	0.006

**Table 3:** Summer survey catches-at-age (shown as proportions) for years for which age-length keys are available.

Age	Proportions caught at age: <i>Merluccius capensis</i> and <i>M. paradoxus</i> combined								
	0	1	2	3	4	5	6	7	8+
1993	0.000	0.049	0.564	0.268	0.058	0.036	0.018	0.006	0.001
1999	0.259	0.111	0.082	0.370	0.121	0.041	0.013	0.002	0.000
2000	0.006	0.082	0.758	0.108	0.030	0.012	0.003	0.000	0.000

**Table 4:** Winter survey catches-at-age (shown as proportions) for years for which age-length keys are available.

Age	Proportions caught at age: <i>Merluccius capensis</i> and <i>M. paradoxus</i> combined								
	0	1	2	3	4	5	6	7	8+
1993	0.000	0.019	0.475	0.364	0.071	0.040	0.021	0.009	0.001

**Table 5:** Commercial length-at-age proportions (shown as percentages) for *M. capensis* and *M. paradoxus* combined off Namibia, for years for which no age-length key is available. Here and in the next two Tables, “Length (in cm)” indicates the minimum of a 1 cm interval.

Length	1997	1998	2001	Length	1997	1998	2001
1	0.000	0.000	0.000	51	0.500	0.645	0.476
2	0.000	0.000	0.000	52	0.426	0.557	0.411
3	0.000	0.001	0.000	53	0.293	0.473	0.307
4	0.000	0.000	0.000	54	0.271	0.326	0.255
5	0.000	0.000	0.001	55	0.241	0.341	0.153
6	0.000	0.001	0.000	56	0.251	0.279	0.127
7	0.001	0.001	0.005	57	0.221	0.229	0.139
8	0.006	0.001	0.034	58	0.181	0.221	0.111
9	0.005	0.007	0.036	59	0.155	0.148	0.100
10	0.018	0.016	0.085	60	0.125	0.145	0.086
11	0.032	0.049	0.210	61	0.130	0.176	0.135
12	0.077	0.086	0.432	62	0.085	0.176	0.092
13	0.044	0.095	0.485	63	0.085	0.105	0.080
14	0.075	0.167	0.595	64	0.041	0.083	0.052
15	0.109	0.258	0.810	65	0.052	0.094	0.046
16	0.130	0.380	1.056	66	0.033	0.081	0.045
17	0.235	0.402	1.372	67	0.035	0.072	0.018
18	0.348	0.529	1.546	68	0.033	0.047	0.020
19	0.618	0.581	1.911	69	0.025	0.044	0.013
20	0.817	0.676	2.182	70	0.014	0.063	0.007
21	1.341	0.872	2.619	71	0.016	0.034	0.012
22	1.953	0.989	3.050	72	0.022	0.029	0.014
23	2.472	1.209	3.192	73	0.016	0.032	0.006
24	3.099	1.404	3.565	74	0.015	0.019	0.010
25	3.904	1.849	3.598	75	0.008	0.010	0.003
26	4.561	2.200	4.185	76	0.007	0.008	0.003
27	4.919	2.808	4.135	77	0.006	0.006	0.002
28	5.474	3.093	4.602	78	0.002	0.004	0.003
29	5.516	3.714	4.582	79	0.003	0.008	0.009
30	5.638	4.678	4.805	80	0.001	0.005	0.001
31	5.117	5.333	4.887	81	0.001	0.002	0.002
32	4.960	5.912	4.632	82	0.001	0.001	0.000
33	4.854	6.026	4.439	83	0.001	0.001	0.000
34	4.507	5.368	3.919	84	0.001	0.000	0.000
35	4.444	6.095	3.923	85	0.000	0.002	0.001
36	4.159	5.783	3.570	86	0.000	0.001	0.001
37	3.833	5.132	3.233	87	0.000	0.001	0.004
38	3.429	4.478	3.200	88	0.000	0.001	0.001
39	3.257	4.231	2.884	89	0.000	0.000	0.000
40	2.721	3.651	2.734	90	0.000	0.000	0.000
41	2.403	3.014	1.914	91	0.000	0.000	0.000
42	2.146	2.667	1.565	92	0.000	0.000	0.000
43	1.925	2.439	1.540	93	0.000	0.000	0.000
44	1.574	1.782	1.245	94	0.000	0.000	0.000
45	1.529	1.752	1.060	95	0.000	0.000	0.000
46	1.328	1.575	0.887	96	0.000	0.000	0.000
47	1.036	1.428	0.862	97	0.000	0.000	0.000
48	0.848	1.210	0.712	98	0.000	0.000	0.000
49	0.665	0.891	0.504	99	0.000	0.000	0.000
50	0.575	0.697	0.451	100	0.000	0.000	0.000

**Table 6:** Summer survey length-at-age proportions (shown as percentages) for *M. capensis* and *M. paradoxus* combined off Namibia, for years for which no age-length key is available.

Length	1990	1991	1992	1994	1995	1996	1997	1998	2001	2002
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.003	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.003
7	0.039	0.000	0.000	0.000	0.000	0.022	0.000	0.002	0.000	0.093
8	0.078	0.000	0.000	0.000	0.000	0.105	0.000	0.014	0.000	0.203
9	0.206	0.000	0.000	0.000	0.000	0.331	0.000	0.235	0.000	1.241
10	0.596	0.000	0.046	0.000	0.000	0.475	0.000	0.475	0.001	2.999
11	1.188	0.000	0.039	0.001	0.000	0.585	0.000	0.685	0.000	5.977
12	0.555	0.000	0.050	0.004	0.000	0.879	0.000	0.416	0.000	6.403
13	0.233	0.000	0.200	0.004	0.000	1.254	0.002	0.850	0.000	4.646
14	0.158	0.000	0.209	0.000	0.036	1.940	0.000	0.931	0.000	2.670
15	0.257	0.000	0.288	0.034	0.321	2.114	0.011	0.272	0.056	1.668
16	0.153	0.063	0.640	0.223	0.996	1.714	0.055	0.236	0.247	1.781
17	0.165	0.154	1.732	0.850	2.157	1.194	0.107	0.235	0.686	1.482
18	0.364	0.460	3.785	1.217	3.131	0.841	0.305	0.522	1.700	2.263
19	1.311	1.566	4.662	2.998	7.327	1.099	0.850	1.403	2.522	4.885
20	4.511	1.904	10.358	5.496	13.897	2.242	1.676	3.981	5.250	10.280
21	8.570	2.199	9.786	9.034	12.940	4.711	2.682	10.670	6.752	12.868
22	10.799	1.862	9.972	10.812	9.808	7.194	3.850	14.257	9.973	11.573
23	11.104	1.810	8.383	10.967	7.830	8.722	4.262	14.415	11.876	7.897
24	11.052	2.776	6.323	9.948	5.564	8.459	4.791	11.004	10.558	5.153
25	7.092	4.267	4.711	7.952	3.566	6.201	4.325	7.263	9.305	4.090
26	6.624	7.185	3.241	5.728	2.833	5.392	4.527	4.703	7.162	2.610
27	5.284	8.697	3.015	5.031	2.667	4.969	5.991	3.354	6.169	2.180
28	4.095	7.299	2.464	3.394	2.458	4.769	6.000	2.247	4.606	1.398
29	5.091	6.997	2.158	2.607	2.372	4.361	7.256	1.602	2.165	0.910
30	3.647	7.038	2.248	2.050	2.125	4.248	7.116	1.460	1.900	0.820
31	2.792	5.018	1.886	1.827	1.894	3.523	6.460	1.199	1.699	0.566
32	2.449	3.817	1.875	1.709	1.835	3.045	5.426	1.194	1.683	0.408
33	1.540	3.104	1.805	1.432	1.598	2.300	4.155	1.259	1.402	0.375
34	1.840	2.841	1.603	1.238	1.364	1.764	3.550	1.257	1.723	0.344
35	1.648	2.568	1.338	1.182	1.127	1.225	2.770	1.299	1.331	0.295
36	1.196	2.367	1.450	0.947	1.013	1.010	2.559	1.230	1.297	0.216
37	1.180	2.548	1.149	0.907	0.893	1.006	2.168	1.125	1.197	0.182
38	0.871	2.108	0.983	0.760	0.740	0.958	1.915	1.029	0.898	0.167
39	0.708	1.720	0.862	0.692	0.725	0.866	1.828	0.982	0.631	0.132
40	0.664	1.756	1.013	0.884	0.803	0.933	1.506	0.924	0.721	0.139
41	0.307	1.581	0.826	0.624	0.844	0.825	1.510	0.883	0.652	0.111
42	0.240	1.335	0.879	0.522	0.743	0.931	1.212	0.890	0.668	0.110
43	0.147	1.382	0.685	0.573	0.749	0.777	0.963	0.661	0.481	0.079
44	0.121	1.466	0.797	0.493	0.642	0.701	1.054	0.695	0.425	0.107
45	0.119	1.453	0.790	0.519	0.485	0.625	0.944	0.551	0.459	0.081
46	0.088	1.357	0.722	0.451	0.525	0.570	1.049	0.521	0.433	0.083
47	0.106	1.193	0.703	0.575	0.396	0.572	0.854	0.461	0.394	0.060
48	0.093	1.106	0.630	0.604	0.339	0.493	0.849	0.415	0.361	0.068
49	0.072	1.024	0.549	0.501	0.286	0.464	0.744	0.343	0.299	0.057
50	0.088	1.037	0.713	0.667	0.323	0.488	0.718	0.317	0.261	0.046



Table 6: continued

Length	1990	1991	1992	1994	1995	1996	1997	1998	2001	2002
51	0.082	0.897	0.524	0.629	0.283	0.452	0.607	0.279	0.195	0.038
52	0.065	0.697	0.533	0.482	0.256	0.411	0.567	0.215	0.267	0.031
53	0.066	0.595	0.422	0.463	0.238	0.346	0.448	0.189	0.153	0.027
54	0.045	0.485	0.352	0.365	0.223	0.335	0.410	0.147	0.146	0.019
55	0.041	0.347	0.391	0.308	0.195	0.252	0.368	0.116	0.190	0.018
56	0.038	0.364	0.309	0.198	0.161	0.231	0.271	0.072	0.238	0.024
57	0.038	0.285	0.258	0.212	0.147	0.182	0.199	0.067	0.144	0.019
58	0.019	0.237	0.243	0.167	0.121	0.144	0.159	0.066	0.132	0.013
59	0.022	0.128	0.212	0.162	0.104	0.116	0.129	0.073	0.156	0.013
60	0.025	0.142	0.235	0.171	0.084	0.100	0.086	0.047	0.031	0.010
61	0.011	0.131	0.134	0.123	0.078	0.080	0.091	0.038	0.053	0.008
62	0.019	0.087	0.129	0.106	0.078	0.067	0.075	0.027	0.053	0.011
63	0.015	0.080	0.077	0.155	0.070	0.059	0.064	0.027	0.050	0.008
64	0.021	0.035	0.083	0.119	0.074	0.057	0.065	0.038	0.021	0.005
65	0.016	0.059	0.092	0.127	0.057	0.050	0.050	0.025	0.042	0.005
66	0.003	0.068	0.072	0.144	0.073	0.028	0.056	0.019	0.045	0.005
67	0.014	0.058	0.052	0.087	0.071	0.030	0.037	0.017	0.032	0.004
68	0.003	0.040	0.041	0.111	0.072	0.028	0.042	0.027	0.027	0.005
69	0.005	0.048	0.029	0.072	0.044	0.024	0.036	0.008	0.014	0.002
70	0.004	0.049	0.030	0.060	0.043	0.028	0.034	0.011	0.015	0.001
71	0.003	0.023	0.041	0.048	0.029	0.020	0.031	0.004	0.013	0.002
72	0.000	0.028	0.023	0.066	0.028	0.012	0.022	0.003	0.013	0.002
73	0.000	0.007	0.014	0.030	0.027	0.008	0.020	0.007	0.009	0.001
74	0.000	0.018	0.020	0.031	0.017	0.006	0.017	0.000	0.002	0.003
75	0.000	0.011	0.019	0.031	0.015	0.003	0.007	0.006	0.001	0.000
76	0.000	0.004	0.014	0.026	0.021	0.009	0.009	0.000	0.003	0.002
77	0.000	0.007	0.014	0.023	0.010	0.003	0.016	0.000	0.004	0.000
78	0.000	0.004	0.021	0.009	0.007	0.007	0.018	0.000	0.005	0.000
79	0.000	0.000	0.008	0.008	0.007	0.005	0.005	0.000	0.000	0.000
80	0.000	0.000	0.018	0.002	0.007	0.000	0.008	0.000	0.001	0.000
81	0.000	0.000	0.007	0.000	0.005	0.003	0.010	0.000	0.000	0.000
82	0.000	0.000	0.009	0.000	0.000	0.000	0.003	0.001	0.001	0.000
83	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.000	0.000
84	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000
85	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
86	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.001	0.000
87	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
88	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
89	0.000	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000
90	0.000	0.003	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000
91	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
92	0.000	0.007	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000
93	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
94	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
95	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
96	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
97	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
98	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
99	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

**Table 7:** Winter survey length-at-age proportions (shown as percentages) for *M. capensis* and *M. paradoxus* combined off Namibia, for years for which no age-length key is available.

Length	1990	1992	1994	1995	1996	Length	1990	1992	1994	1995	1996
1	0.000	0.000	0.000	0.000	0.000	51	0.199	0.249	0.649	0.327	0.171
2	0.000	0.000	0.000	0.000	0.000	52	0.180	0.235	0.665	0.318	0.166
3	0.000	0.000	0.000	0.000	0.000	53	0.124	0.193	0.525	0.324	0.131
4	0.000	0.000	0.000	0.000	0.000	54	0.177	0.213	0.436	0.255	0.113
5	0.000	0.000	0.000	0.000	0.000	55	0.102	0.228	0.453	0.211	0.098
6	0.000	0.000	0.000	0.000	0.005	56	0.065	0.177	0.301	0.159	0.081
7	0.000	0.000	0.000	0.010	0.139	57	0.053	0.178	0.268	0.156	0.067
8	0.000	0.000	0.000	0.213	0.553	58	0.045	0.168	0.198	0.098	0.051
9	0.000	0.000	0.000	0.912	0.853	59	0.064	0.125	0.172	0.096	0.041
10	0.000	0.004	0.000	1.694	0.208	60	0.045	0.168	0.179	0.097	0.025
11	0.026	0.004	0.000	1.914	0.404	61	0.035	0.109	0.146	0.071	0.032
12	0.045	0.002	0.000	2.557	0.217	62	0.025	0.141	0.151	0.071	0.030
13	0.008	0.024	0.000	3.512	0.230	63	0.008	0.148	0.125	0.033	0.019
14	0.058	0.053	0.000	5.433	0.689	64	0.033	0.098	0.152	0.039	0.015
15	0.093	0.160	0.000	8.779	3.009	65	0.016	0.088	0.140	0.033	0.013
16	0.316	0.667	0.000	7.495	2.431	66	0.018	0.091	0.128	0.039	0.007
17	0.652	0.979	0.107	3.880	2.626	67	0.035	0.070	0.121	0.031	0.019
18	1.561	1.664	0.320	2.292	3.188	68	0.018	0.069	0.108	0.022	0.013
19	2.011	3.787	0.787	2.729	3.869	69	0.013	0.059	0.113	0.019	0.009
20	2.162	7.941	2.790	5.287	6.778	70	0.006	0.056	0.119	0.030	0.006
21	2.244	12.517	4.101	6.459	7.013	71	0.008	0.028	0.077	0.016	0.002
22	1.857	13.029	5.356	7.003	8.115	72	0.014	0.032	0.074	0.012	0.008
23	2.601	11.021	6.166	5.302	7.947	73	0.003	0.030	0.077	0.010	0.002
24	4.556	9.719	7.379	4.843	8.737	74	0.004	0.034	0.045	0.023	0.000
25	6.413	7.226	6.990	3.259	8.151	75	0.004	0.017	0.032	0.014	0.003
26	8.690	5.095	7.275	2.535	6.358	76	0.000	0.010	0.044	0.009	0.003
27	8.489	3.874	7.047	2.113	4.275	77	0.000	0.007	0.035	0.006	0.000
28	7.657	2.716	5.750	1.909	3.160	78	0.000	0.007	0.026	0.003	0.000
29	7.078	2.355	4.651	1.641	2.118	79	0.002	0.010	0.038	0.006	0.003
30	6.883	1.932	4.095	1.476	2.115	80	0.000	0.002	0.030	0.007	0.000
31	5.674	1.653	3.461	1.268	1.547	81	0.000	0.000	0.010	0.006	0.000
32	4.960	1.390	2.662	1.335	1.386	82	0.000	0.000	0.017	0.003	0.000
33	4.009	1.059	2.309	0.989	1.292	83	0.000	0.002	0.012	0.000	0.000
34	3.483	0.921	2.180	0.852	1.236	84	0.000	0.000	0.000	0.000	0.000
35	2.704	0.901	2.080	0.920	1.215	85	0.000	0.000	0.003	0.000	0.000
36	2.212	0.697	1.936	0.914	1.219	86	0.000	0.000	0.000	0.000	0.000
37	2.022	0.612	1.971	0.834	1.267	87	0.000	0.000	0.000	0.000	0.000
38	1.599	0.558	1.700	0.899	1.047	88	0.000	0.000	0.000	0.000	0.000
39	1.236	0.426	1.598	0.651	0.868	89	0.000	0.000	0.000	0.000	0.000
40	1.136	0.519	1.526	0.719	0.823	90	0.000	0.000	0.000	0.000	0.000
41	1.043	0.440	1.337	0.624	0.589	91	0.000	0.000	0.000	0.000	0.000
42	0.923	0.446	1.230	0.570	0.513	92	0.000	0.000	0.000	0.000	0.000
43	0.617	0.386	1.165	0.518	0.441	93	0.000	0.000	0.000	0.000	0.000
44	0.678	0.424	1.168	0.532	0.469	94	0.000	0.000	0.000	0.000	0.000
45	0.652	0.393	1.044	0.507	0.387	95	0.000	0.000	0.006	0.000	0.000
46	0.594	0.324	0.967	0.462	0.347	96	0.000	0.000	0.000	0.000	0.000
47	0.501	0.306	0.897	0.448	0.313	97	0.000	0.000	0.000	0.000	0.000
48	0.466	0.271	0.792	0.417	0.263	98	0.000	0.000	0.000	0.000	0.000
49	0.408	0.243	0.777	0.404	0.232	99	0.000	0.000	0.000	0.000	0.000
50	0.386	0.223	0.711	0.344	0.228	100	0.000	0.000	0.000	0.000	0.000

**Table 8:** Estimates of the parameter values of the Von Bertalanffy growth equation for the Namibian hake.

	All data
$L_{\infty}$ (cm)	120.0
$\kappa$ (yr <sup>-1</sup> )	0.089
$t_0$ (yr)	-0.318

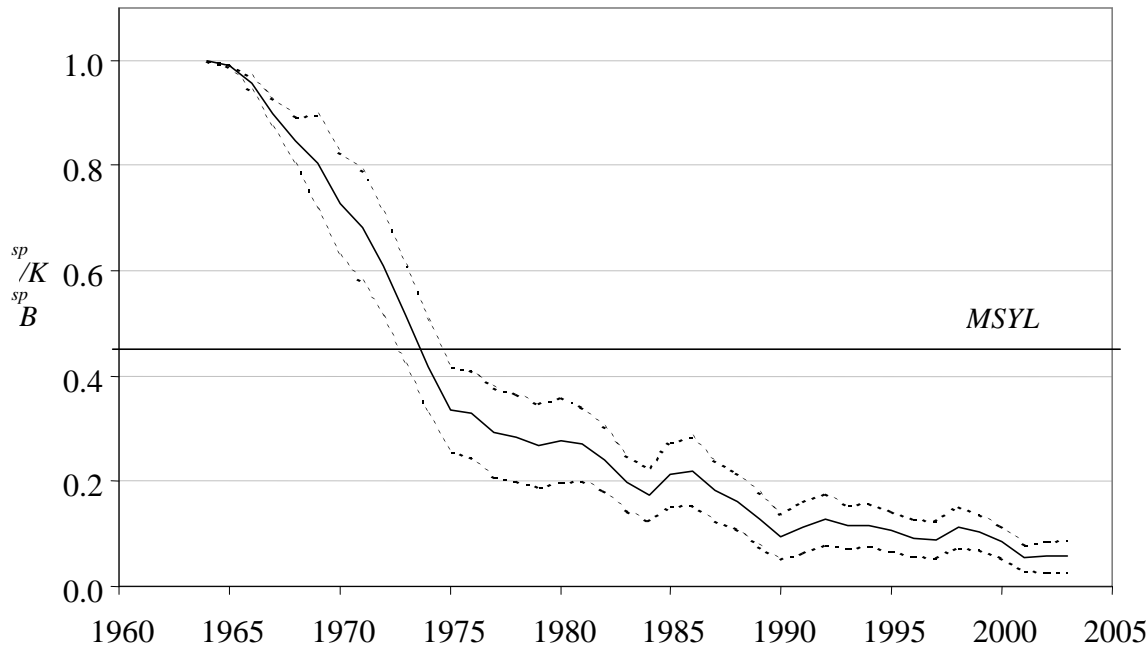
**Table 9:** Estimates of management quantities for the Reference Case (1a) assessment and 8 sensitivities for the Namibian hake resource. Figures in parenthesis are Hessian-based CV's.

	1a) Reference Case					1b) Von Bert fixed, $q = 1.0$					1c) Von Bert fixed, $q = 0.7$					1d) Von Bert fixed, $q = 0.4$					1e) Von Bert fixed, $q$ estimated, $h = 0.5$																								
Total negative loglikelihood	17.3					18.1					20.3					24.3					26.4																								
-lnL : CPUE	-40.8					-39.3					-36.6					-31.0					-29.7																								
-lnL: Survey	-13.1					-12.5					-12.2					-13.1					-15.3																								
-lnL: CAA com	-28.2					-30.3					-32.7					-34.8					-35.7																								
-lnL: LAA com	-3.5					-1.6					0.6					2.5					4.7																								
-lnL: LAA surv	72.5					70.9					69.6					68.6					69.1																								
-lnL: SR Residuals	20.9					21.1					21.7					22.4					23.1																								
negpen	0.0					0.0					0.0					0.0					0.0																								
$K^{SP}$	4508 (662)					4816 (765)					5489 (1077)					6600 (2218)					3073																								
$K^{Est}$	3876 (614)					4398 (662)					5198 (1016)					6369 (2256)					3261																								
$B^{SP}$	261 (71)					351 (69)					527 (128)					1131 (413)					2039																								
$B^{Est}$	228 (68)					308 (78)					475 (148)					1077 (460)					2133																								
$h$	0.294 (0.042)					0.259 (0.027)					0.230 (0.029)					0.215 (0.049)					0.500																								
$MSYL^{SP}$	2030 (361)					2248 (408)					2647 (591)					3238 (1265)					1121																								
$MSYL^{Est}$	2014 (375)					2202 (434)					2581 (611)					3182 (1255)					1168																								
$MSY$	218 (52)					171 (50)					112 (83)					73 (201)					371																								
$B^{SP}/K^{SP}$	0.058 (0.015)					0.073 (0.017)					0.096 (0.032)					0.171 (0.109)					0.664																								
$B^{Est}/K^{Est}$	0.059 (0.016)					0.070 (0.021)					0.091 (0.037)					0.169 (0.122)					0.654																								
$B^{SP}/MSYL^{SP}$	0.129 (0.032)					0.156 (0.039)					0.199 (0.072)					0.349 (0.242)					1.819																								
$B^{Est}/MSYL^{Est}$	0.113 (0.031)					0.140 (0.039)					0.184 (0.073)					0.339 (0.248)					1.826																								
$MSYL^{SP}/K^{SP}$	0.450 (0.019)					0.467 (0.014)					0.482 (0.016)					0.491 (0.029)					0.365																								
$MSYL^{Est}/K^{Est}$	0.520 (0.057)					0.501 (0.054)					0.497 (0.054)					0.500 (0.053)					0.358																								
$M$	0.635 (0.054)					0.692 (0.033)					0.735 (0.041)					0.760 (0.058)					0.663																								
Age	$S_{pre1,a}$	$S_{pre2,a}$	$S_{post,a}$	$S_{num,a}$	$S_{wint,a}$	$S_{pre1,a}$	$S_{pre2,a}$	$S_{post,a}$	$S_{num,a}$	$S_{wint,a}$	$S_{pre1,a}$	$S_{pre2,a}$	$S_{post,a}$	$S_{num,a}$	$S_{wint,a}$	$S_{pre1,a}$	$S_{pre2,a}$	$S_{post,a}$	$S_{num,a}$	$S_{wint,a}$	$S_{pre1,a}$	$S_{pre2,a}$	$S_{post,a}$	$S_{num,a}$	$S_{wint,a}$																				
0	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.001																				
1	0.007	0.000	0.009	0.125	0.098	0.007	0.000	0.008	0.117	0.089	0.006	0.000	0.007	0.111	0.087	0.006	0.000	0.007	0.105	0.088	0.007	0.000	0.008	0.113	0.101																				
2	0.117	0.734	0.081	1.000	0.949	0.107	0.707	0.074	1.000	0.904	0.101	0.692	0.069	1.000	0.899	0.099	0.689	0.069	1.000	0.907	0.098	0.671	0.088	1.000	0.937																				
3	0.704	1.000	0.465	1.000	1.000	0.686	1.000	0.439	1.000	1.000	0.679	1.000	0.428	1.000	1.000	0.685	1.000	0.438	1.000	1.000	0.636	1.000	0.525	1.000	1.000																				
4	0.978	1.000	0.896	0.791	0.825	0.977	1.000	0.885	0.842	0.836	0.977	1.000	0.882	0.859	0.855	0.978	1.000	0.892	0.852	0.869	0.967	1.000	0.927	0.772	0.805																				
5	1.000	1.000	0.988	0.625	0.681	1.000	1.000	0.987	0.709	0.698	1.000	1.000	0.987	0.737	0.730	1.000	1.000	0.989	0.726	0.754	1.000	1.000	0.993	0.595	0.647																				
6	0.677	0.973	0.999	0.494	0.562	0.721	0.995	0.999	0.597	0.583	0.743	0.985	0.999	0.633	0.623	0.746	0.953	0.999	0.619	0.654	1.000	1.000	0.999	0.459	0.520																				
7	0.457	0.947	1.000	0.390	0.463	0.519	0.990	1.000	0.503	0.487	0.552	0.971	1.000	0.543	0.532	0.556	0.908	1.000	0.527	0.567	0.998	1.000	1.000	0.354	0.418																				
8+	0.309	0.922	1.000	0.309	0.382	0.374	0.984	1.000	0.423	0.407	0.409	0.957	1.000	0.466	0.454	0.414	0.865	1.000	0.449	0.492	0.996	1.000	1.000	0.273	0.336																				
Commercial:	Sigma's		$q$ 's ( $\times 10^3$ )			Sigma's		$q$ 's ( $\times 10^3$ )			Sigma's		$q$ 's ( $\times 10^3$ )			Sigma's		$q$ 's ( $\times 10^3$ )			Sigma's		$q$ 's ( $\times 10^3$ )																						
CPUE ICSEAF (1.3+1.4)	0.139	(0.018)	0.309	(0.058)		0.152	(0.021)	0.262	(0.044)		0.169	(0.027)	0.212	(0.046)		0.197	(0.040)	0.162	(0.065)		0.140		0.377																						
CPUE ICSEAF (1.5)	0.181	(0.014)	0.472	(0.089)		0.189	(0.016)	0.400	(0.067)		0.202	(0.021)	0.323	(0.070)		0.223	(0.032)	0.247	(0.098)		0.178		0.575																						
CPUE GLM	0.189	(0.026)	2.450	(0.556)		0.197	(0.034)	1.937	(0.341)		0.218	(0.055)	1.364	(0.271)		0.286	(0.112)	0.716	(0.163)		0.434		0.469																						
Survey sigma's:																																													
Spanish winter	0.293 (0.019)					0.300 (0.017)					0.313 (0.019)					0.339 (0.030)					0.406																								
Spanish summer	0.573 (0.036)					0.552 (0.029)					0.529 (0.029)					0.502 (0.031)					0.472																								
Survey $q$ 's:																																													
Spanish winter	0.808 (0.169)					0.657 (0.059)					0.491 (0.059)					0.323 (0.071)					0.485																								
Spanish summer	1.369 (0.297)					1.024 (0.094)					0.748 (0.101)					0.492 (0.129)					0.722																								
Nansen summer	1.383 (0.310)					Com. trawlers $q$ 's: 1.528 (0.342)					1.000 (0.000)					1.111 (0.033)					0.379																								
Nansen winter	1.297 (0.303)					1.433 (0.335)					1.000 (0.000)					1.104 (0.033)					0.700 (0.000)					0.778 (0.023)					0.400 (0.000)					0.443 (0.014)					0.379				
Signas:																																													
LAA	CAA					LAA					CAA					LAA					CAA																								
commercial:	0.037 (0.007)					0.118 (0.006)					0.039 (0.007)					0.117 (0.006)					0.042 (0.008)					0.115 (0.006)					0.045 (0.008)					0.113 (0.005)					0.048				
summer survey	0.078 (0.003)					0.143 (0.021)					0.077 (0.003)					0.145 (0.020)					0.076 (0.003)					0.146 (0.020)					0.075 (0.003)					0.144 (0.019)					0.075				
winter survey	0.065 (0.004)					0.087 (0.021)					0.066 (0.004)					0.087 (0.021)					0.065 (0.004)					0.088 (0.021)					0.065 (0.004)					0.088 (0.021)					0.066				
LAA "spread": $\beta$	0.137 (0.010)					0.135 (0.010)					0.134 (0.010)					0.133 (0.010)					0.133 (0.010)					0.132																			
Nansen survey add. variance	0.073 (0.063)					0.078 (0.065)					0.082 (0.071)					0.073 (0.082)					0.073 (0.082)					0.053																			

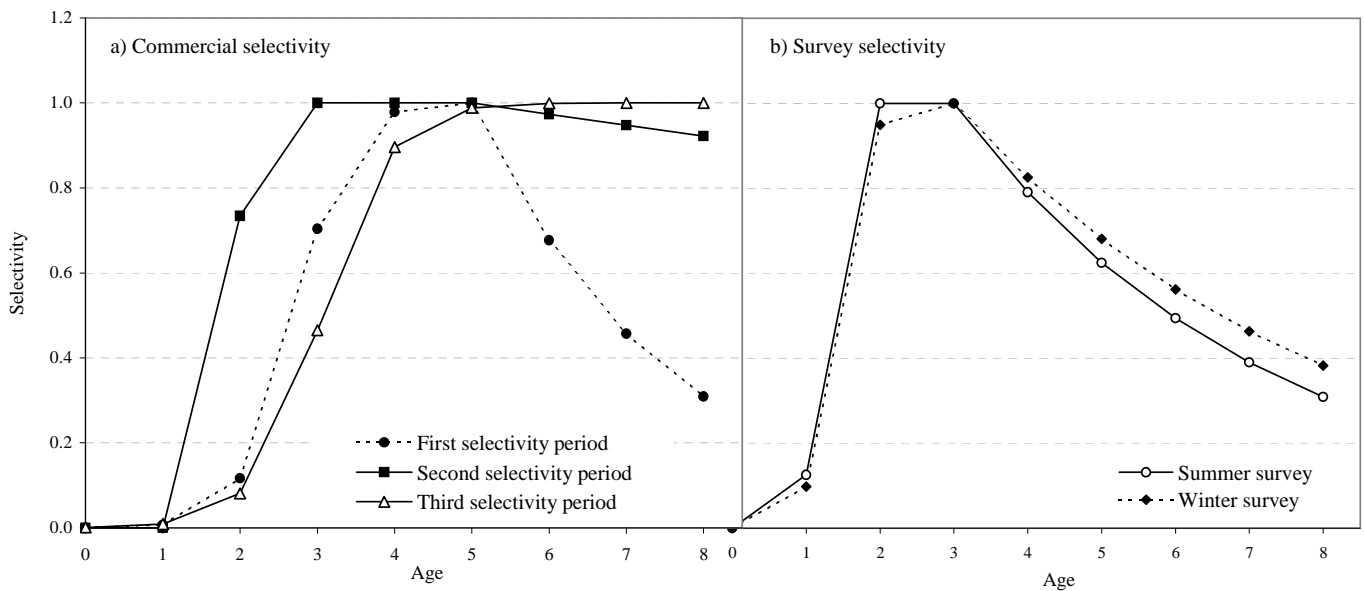


Table 9: continued

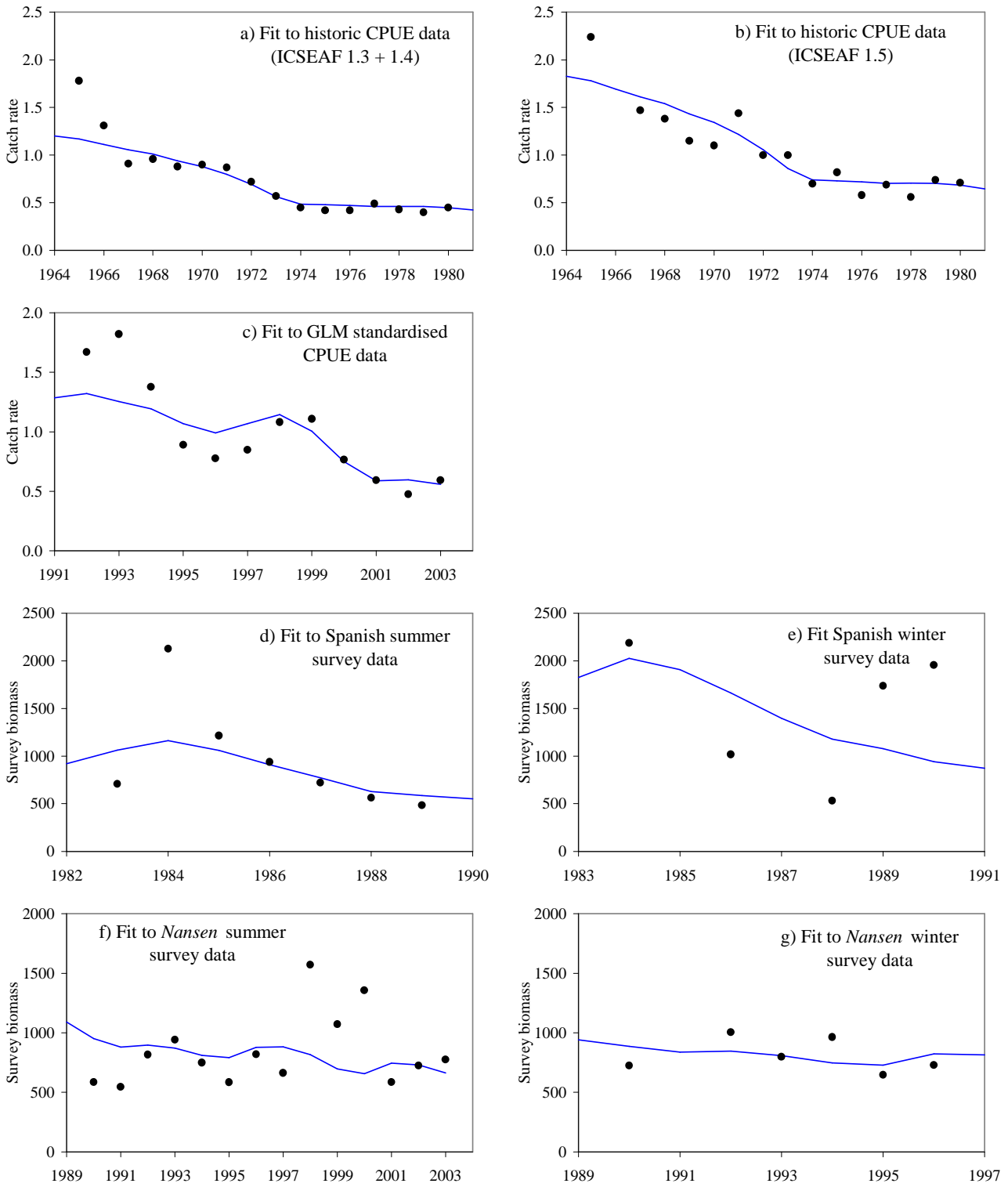
	1a) Reference Case	5a) Including extra CPUE series (all years)	5b) Including extra CPUE series from 1985 onwards	5c) Including extra CPUE series from 1985 onwards, with $q_{extra} = p * q_{GLM}$	6) Including 2004 survey biomass estimate
Total negative loglikelihood	17.3	-1.6	18.0	20.7	28.0
-lnL : CPUE	-40.8	-63.9	-40.7	-37.3	-40.9
-lnL : Survey	-13.1	-15.6	-15.5	-14.7	-12.5
-lnL : CAA.com	-28.2	-32.8	-34.3	-36.9	-37.2
-lnL : LAA.com	-3.5	4.5	4.3	5.4	7.1
-lnL : LAA.surv	72.5	72.1	70.3	68.8	81.7
-lnL : SR Residuals	20.9	22.9	23.1	24.6	21.4
negpen	0.0	0.0	0.0	0.0	0.0
$K^{sp}$	4508 (662)	2906 (437)	2905 (255)	2904 (269)	4627 (641)
$K^{wp}$	3876 (614)	3036 (298)	3069 (277)	3071 (293)	4008 (606)
$B^{sp}$	261 (71)	1919 (306)	2026 (333)	2002 (324)	283 (73)
$B^{wp}$	228 (68)	2076 (328)	2185 (349)	2071 (329)	182 (68)
$h$	0.294 (0.042)	0.555 (0.094)	0.567 (0.102)	0.610 (0.132)	0.283 (0.036)
$MSTL^{sp}$	2030 (361)	1007 (240)	990 (175)	939 (205)	2112 (345)
$MSTL^{ws}$	2014 (375)	1098 (246)	1083 (179)	998 (198)	1875 (350)
$MSTL^{sp}$	218 (52)	363 (33)	382 (52)	420 (68)	211 (32)
$B^{sp}/K^{sp}$	0.058 (0.015)	0.660 (0.095)	0.698 (0.108)	0.689 (0.099)	0.061 (0.017)
$B^{ws}/K^{ws}$	0.059 (0.016)	0.684 (0.118)	0.712 (0.105)	0.675 (0.093)	0.045 (0.016)
$B^{sp}/MSTL^{sp}$	1.129 (0.032)	1.906 (0.396)	2.047 (0.465)	2.132 (0.529)	0.134 (0.037)
$B^{ws}/MSTL^{ws}$	0.113 (0.031)	1.892 (0.360)	2.018 (0.420)	2.075 (0.461)	0.097 (0.036)
$MSTL^{sp}/K^{sp}$	0.450 (0.019)	0.346 (0.034)	0.341 (0.036)	0.323 (0.047)	0.457 (0.016)
$MSTL^{ws}/K^{ws}$	0.520 (0.057)	0.362 (0.085)	0.353 (0.032)	0.325 (0.040)	0.468 (0.053)
$M$	0.635 (0.054)	0.621 (0.031)	0.637 (0.029)	0.646 (0.030)	0.644 (0.052)
Commercial:					
CPUE ICSEAF (1,3+1,4)	0.139 (0.018)	0.309 (0.058)	0.125 (0.018)	0.141 (0.027)	0.143 (0.019)
CPUE ICSEAF (1,5)	0.181 (0.014)	0.472 (0.089)	0.168 (0.013)	0.178 (0.020)	0.185 (0.015)
CPUE GLM	0.189 (0.026)	2.450 (0.556)	0.430 (0.045)	0.376 (0.041)	0.182 (0.031)
CPUE 1975-1994		0.149 (0.016)	0.235 (0.035)	0.324 (0.031)	2.589 (0.544)
Survey signals:					
Spanish winter	0.293 (0.019)	0.405 (0.027)	0.415 (0.035)	0.425 (0.036)	0.294 (0.019)
Spanish summer	0.573 (0.036)	0.430 (0.029)	0.446 (0.032)	0.469 (0.032)	0.564 (0.035)
Survey $q$ 's:					
Spanish winter	0.808 (0.169)	0.697 (0.118)	0.565 (0.121)	0.451 (0.089)	0.757 (0.152)
Spanish summer	1.369 (0.297)	1.020 (0.159)	0.839 (0.167)	0.681 (0.127)	1.317 (0.278)
Mansen summer	1.383 (0.310)	1.528 (0.342)	0.448 (0.078)	0.402 (0.075)	1.360 (0.291)
Mansen winter	1.297 (0.303)	1.433 (0.335)	0.491 (0.085)	0.380 (0.072)	1.505 (0.322)
Survey signals:					
commercial	0.037 (0.007)	0.118 (0.006)	0.047 (0.004)	0.049 (0.004)	0.049 (0.004)
summer survey	0.078 (0.003)	0.143 (0.021)	0.076 (0.002)	0.075 (0.002)	0.079 (0.002)
winter survey	0.065 (0.004)	0.087 (0.021)	0.067 (0.004)	0.066 (0.004)	0.079 (0.002)
LAA <sup>spread</sup> , $\beta$	0.137 (0.010)	0.133 (0.010)	0.134 (0.010)	0.133 (0.010)	0.135 (0.010)
Mansen survey add. variance	0.073 (0.063)	0.050 (0.054)	0.051 (0.055)	0.058 (0.057)	0.079 (0.066)



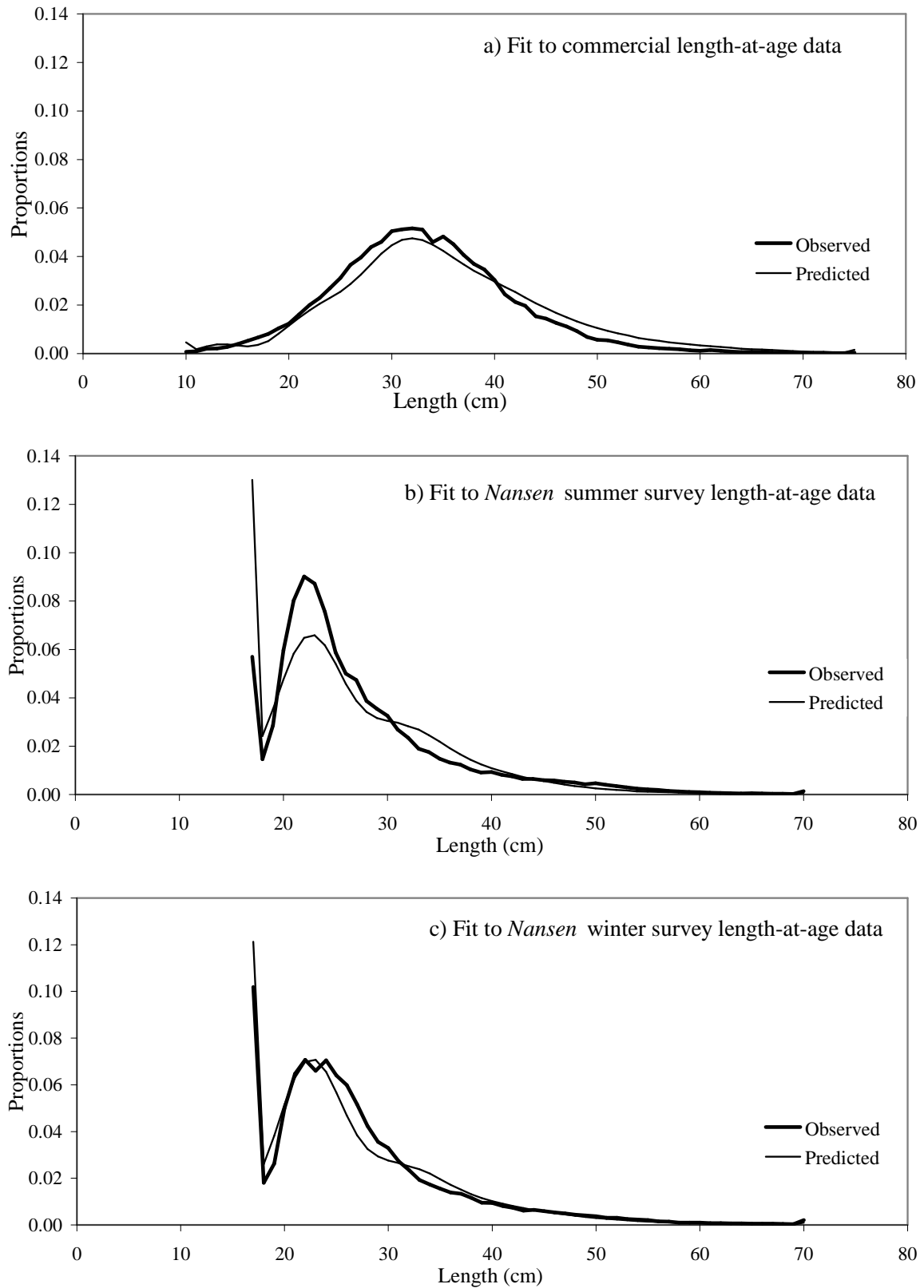
**Fig. 1:** Time series of spawning biomass for the Namibian hake resource as estimated from the Reference Case (1a) assessment. The biomass is expressed in terms of its pre-exploitation equilibrium level. The Hessian-based 95% CIs for the spawning biomass (dashed lines) and estimated MSYL are also shown.



**Fig. 2:** a) Commercial and b) survey fishing selectivities as estimated for the Reference Case (1a) assessment. The commercial selectivities are shown for the each of the three selectivity periods (1964 to 1973, 1984 to 1989 and 1990 to 2003). A selectivity is estimated separately for the summer and winter surveys.

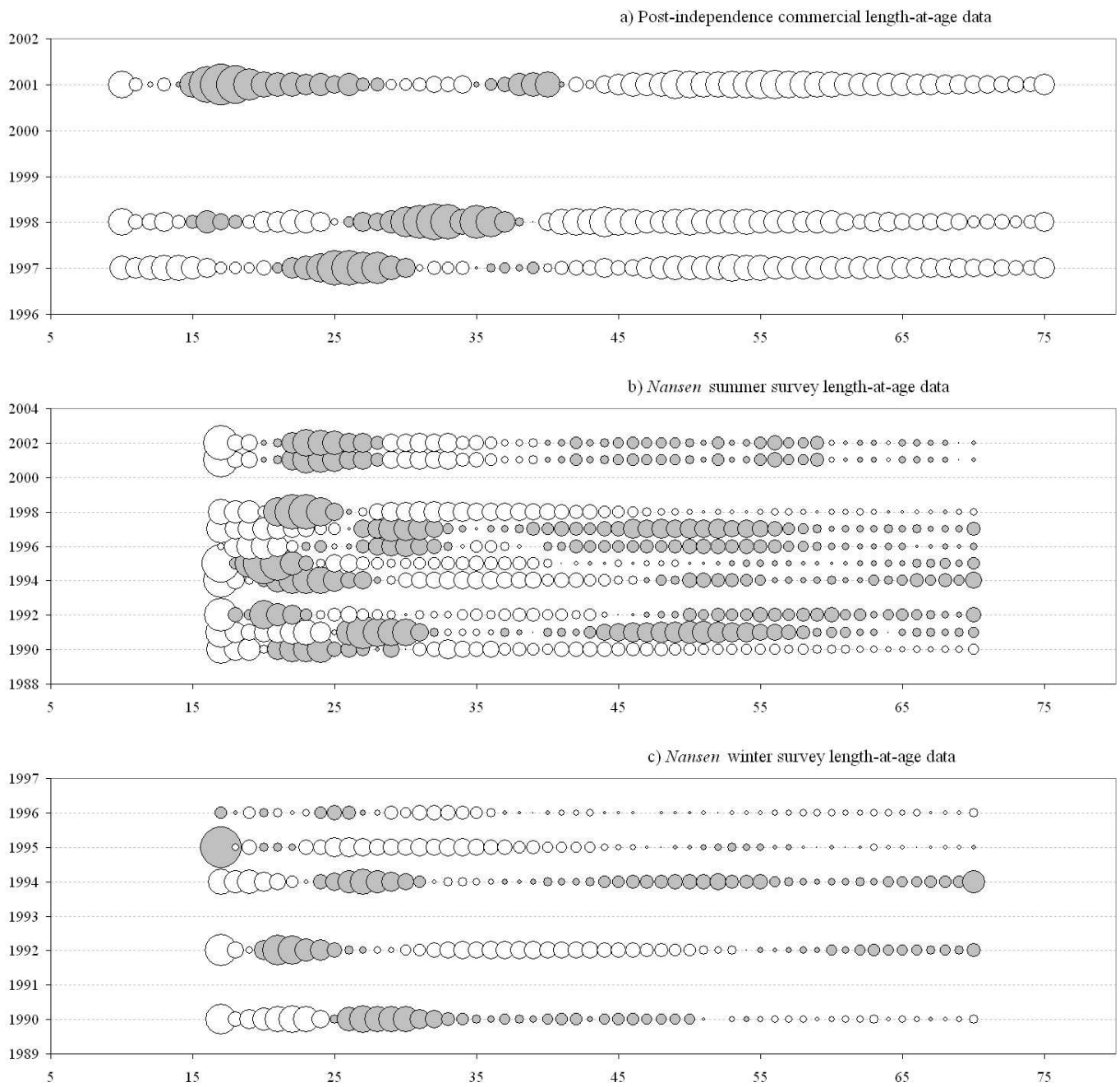


**Fig. 3:** Reference Case (1a) model fits to the CPUE and survey abundance indices for the Namibian hake resource.

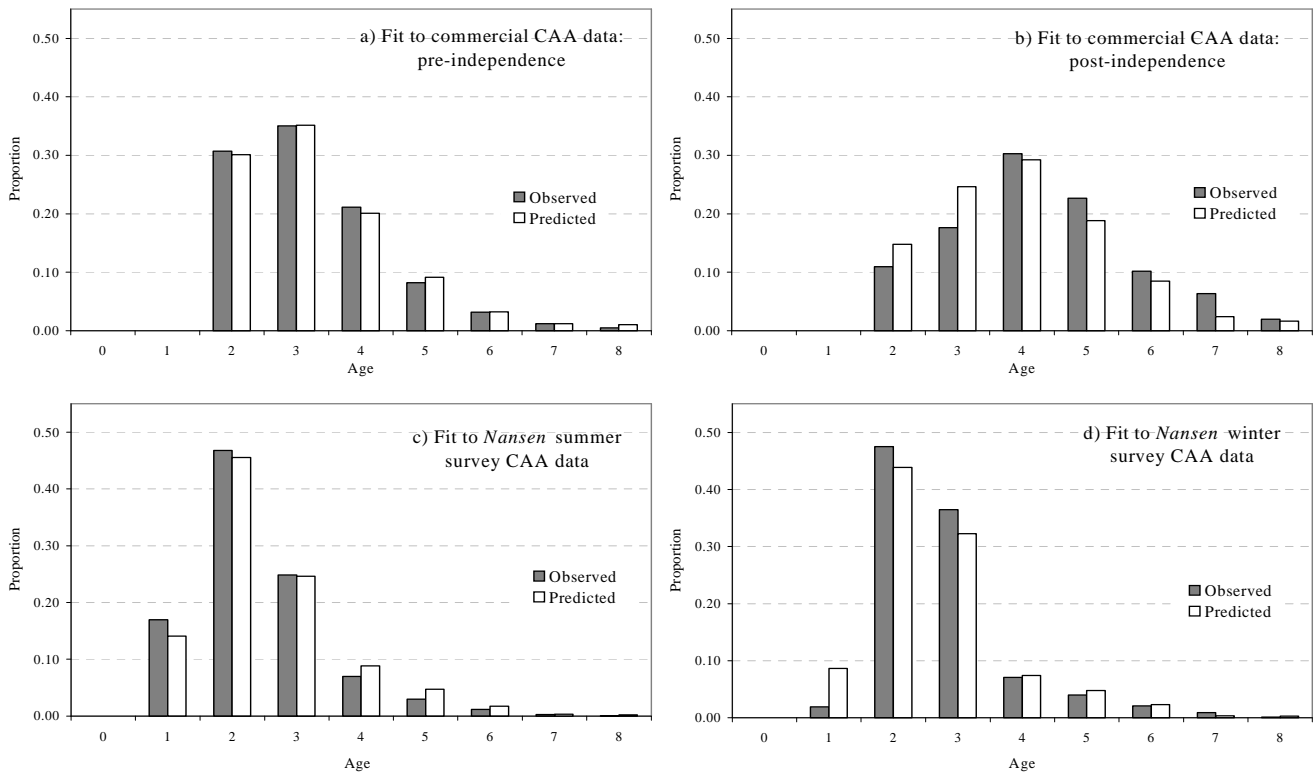


**Fig. 4:** Reference Case (1a) model fits to length-at-age proportion data, as averaged over all the years with data for a) the commercial, b) the summer survey and c) the winter survey data. The “spikes” at the two ends of the plots reflect minus- and plus-groups.

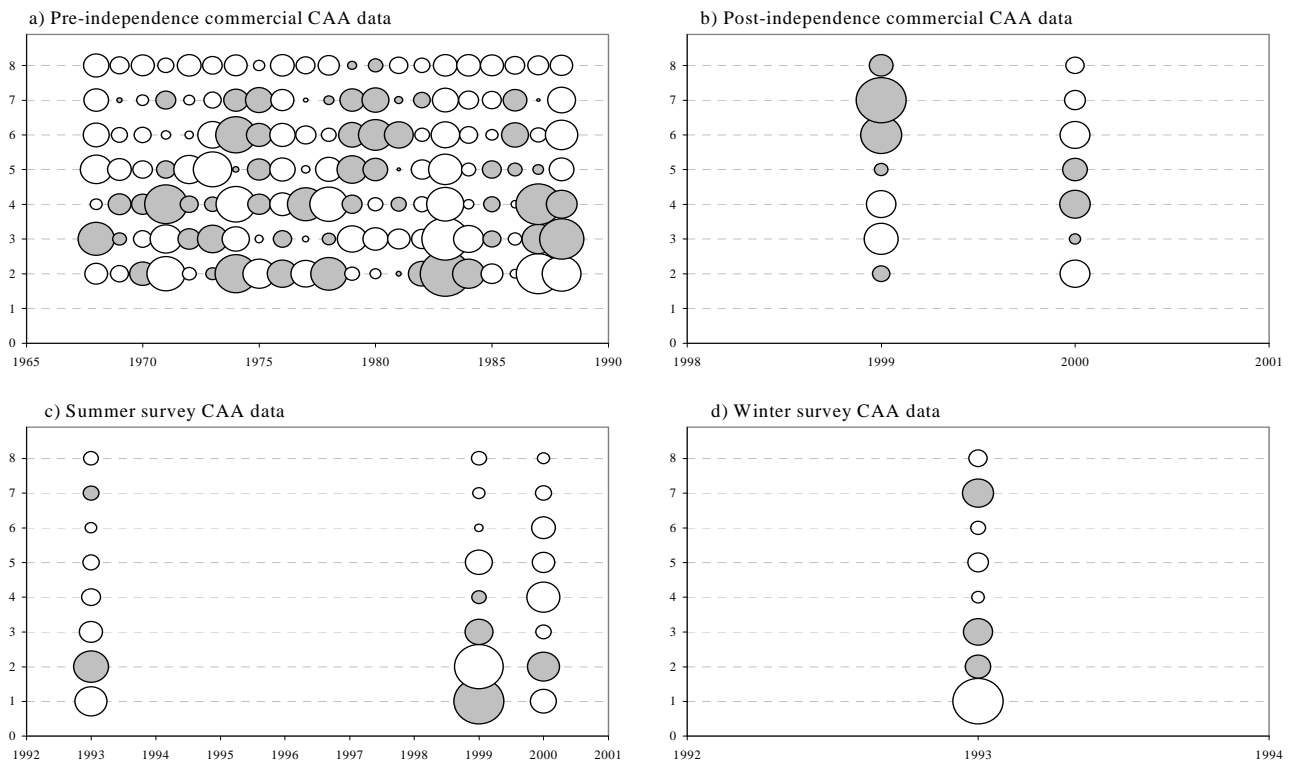




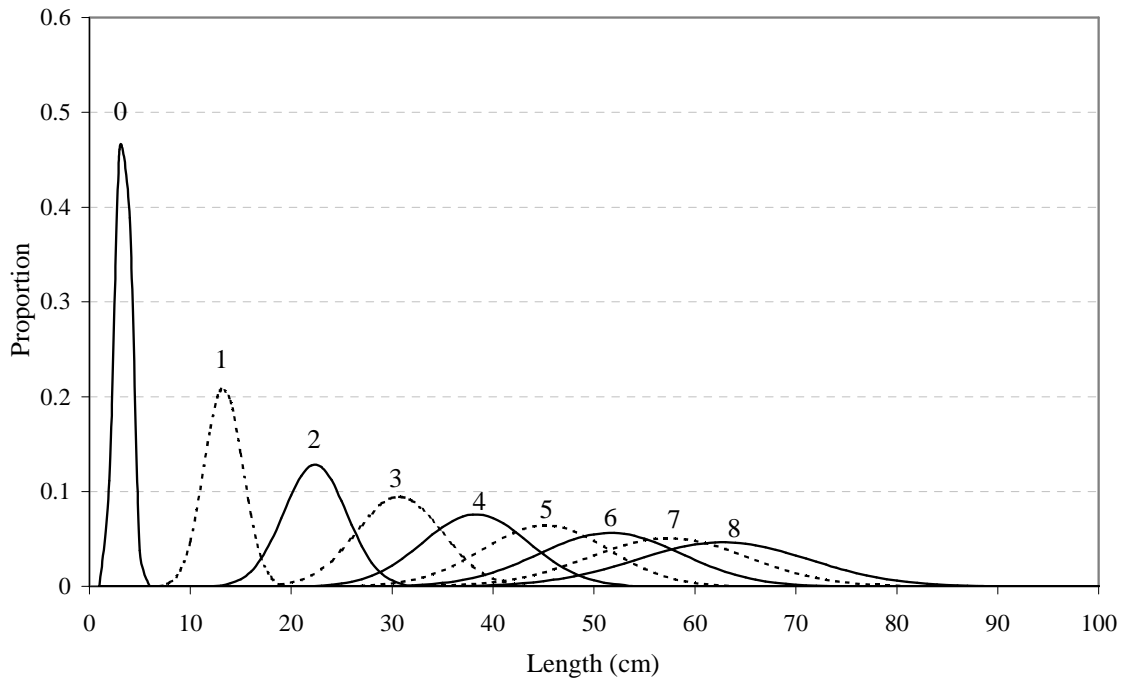
**Fig. 5:** “Bubble plots” of the length-at-age residuals for the Reference Case (1a) assessment for a) the commercial, b) the summer survey and c) the winter survey data. The size (area) of the bubble is proportional to the corresponding standardized residual. For positive residuals, the bubbles are gray and for negative residuals, the bubbles are white.



**Fig. 6:** Reference Case (1a) model fits to catch-at-age proportion data, as averaged over all the years with data for a) the commercial pre-independence, b) the commercial post-independence, c) the *Nansen* summer survey and d) the *Nansen* winter survey data.



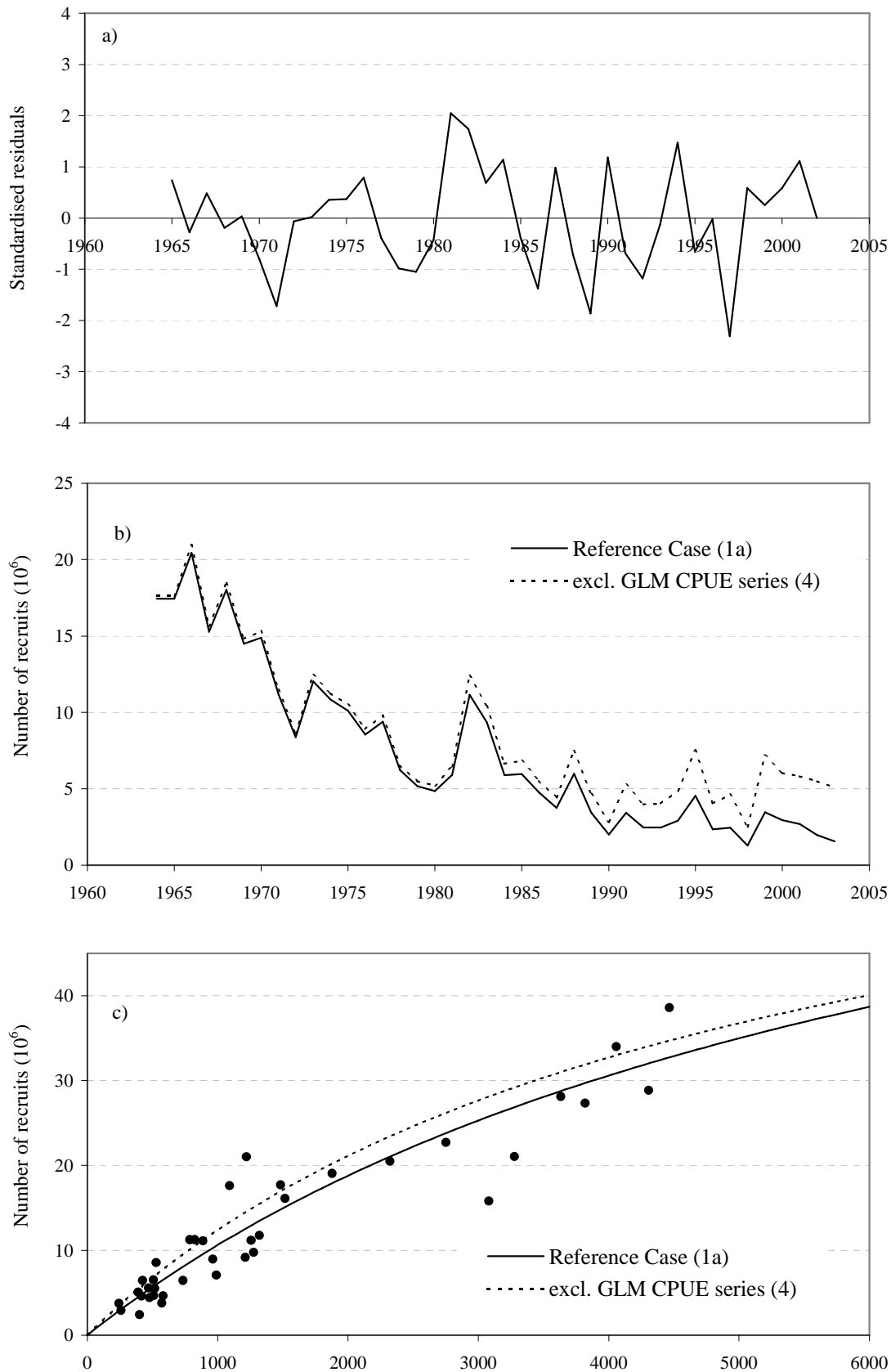
**Fig. 7:** “Bubble plots” of the catch-at-age residuals for the Reference Case (1a) assessment. The size (area) of the bubble is proportional to the corresponding standardized residual. For positive residuals, the bubbles are gray and for negative residuals, the bubbles are white.



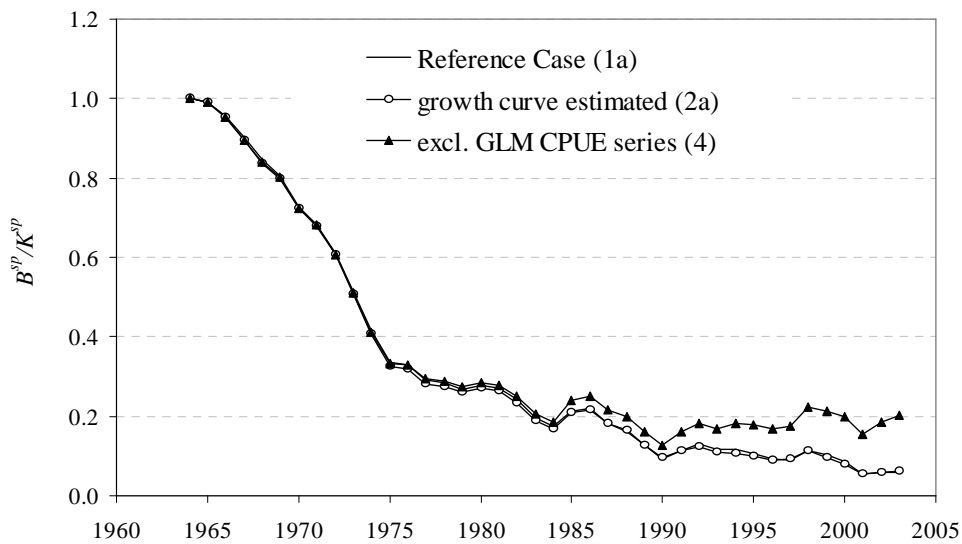
**Fig. 8:** Reference Case (1a) model estimated length-at-age distributions (for ages 0 to 8) for the Namibian hake resource.



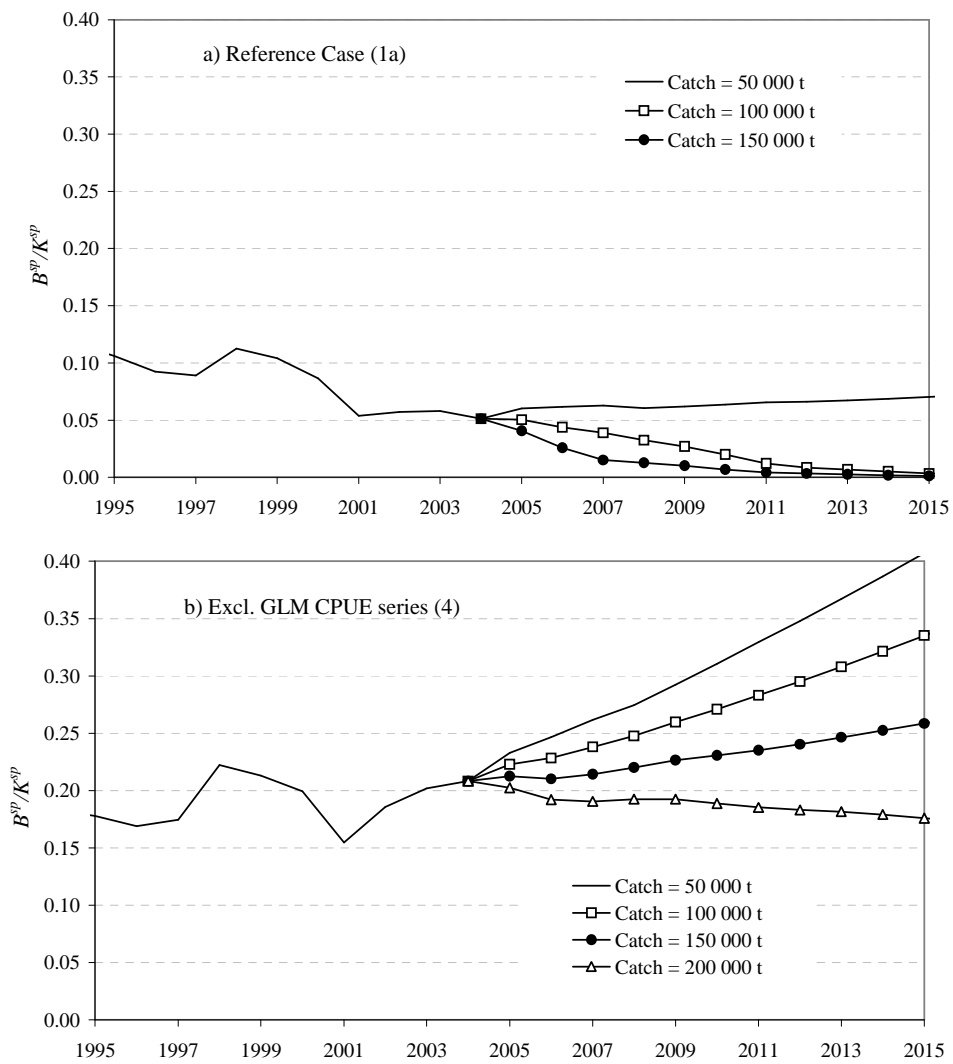
**Fig. 9:** Estimated mean length-at-age from the Von Bertalanffy growth equation used in the Reference Case (1a) assessment and from the straight line estimated in sensitivity (2a).



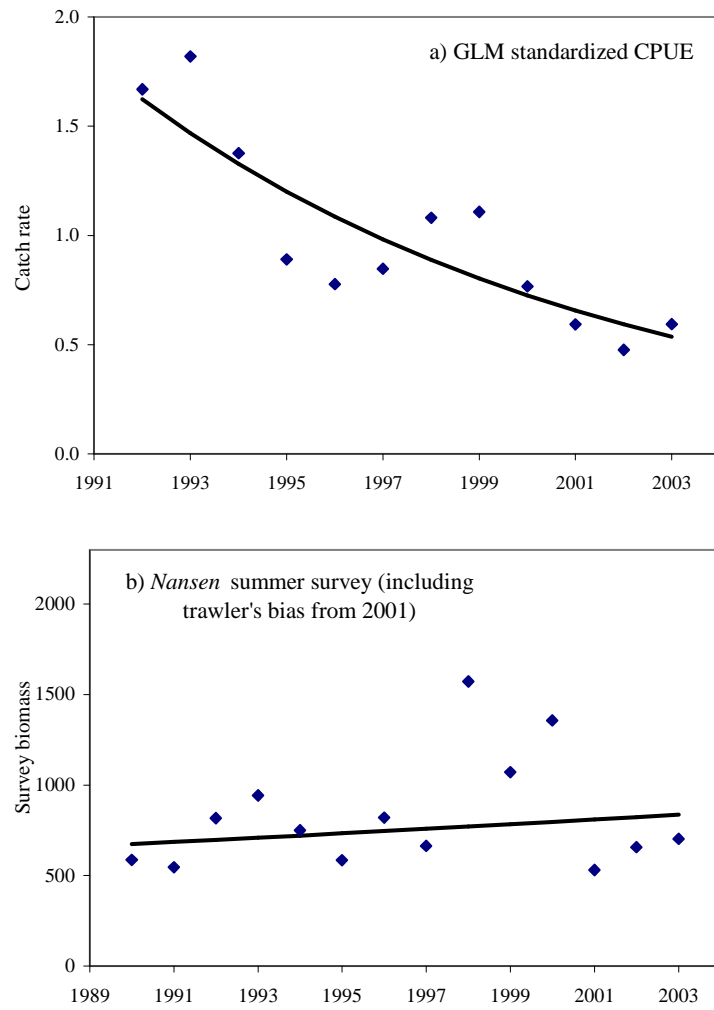
**Fig. 10:** Time-series of the Reference Case (1a) model estimated standardised recruitment residuals (a) and recruitment (b) for the Namibian hake resource; (c) shows the estimated stock-recruitment relationship. The time-series of recruitment and the stock-recruitment relationship are also shown for the sensitivity (4) for which the GLM standardized CPUE series is not included in the model fitting procedure.



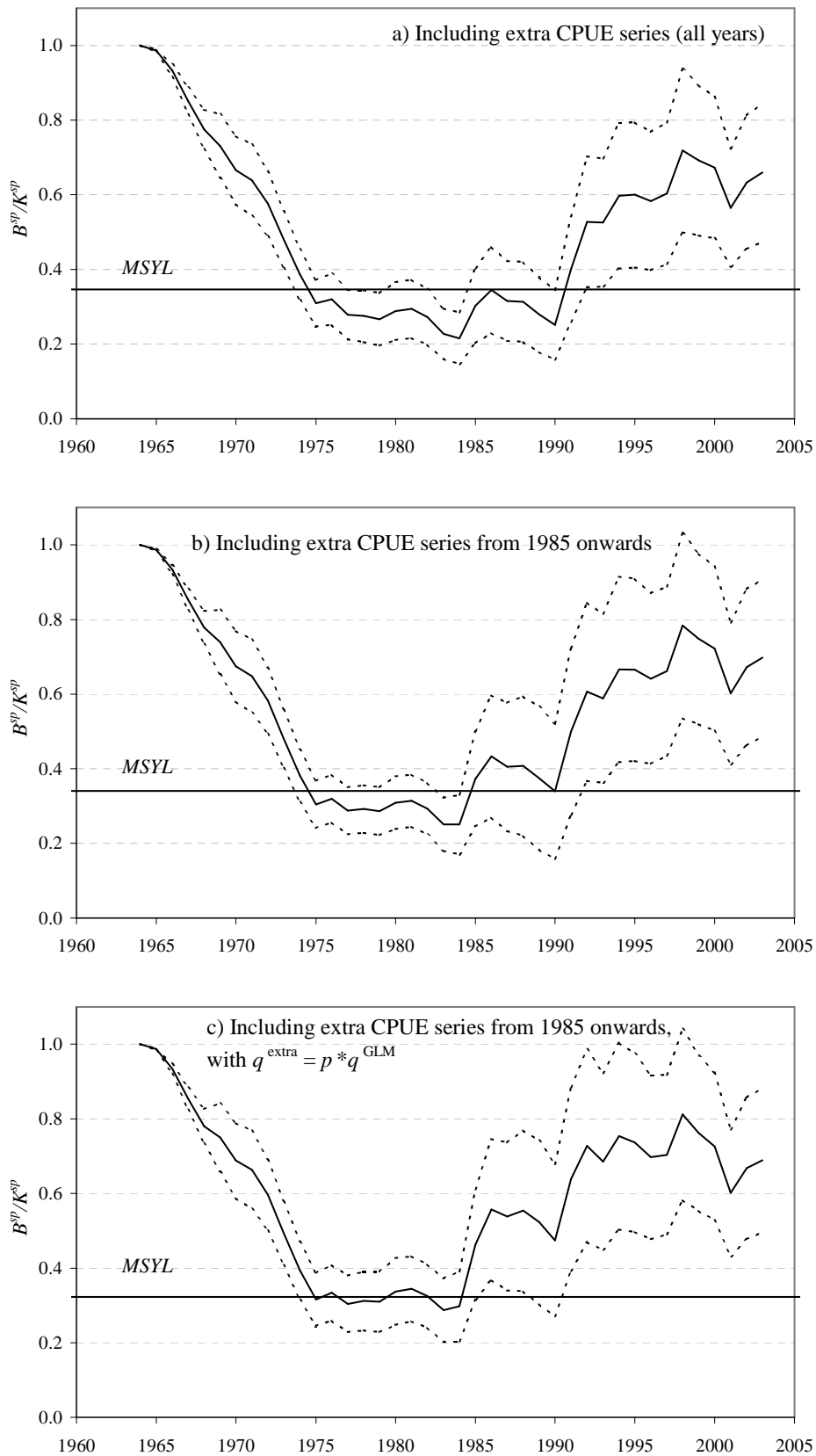
**Fig. 11:** Comparison of spawning biomass trajectories for the Reference Case (1a) and two sensitivities (see text for details) for the Namibian hake resource. The biomass is expressed in terms of its pre-exploitation equilibrium level.



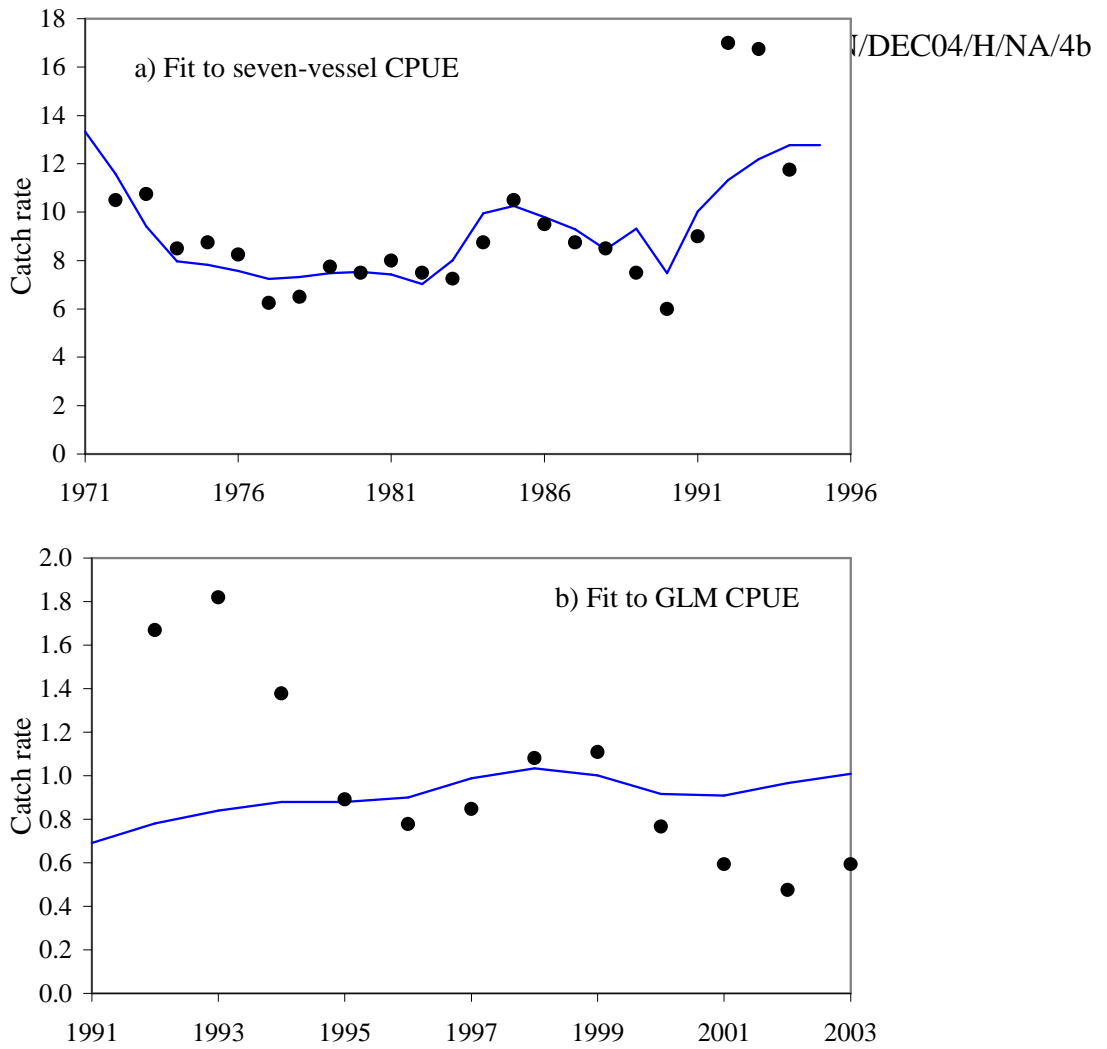
**Fig. 12:** Projected spawning biomass under selected constant catch strategies for a) the Reference Case (1a) assessment and b) sensitivity (4), excluding the GLM standardised CPUE series in the model fitting procedure.



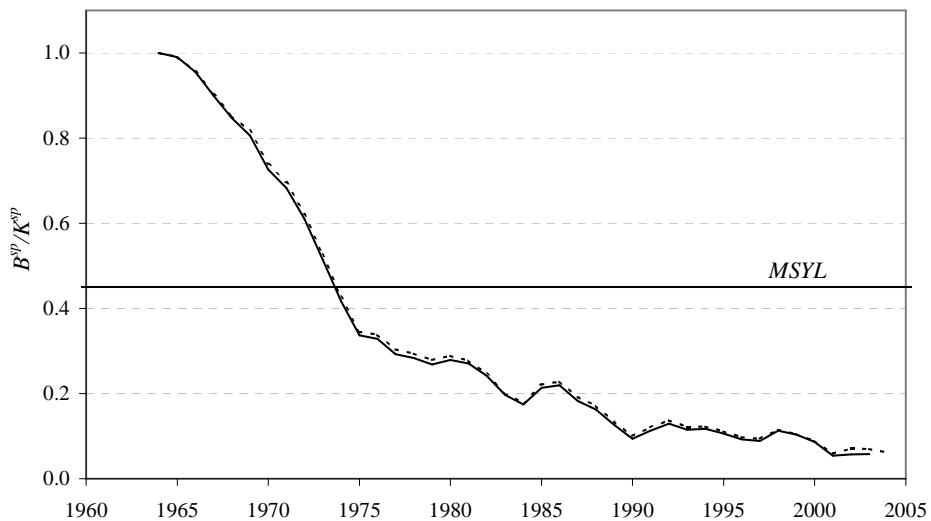
**Fig. 13:** Log-linear regression fits to a) the GLM standardized CPUE series and b) the *Nansen* summer survey (the trawler surveys in this series have been recalibrated to adjust for their relative bias compared to the *Nansen*).



**Fig. 14:** Spawning biomass trajectories for the three sensitivities which include the seven-vessel CPUE series (5a, b and c) for the Namibian hake resource. The biomass is expressed in terms of its pre-exploitation equilibrium level.



**Fig. 15:** Case 5a model fits to the seven-vessel and GLM-standardised CPUE series for the Namibian hake resource.



**Fig. 16:** Comparison of spawning biomass trajectories for the Reference Case (1a) (full curve) and the sensitivity which includes the 2004 survey biomass estimates (6) (dashed curve) for the Namibian hake resource. The biomass is expressed in terms of its pre-exploitation equilibrium level.



## Appendix 1

### The Age-Structured Production Model Including both Catch-at-Age and Catch-at-Length Information

#### A.1 Population Dynamics

##### A.1.1 Numbers-at-age

The Namibian hake resource dynamics are modelled by the following set of population dynamics equations:

$$N_{y+1,0} = R_{y+1} \quad \text{A.1}$$

$$N_{y+1,a+1} = \left( N_{y,a} e^{-M_a/2} - \sum_f C_{y,a} \right) e^{-M_a/2} \quad \text{for } 0 \leq a \leq m-2 \quad \text{A.2}$$

$$N_{y+1,m} = \left( N_{y,m-1} e^{-M_{m-1}/2} - \sum_f C_{y,m-1} \right) e^{-M_{m-1}/2} + \left( N_{y,m} e^{-M_m/2} - \sum_f C_{y,m} \right) e^{-M_m/2} \quad \text{A.3}$$

where

$N_{y,a}$  is the number of fish of age  $a$  at the start of year  $y$ ,

$R_y$  is the recruitment (number of 0-year-old fish) at the start of year  $y$ ,

$M_a$  denotes the natural mortality rate on fish of age  $a$ ,

$C_{y,a}$  is the number of hake of age  $a$  caught in year  $y$ , and

$m$  is the maximum age considered (taken to be a plus-group).

These equations simply state that for a closed population, with no immigration and emigration, the only sources of loss are natural mortality (predation, disease, etc.) and fishing mortality (catch). They reflect Pope's approximation (Pope, 1972) (the catches are assumed to be taken as a pulse in the middle of the year) rather than the more customary Baranov catch equations (Baranov, 1918) (catches are incorporated in the form of a continuous fishing mortality). Pope's approximation has been used in order to speed computations. As long as mortality rates are not too high, the differences between the Baranov and Pope formulation will be minimal. Tests showed this approximation to be adequate for the hake stocks (A. Punt, pers. commn).

##### A.1.2 Recruitment

Tomorrow's recruitment depends upon the reproductive output of today's fish. The number of recruits (i.e. new zero-year old fish) at the start of year  $y$  is assumed to be related to the spawning stock size (i.e. the biomass of mature fish) by a Beverton-Holt stock-recruitment relationship (Beverton and Holt, 1957), allowing for annual fluctuations:

$$R_y = \frac{\alpha B_y^{sp}}{\beta + B_y^{sp}} e^{(\zeta_y - \sigma_R^2/2)} \quad \text{A.4}$$

where

$\alpha$  and  $\beta$  are spawning biomass-recruitment relationship parameters,  $\alpha$  being the maximum number of recruits produced, and  $\beta$  the spawning stock needed to produce a recruitment equal to  $\alpha/2$ , in the deterministic case,

$\zeta_y$  reflects fluctuation about the expected recruitment for year  $y$ , which is assumed to be normally distributed with standard deviation  $\sigma_R$  (which is input in the applications considered here); these residuals are treated as estimable parameters in the model fitting process. Estimating the stock-recruitment residuals is made possible by the availability of catch-at-age data, which give some indication of the age-structure of the population. The  $-\sigma_R^2/2$  term is to correct for bias given the skewness of the log-normal distribution; it ensures that, on average, recruitments will be as indicated by the deterministic component of the stock-recruitment relationship (Geromont and Butterworth, 1998).

$B_y^{sp}$  is the spawning biomass at the start of year  $y$ , computed as:

$$B_y^{sp} = \sum_{a=1}^m f_a w_a N_{y,a} \quad \text{A.5}$$

where

$w_a$  is the begin-year mass of fish of age  $a$  and

$f_a$  is the proportion of fish of age  $a$  that are mature.

In order to work with estimable parameters that are more meaningful biologically, the stock-recruitment relationship is re-parameterised in terms of the pre-exploitation equilibrium spawning biomass,  $K^{sp}$ , and the “steepness”,  $h$ , of the stock-recruitment relationship, which is the proportion of the virgin recruitment that is realised at a spawning biomass level of 20% of the virgin spawning biomass:

$$\alpha = \frac{4hR_1}{5h-1} \quad \text{A.6}$$

and

$$\beta = \frac{K^{sp}(1-h)}{5h-1} \quad \text{A.7}$$

where

$$R_1 = K^{sp} / \left[ \sum_{a=1}^{m-1} f_a w_a \exp\left(-\sum_{a'=0}^{a-1} M_{a'}\right) + f_m w_m \frac{\exp\left(-\sum_{a'=0}^{a-1} M_{a'}\right)}{1 - \exp(-M_m)} \right] \quad \text{4.8}$$

In the fitting procedure, both  $h$  and  $K^{sp}$  are estimated. The steepness parameter is an important parameter, as the overall potential yield of an ASPM depends primarily on the steepness of the stock-recruitment curve and on the natural mortality rate.

### A.1.3 Total catch and catches-at-age

The catch by mass in year  $y$  is given by:

$$C_y = \sum_{a=0}^m w_{a+1/2} C_{y,a} = \sum_{a=0}^m w_{a+1/2} N_{y,a} e^{-M_a/2} S_{y,a} F_y \quad \text{A.9}$$

where

$w_{a+1/2}$  denotes the mid-year mass of fish of age  $a$ ,

$C_{y,a}$  is the catch-at-age, i.e. the number of fish of age  $a$ , caught in year  $y$ ,

$S_{y,a}$  is the commercial selectivity (i.e. vulnerability to fishing gear) at age  $a$  for year  $y$ ; when  $S_{y,a} = 1$ , the age-class  $a$  is said to be fully selected, and

$F_y$  is the fished proportion for a fully selected age class  $a$ .

The model estimate of the mid-year exploitable (“available”) component of biomass for each fleet is calculated by converting the numbers-at-age into mid-year mass-at-age (using the mid-year individual weights) and applying natural and fishing mortality for half the year:

$$B_y^{ex} = \sum_{a=0}^m w_{a+1/2} S_{y,a} N_{y,a} \exp(-M_a/2) (1 - \sum_f S_{y,a} F_y / 2) \quad \text{A.10}$$

whereas for survey estimates of biomass at the start of the year (summer):

$$B_y^{surv} = \sum_{a=0}^m w_a S_a^{surv} N_{y,a} \quad \text{A.11}$$

and in mid-year (winter):

$$B_y^{surv} = \sum_{a=0}^m w_{a+1/2} S_a^{surv} N_{y,a} \exp(-M_a/2) (1 - \sum_f S_{y,a} F_y / 2) \quad \text{A.12}$$

where

$S_a^{surv}$  is the survey selectivity.

It is assumed that the resource is at the deterministic equilibrium that corresponds to an absence of harvesting at the start of the initial year considered, i.e.  $B_{y_0}^{sp} = K^{sp}$ .

## A.2 The likelihood function

The model is fitted to CPUE and survey abundance indices, commercial and survey catch-at-age data, commercial and survey catch-at-length data, as well as stock-recruitment residuals to estimate model parameters. Contributions by each of these to the negative of the log-likelihood ( $-\ln L$ ) are as follows.

### A.2.1 CPUE relative abundance data

The likelihood is calculated assuming that the observed abundance index is log-normally distributed about its expected value:

$$I_y^i = \hat{I}_y^i \exp(\varepsilon_y^i) \quad \text{or} \quad \varepsilon_y^i = \ln(I_y^i) - \ln(\hat{I}_y^i) \quad \text{A.13}$$

where

$I_y^i$  is the abundance index for year  $y$  and series  $i$ ,

$\hat{I}_y^i = \hat{q}^i \hat{B}_y^{ex}$  is the corresponding model estimate, where  $\hat{B}_y^{ex}$  is the model estimate of exploitable resource biomass, given by equation A.10,

$\hat{q}^i$  is the constant of proportionality for abundance series  $i$ , and

$\varepsilon_y^i$  from  $N\left(0, (\sigma_y^i)^2\right)$ .

The contribution of the CPUE data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\ln L^{CPUE} = \sum_i w^i \sum_y \left[ \ln \sigma_y^i + (\varepsilon_y^i)^2 / 2(\sigma_y^i)^2 \right] \quad \text{A.14}$$

where

$w^i$  is a series weighting factor (see below), and

$\sigma_y^i$  is the standard deviation of the residuals for the logarithms of index  $i$  in year  $y$ .

Homoscedasticity of residuals is usually assumed, so that  $\sigma_y^i = \sigma^i$  is estimated in the fitting procedure by its maximum likelihood value:

$$\hat{\sigma}^i = \sqrt{1/n_i \sum_y (\ln(I_y^i) - \ln(\hat{q}^i \hat{B}_y^{ex}))^2} \quad \text{A.15}$$

where

$n_i$  is the number of data points for abundance index  $i$ .

The catchability coefficient  $q^i$  for abundance index  $i$  is estimated by its maximum likelihood value which is given by:

$$\ln \hat{q}^i = 1/n_i \sum_y (\ln I_y^i - \ln \hat{B}_y^{ex}) \quad \text{A.16}$$

In the Namibian hake case, equation A.14 is used to compute the log-likelihood contributions from two ICSEAF CPUE series from 1965 onwards, and the more recent CPUE series from the post-independence Namibian fishery. The first two series are each downweighted by 50% (i.e.  $w^i=0.5$ ), since they are treated as equivalent indices of the same quantity. The more recent CPUE series is given a weight  $w^i = 1$ .

The selectivity functions used to compute  $B_y^{ex}$  from equation A.10 and hence  $I_y^i$ , are taken to differ pre- and post-independence, as the latter period was marked by exclusion of vessels from fishing in waters shallower than 200 m (to better avoid juveniles). This function is taken to be independent of time for the latter of these two periods, for which it is denoted in Tables by  $S_{2002,a}$ . For the earlier period, the catch-at-age data indicate a trend over time towards selecting younger fish. Selectivity has therefore been modelled by a form  $S_{1964,a}$  which remains constant until 1973, after which it changes in a linear fashion to reach a form in 1984 which remains constant from then until the end of this earlier period in 1989 ( $S_{1989,a}$ ).

### A.2.2 Survey abundance data

Data from the Spanish surveys over the 1983-1990 period are treated as relative abundance indices in exactly the same manner as the CPUE series above, with log-likelihood contributions being provided by equation A.14. The only differences are that the survey selectivity function  $S_a^{surv}$  replaces the commercial selectivity  $S_{y,a}$  and that account is also taken of the begin- or mid-year nature of the survey.

For the *Nansen* surveys, an estimate of sampling variance is available for each surveys and the associated  $\sigma_y^i$  is generally taken to be given by the corresponding survey CV. However, these estimates likely fail to include all sources of variability, and unrealistically high precision (low variance and hence high weight) could be accorded to these indices. The procedure adopted takes into account an additional variance for each survey  $(\sigma_A^i)^2$  as follows.

$$- \ln L^{surv} = \sum_i \sum_y \left[ \frac{(\varepsilon_y^i)}{\left[ 2 \left( (\sigma_y^i)^2 + (\sigma_A^i)^2 \right) \right]} + \ln \sqrt{(\sigma_y^i)^2 + (\sigma_A^i)^2} \right] \quad \text{A.17}$$

where

$\sigma_y^i$  is the (sampling) standard error of the estimate for survey series  $i$  in year  $y$ , which is input, and

$\sigma_A^i$  is the square root of the additional variance for survey series  $i$ , and is assumed to be the same for each series  $i$ .

$\sigma_A^i$  is treated as another search parameter in the minimisation process. This procedure is carried out enforcing the constraint that  $(\sigma_A^i)^2 \geq 0$ , i.e. the overall variance cannot be less than its externally input component.

From 2000 onwards, the surveys have been conducted by commercial trawlers and a different value for the coefficient of proportionality  $q$  ("catchability" or multiplicative bias) is assumed to apply to the surveys conducted by these trawlers. Calibration experiments have been conducted between the *Nansen* and three of the trawlers used in the research surveys in order to provide a basis to relate the catchability coefficients of the two types of vessel ( $q_{Nansen}$  and  $q_{trawler}$ ) (Butterworth *et al.*, 2001). The means of the log CPUE ratios between the *Nansen* and the trawler in these three experiments were (T. Stromme, pers. commn):

<i>Nansen vs Oshakati</i>	:	-0.2237	s.e. = 0.0713
<i>Nansen vs Garoga</i>	:	+0.0567	s.e. = 0.0507
<i>Nansen vs Ribadeo</i>	:	-0.1900	s.e. = 0.0494

Applying inverse variance weighting to these results gives the following estimates:

$$\Delta \ln q = 0.100 \quad \text{with } \sigma_{\Delta \ln q} = 0.032.$$

where

$$\ln q_{\text{trawler}} = \ln q_{\text{Nansen}} + \Delta \ln q \quad \text{A.18}$$

The following contribution is therefore added to the negative log-likelihood in the assessment:

$$- \ln L^{q\text{-ch}} = (\ln q_{\text{trawler}} - \ln q_{\text{Nansen}} - \ln q)^2 / 2\sigma_{\Delta \ln q}^2 \quad \text{A.19}$$

Note that,  $q_{\text{trawler}}$  above refers to the average value for the commercial trawlers, so in order to allow for inter-trawler variability in  $q$ , the sampling CVs from the surveys from 2000 onwards ( $\sigma_y$ ) are increased as shown below to account for the standard deviation of 0.153 of the three means above about their global weighted mean, i.e.:

$$\sigma_y^2 \rightarrow \sigma_y^2 + 0.153^2 \quad \text{A.20}$$

Equation A.16 for the maximum likelihood estimate of  $q$  no longer applies with the addition of the term in equation A.19 to the log-likelihood, but closed-form solutions are still available for the maximum likelihood estimates of  $q_{\text{Nansen}}$  and  $q_{\text{trawler}}$  from setting partial derivatives of the total log-likelihood with respect to these two variables equal to zero.

### A.2.3 Commercial catches-at-age

The contribution of the catch-at-age data to the negative of the log-likelihood function when assuming an “adjusted” log-normal error distribution is given by:

$$- \ln L^{\text{age}} = \sum_f \sum_y \sum_a \left[ \ln \left( \sigma_{\text{com}} / \sqrt{p_{y,a}} \right) + p_{y,a} \left( \ln p_{y,a} - \ln \hat{p}_{y,a} \right)^2 / 2 \left( \sigma_{\text{com}} \right)^2 \right] \quad \text{A.21}$$

where

$p_{y,a} = C_{y,a} / \sum_a C_{y,a}$  is the observed proportion of fish caught in year  $y$  that are of age  $a$ ,

$\hat{p}_{y,a} = \hat{C}_{y,a} / \sum_a \hat{C}_{y,a}$  is the model-predicted proportion of fish caught in year  $y$  that are of age  $a$ ,

where

$$\hat{C}_{y,a} = N_{y,a} e^{-M_a/2} S_{y,a} F_y \quad \text{A.22}$$

and

$\sigma_{\text{com}}$  is the standard deviation associated with the catch-at-age data, which is estimated in the fitting procedure by:

$$\hat{\sigma}_{com} = \sqrt{\frac{\sum_y \sum_a p_{y,a} (\ln p_{y,a} - \ln \hat{p}_{y,a})^2}{\sum_y \sum_a 1}} \quad \text{A.23}$$

The log-normal error distribution underlying equation A.21 is chosen on the grounds that (assuming no ageing error) variability is likely dominated by a combination of interannual variation in the distribution of fishing effort, and fluctuations (partly as a consequence of such variations) in selectivity-at-age, which suggests that the assumption of a constant coefficient of variation is appropriate. However, for ages poorly represented in the sample, sampling variability considerations must at some stage start to dominate the variance. To take this into account in a simple manner, motivated by binomial distribution properties, Punt (pers. commn) advocates weighting by the observed proportions (as in equation A.21) so that undue importance is not attached to data based upon a few samples only.

Commercial catches-at-age are incorporated in the likelihood function using equation A.21, for which the summation over age  $a$  is taken from age 2 (considered as a minus group) to age 8 (a plus group). The ages for the minus- and plus-groups are chosen so that few fish (approximately less than 1% of the total sampled) fall outside this age range.

#### A.3.4 Survey catches-at-age

The survey catches-at-age are incorporated into the negative of the log-likelihood in an analogous manner to the commercial catches-at-age, assuming an adjusted log-normal error distribution (equation A.21) with:

$$p_{y,a}^{surv} = C_{y,a}^{surv} / \sum_a C_{y,a'}^{surv} \quad \text{is the observed proportion of fish of age } a \text{ from survey } surv \text{ in year } y,$$

$\hat{p}_{y,a}^{surv}$  is the expected proportion of fish of age  $a$  in year  $y$  in the survey  $surv$ , given by:

$$\hat{p}_{y,a}^{surv} = \frac{S_a^{surv} N_{y,a}}{\sum_{a'=0}^m S_{a'}^{surv} N_{y,a'}} \quad \text{A.24}$$

for begin-year (summer) surveys, or

$$\hat{p}_{y,a}^{surv} = \frac{S_a^{surv} N_{y,a} \exp(-M_a/2) \left(1 - \sum_f S_{y,a}^f F_y^f / 2\right)}{\sum_{a'=0}^m S_{a'}^{surv} N_{y,a'} \exp(-M_{a'}/2) \left(1 - \sum_f S_{y,a'}^f F_y^f / 2\right)} \quad \text{A.25}$$

for mid-year (winter) surveys.

#### A.2.5 Commercial and survey catch-at-length

The predicted annual catches-at-age (by number) made by each fleet ( $\hat{C}_{y,a}^f$ ), given by a fleet-specific form of equation A.22, are converted into predicted proportions of catch of age  $a$ :

$$\hat{p}_{y,a}^f = \hat{C}_{y,a}^f / \sum_{a'} \hat{C}_{y,a'}^f \quad \text{A.26}$$

The proportions-at-age are then converted into proportions-at-length using the von Bertalanffy growth equation, assuming that the length-at-age distribution remains constant over time:

$$\hat{p}_{y,l}^f = \sum_a \hat{p}_{y,a}^f A_{a,l} \quad \text{A.27}$$

where  $A_{a,l}$  is the proportion of fish of age  $a$  that fall in the length group  $l$  (thus  $\sum_l A_{a,l} = 1$  for all ages  $a$ ).

The matrix  $A$  is calculated under the assumption that length-at-age is normally distributed about a mean given by the Von Bertalanffy equation (Brandão *et al.*, 2002), i.e.:

$$L_a \sim N^* \left[ L_\infty \left( 1 - e^{-k(a-t_0)} \right); \theta_a^2 \right] \quad \text{A.28}$$

where

$N^*$  is the normal distribution truncated at  $\pm 3$  standard deviations, and

$\theta_a$  is the standard deviation of length-at-age  $a$ , which is modelled to be proportional to the expected length-at-age  $a$ , i.e.:

$$\theta_a = \beta L_\infty \left( 1 - e^{-k(a-t_0)} \right) \quad \text{A.29}$$

with  $\beta$  a parameter estimated in the model fitting process.

In this analysis, the growth curve and the extent of variability about it have been assumed to be constant over time.

Note that since the model of the population's dynamics is based upon a one-year time step, the value of  $\beta$  and hence the  $\theta_a$ 's estimated will reflect the real variability of the length-at-age as well as the 'spread' that arises from the fact that fish in the same annual cohort are not all spawned at exactly the same time, and that catching takes place throughout the year so that there are differences in the age (in terms of fractions of a year) of fish allocated to the same cohort.

The following term is then added to the negative log-likelihood:

$$-\ln L^{\text{length}} = w_{\text{len}} \sum_f \sum_y \sum_l \left[ \ln \left( \sigma_{\text{len}}^f / \sqrt{p_{y,l}^f} \right) + p_{y,l}^f \left( \ln p_{y,l}^f - \ln \hat{p}_{y,l}^f \right)^2 / 2 \left( \sigma_{\text{len}}^f \right)^2 \right] \quad \text{A.30}$$

where

$p_{y,l}^f$  is the observed proportion (by number) in length group  $l$  in the catch in year  $y$  for fleet  $f$ , and

$\sigma_{\text{len}}^f$  is the standard deviation associated with the length-at-age data for fleet  $f$ , which is estimated in the fitting procedure by:

$$\hat{\sigma}_{\text{len}}^f = \sqrt{\sum_y \sum_l p_{y,l}^f \left( \ln p_{y,l}^f - \ln \hat{p}_{y,l}^f \right)^2 / \sum_y \sum_l 1} \quad \text{A.31}$$

Equation A.30 makes the assumption that proportion-at-length data are log-normally distributed about their model-predicted values. The associated variance is taken to be inversely proportional to  $p_{y,l}^f$  to downweight contributions from observed small proportions which will correspond to small predicted sample sizes.



The  $w_{len}$  weighting factor may be set at a value less than 1 to downweight the contribution of the catch-at-length data to the overall negative log-likelihood compared to that of the CPUE and survey data. The reason that this factor is introduced is that the  $p_{y,l}^f$  data for a given year show evidence of strong positive correlation, and so are not as informative as the independence assumption underlying the form of equation A.30 would otherwise suggest. In this assessment  $w_{len} = 1/6$ . This is based upon the ratio of the number of length groups considered (about 60) and the 8 cohorts present in the resource, which roughly reflect the actual number of degrees of freedom in these data.

### A.2.6 Stock-recruitment function residuals

The stock-recruitment residuals are assumed to be log-normally distributed and serially correlated. Thus, the contribution of the recruitment residuals to the negative of the log-likelihood function is given by:

$$- \ell n L^{SR} = \sum_{y=y1+1}^{y2} \left[ \ell n \sigma_R + \left( \frac{\zeta_y - \rho \zeta_{y-1}}{\sqrt{1 - \rho^2}} \right)^2 / 2\sigma_R^2 \right] \quad \text{A.32}$$

where

$\zeta_y = \rho \zeta_{y-1} + \sqrt{1 - \rho^2} \varepsilon_y$  is the recruitment residual for year  $y$ , which is estimated for year  $y1$  to  $y2$  (see equation 4.4),

$\varepsilon_y$  from  $N(0, (\sigma_R)^2)$

$\sigma_R$  is the standard deviation of the log-residuals, which is input, and

$\rho$  is the serial correlation coefficient, which is input.

Because of the maximum-likelihood nature of the assessment methodology applied to incorporate recruitment fluctuations, the value of  $\sigma_R$  cannot be estimated from the data but must be independently specified. Indeed, the penalised likelihood formulation used will always yield a maximum for the deterministic limit of  $\sigma_R \rightarrow 0$ . This is problematic particularly in cases where estimates of stock status and productivity are strongly dependent on the value chosen for  $\sigma_R$ . One would need to adopt fully Bayesian methodology, together with a prior for  $\sigma_R$  to deal properly with this difficulty.

In the interest of simplicity, equation A.32 omits a term in  $\zeta_{y1}$  for the case when serial correlation is assumed ( $\rho \neq 0$ ), which is generally of little quantitative consequence to values estimated.

However, for the applications reported here for Namibian hake, the stock-recruitment residuals have been assumed not to be serially correlated, i.e.  $\rho = 0$ .  $\sigma_R$  is furthermore fixed at 0.25 for the Reference Case assessment.

### A.3 Estimation of precision

Coefficients of variation (CVs) and probability intervals have been evaluated using the Hessian-based approximation, which involves replacing the log-likelihood surface in the vicinity of its global maximum by a quadratic form (Rice, 1995). The delta method is used by ADMB to compute CVs for quantities that are functions of estimable

parameters of the model. This approach does not give exact answers. However, likelihood profile calculations are very time consuming, and the Hessian-based results regarding precision were considered adequate for the purpose to which such results were used here.

## A.4 Model parameters

### A.4.1 Estimable parameters

In addition to the virgin spawning biomass ( $K^{sp}$ ) and the “steepness” of the stock-recruitment relationship ( $h$ ), the following parameters are also estimated in some of the model fits undertaken.

#### Natural mortality:

Natural mortality ( $M_a$ ) is assumed either to be independent of age or age-specific, and input (fixed) or estimated using the following functional form in the latter case:

$$M_a = \begin{cases} M_2 & \text{for } a \leq 1 \\ \alpha^M + \frac{\beta^M}{a+1} & \text{for } a \geq 2 \end{cases} \quad \text{A.33}$$

$M_0$  and  $M_1$  are set equal to  $M_2$  as there is virtually no information in the data (hake of ages below 2 are hardly caught) taken into account in the likelihood which would allow independent estimation of  $M_0$  and  $M_1$ .

In the Reference Case assessment,  $M$  is taken to be independent of age.

#### Fishing selectivity-at-age:

The commercial and survey fishing selectivity take the form of a logistic curve:

$$S_a^f = \begin{cases} 0 & \text{for } a = 0 \\ \left[ 1 + \exp\left(-\left(a - a_c^f\right) / \delta^f\right) \right]^{-1} & \text{for } a \geq 1 \end{cases} \quad \text{A.34}$$

where

$a_c^f$  years is the age-at-50% selectivity,

$\delta^f$  year<sup>-1</sup> defines the steepness of the ascending limb of the selectivity curve.

The selectivity is sometimes modified to include a decrease in selectivity at older ages, as follows:

$$S_a \rightarrow S_a \exp\left(-s \left(a - a_{slope}\right)\right) \quad \text{for } a > a_{slope}, \quad \text{A.35}$$

where

$s$  measures the rate of decrease in selectivity with age for fish older than  $a_{slope}$ .

### A.4.2 Input parameters

#### Age-at-maturity:

The proportion of fish of age  $a$  that are mature is approximated by  $f_a = 1$  for  $a \geq a_{mat}$  years, where  $a_{mat} = 4$  for the Namibian hake.

**Weight-at-age:**

The weight-at-age (begin and mid-year) is calculated from the combination of the Von Bertalanffy growth equation and the mass-at-length function.