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A stock assessment of the Namibian Horse Mackerel (*Trachurus trachurus capensis*) based on an age-structured production model

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The horse mackerel stock has been assessed using a fleet-segregated age-structured production model. The trend information of the CPUE and acoustic biomass survey, as well as catch-at-age data were used to estimate the current stock status. Results, based on these data, indicated that the stock is at a low level, below the maximum sustainable yield level, and would not be able to support catches as high as 350 000 tonnes, until the stock is rebuilt to its MSY level.

Introduction

The Namibian horsemackerel stock was assessed on an annual basis over the years 1970 to 1990 by the International Commission for Southeast Atlantic Fisheries (ICSEAF) through its Standing Committee on Stock Assessment. This function was taken over in 1990 by the Ministry of Fisheries and Marine Resources, Namibia. During the ICSEAF period the stock was assessed mainly by means of the Virtual Population Analysis (VPA) using internationally derived age-length keys.

Catches were low in the first ten years of exploitation (1961-1970) averaging at 62 700 tonnes a year. During the following decade (1971-1980) average catches increased substantially to 339 200 tonnes. A total allowable catch (TAC) of 500 000 tonnes was implemented in 1980. This regulatory measure was imposed only on the foreign midwater fleet, while unrestricted catches were allowed for the local purse-seine fleet. The only limiting factor to the purse-seine fleet, at this time, was the closure of the fishing season in August. During the period from 1980 to 1990 an average of 536 800 tonnes were caught annually, which decreased to an average of 358 800 tonnes over the period 1991 to 2000.

Methods

Data used in the assessment

Catch statistics (Appendix 1: Table I) are available from 1961 – 2003 and the assessment was conducted for that period assuming no exploitation prior to 1961. Horse mackerel is exploited by two fleets, the midwater trawl and pelagic purseine fleet. Although a combined TAC is given, catches are recorded separately for these fisheries, which make it possible to model their effects separately. The plus-group was taken as age 7. Catch-at-age for the two fisheries as well as the catch samples during the acoustic surveys were included in the assessment (Appendix 1: Figure 1). These catch-at-age matrices are based on one single age-length key (1996), which was constructed at the Horse mackerel ageing workshop.

Catch-at-age matrices are normalized to proportions before use in the assessment. Young and old age classes, which had less than 10% of the catch were pooled into a minus and a plus group for both fisheries and the survey (Appendix 1: Table II, III and IV).

Pelagic fishery: Minus group: 0 Plus group: 3

Midwater fishery: Minus group: 1 Plus group: 5

Survey: Minus group: 0 Plus group:3

Age-at-maturity was calculated from length at maturity using the Namibian age-length-key (Appendix 1: Table V).

Commercial catch rates of the midwater fishing fleet are used to calculate the annual CPUE in terms of tonnes per hour. This data are available for the period 1991 to 2004. The data set for 2004 includes month from January to May. The CPUE data was attempted to be standardized using the general linearized model, but only about 20% of the variation could be explained by this model. Therefore the overall 5% trimmed mean of the annual commercial catch rate data was calculated to avoid any extreme outliers that may be influencing the overall mean. The general trend, over this time period, shows a steady decline (Figure 1, Appendix 1: Table VI). Horse mackerel is an aggregating species. A slight decline in stock abundance could be masked by this aggregating behaviour and would therefore not be reflected in the CPUE trend. Therefore, once a decline in the CPUE trend is observed for this kind of species, it needs to be appreciated that the extent of the decline in abundance is even greater. This CPUE series is used for determining the current state of the horse mackerel stock off Namibia. The information that is provided by the trend in the CPUE series since 1991 is used, rather than the CPUE of the previous year only.

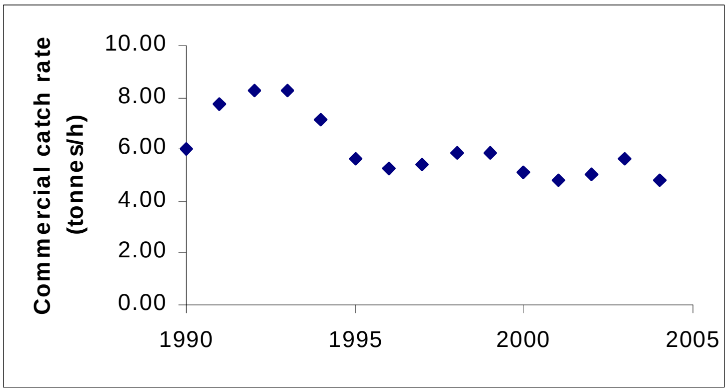


Figure 1: *The overall 5% trimmed mean of the commercial catch rate data from 1990-2003.*

The trend information in the relative biomass estimate that is obtained by an annual acoustic survey (Figure 2 Appendix 1: Table VII) is also used to determine the current state of the stock. Only estimates since 1999 have been used, as since then the surveys have been standardized. Although the last two surveys indicated a possible increase in biomass, the overall trend from 1991 to 2004 is negative.

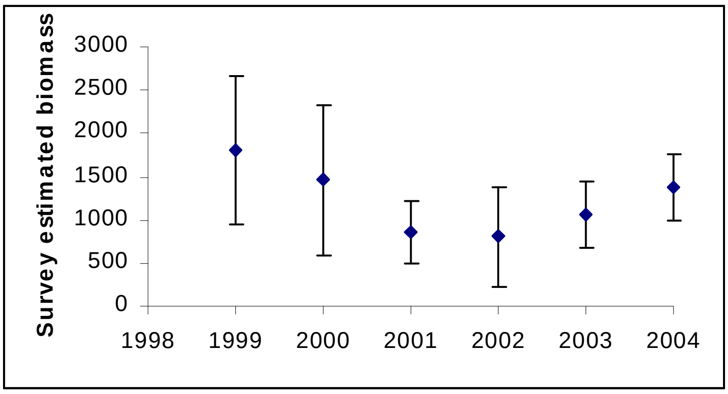


Figure 2: *Acoustic biomass values (tonnes) from 1999 to 2004. The 95% confidence intervals have been calculated from their sampling CV's only.*

The Age-Structured Production Model (ASPM)

An age-structured production model is implemented and fitted to the indices of abundance (commercial CPUE series and survey biomass estimates). This model (Appendix 2) is fitted to the midwater CPUE series and the survey biomass estimates. The fitting process assumes that the survey biomass and the CPUE series provide an index of relative abundance, and minimizes the log-likelihood function. The fitting process also relates commercial catch-at-age data, for both purse-seine and midwater fishery, to estimate selectivity and recruitment residuals for the years that this data is available. The log-likelihood function is a function of the unexploited equilibrium spawner-biomass, B_0^{sp}, the steepness parameter, *h* (which is the fraction of the recruitment at the unexploited equilibrium level of spawning biomass to be expected when this biomass is reduced to 20% of B_0^{sp}), the natural mortality *M*, and the constant of proportionality *q* for each CPUE index. The model estimates B_0^{sp}, the individual selectivities-at-age and the slope when the selectivity is lower than one for older ages. The parameters *h*, *M* and *q* (survey) are set externally as the information in the data is not sufficient to estimate these parameters within the fitting process. The proportional factor (*q*) for the commercial CPUE data is calculated within the model. The log-likelihood function is minimized with respect to B_0^{sp}, and the catch-at-age data, for various combinations of natural mortality in the range between 0.3-0.5 per year and various values for the steepness parameter (0.4-0.8) and various values for the survey catchability, between 1-2. In the light of the results of the above combinations, a base case is chosen.

The criteria for the base case is the maximum log-likelihood, which provides the best fit to the data. However, if the corresponding parameters *h*, *M* and *q* are unrealistic for the horse mackerel, then combinations close to the maximum obtained log-likelihood are chosen as a basis for projections. Confidence intervals are obtained for the different management quantities using MCMC function in the ADMB software.

Results

The results shown in Figures 3-7 are based on the base case that was chosen with q=2, M=0.3 and h=0.6.

The decline of the estimated midwater fleet exploitable biomass with the total annual catches taken off the stock from 1972 to 2005 is illustrated by Figure 3. The certainty of this decline is dependent on the goodness of fit of various estimated to observed data

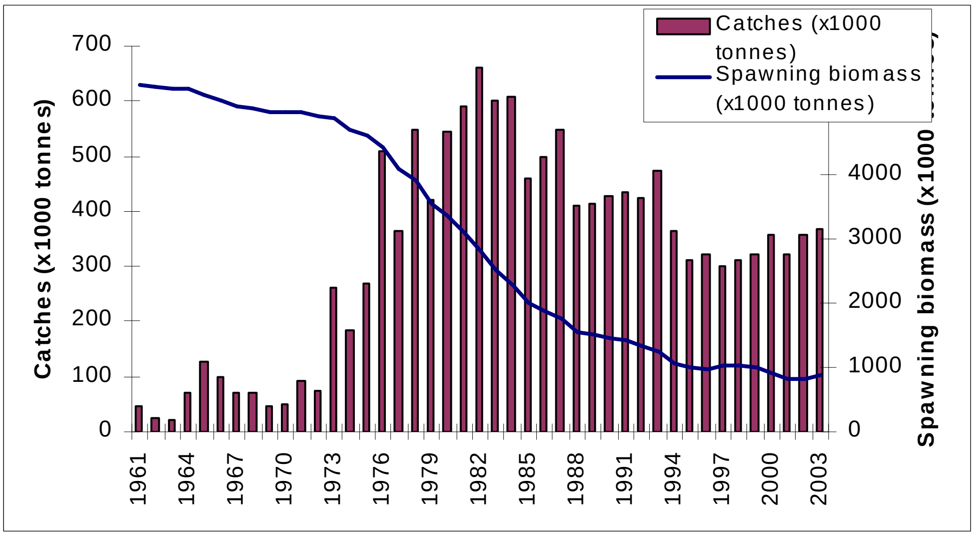
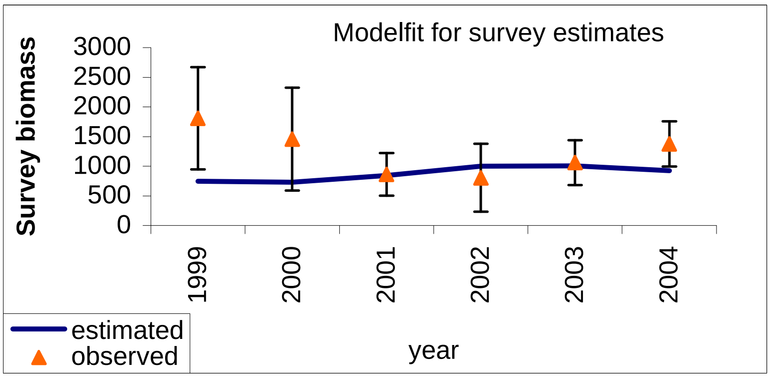


Figure 3: *The estimated decline of the spawning biomass with the catches taken by the pelagic and midwater fishing fleet from 1972 to 2003.*

Figure 4 and 5 show the model fit of the estimated abundance trends (CPUE and acoustic biomass) and the estimated average proportions of catch-at-age to the observed values, respectively. The estimated CPUE trend follows the observed trend well, but the model could not reproduce the observed acoustic biomass trend closely.



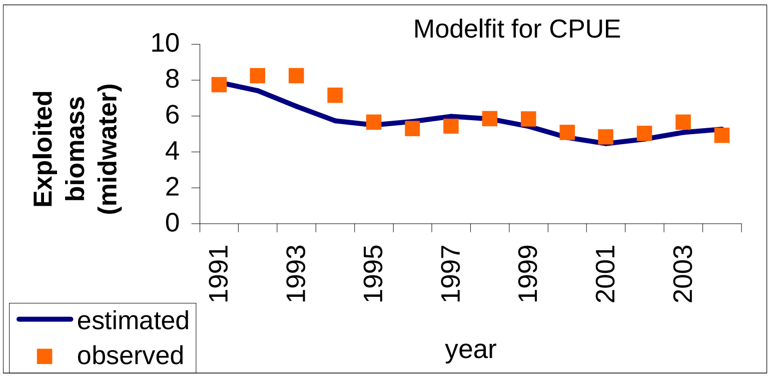


Figure 4: *Estimated acoustic biomass and CPUE trend fitted to the observed values.*

The estimated catch-at-age arrays are fitted reasonably for the commercial fleet (Figure 5), with the exception of the slight overestimation of the one year-old fish and a slight underestimation of the two-year old fish caught by the midwater fleet. However, the model could not estimate the average proportion of catch-at-age for the survey well. The zero year old fish are overestimated for the survey, whereas the two-year old fish are slightly underestimated.

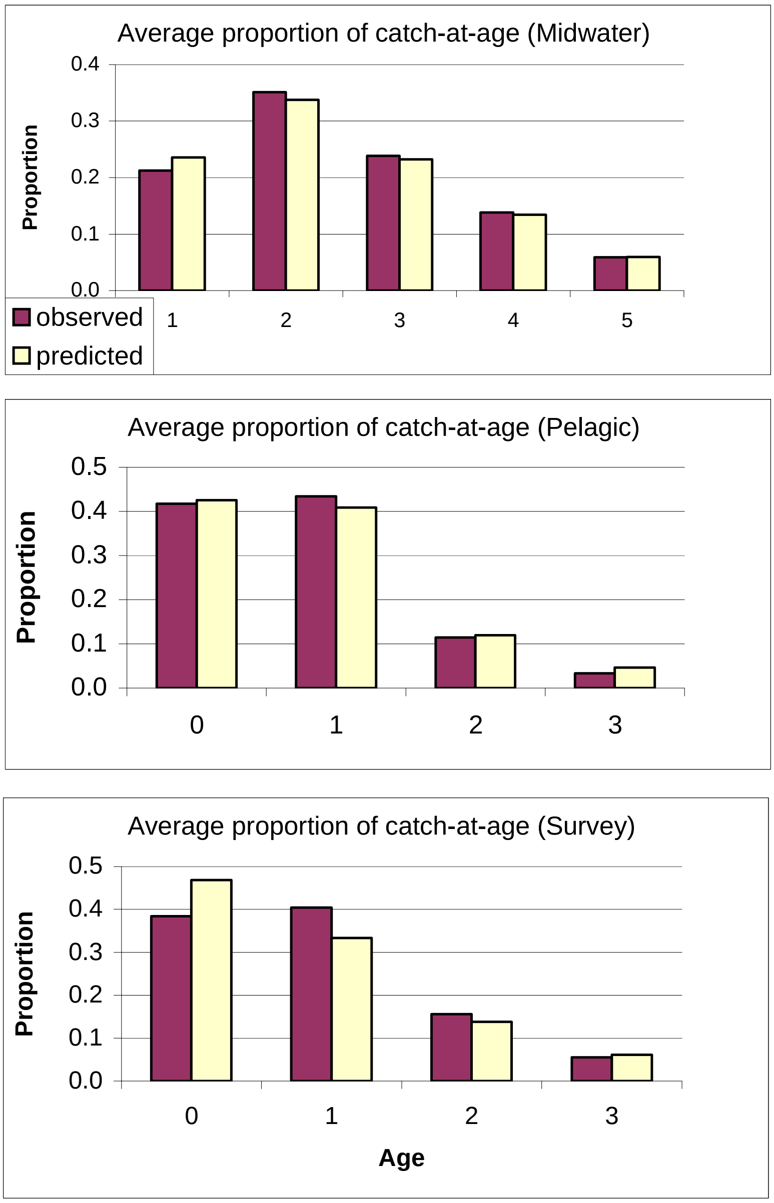


Figure 5: *Average estimated and observed proportion of fish caught for the different fleets as well as the surveys.*

Figure 6 shows the residuals of annual individual proportions of catch-at-age. Although no clear patterns can be identified, some residuals in the pelagic and survey data appear to be high.

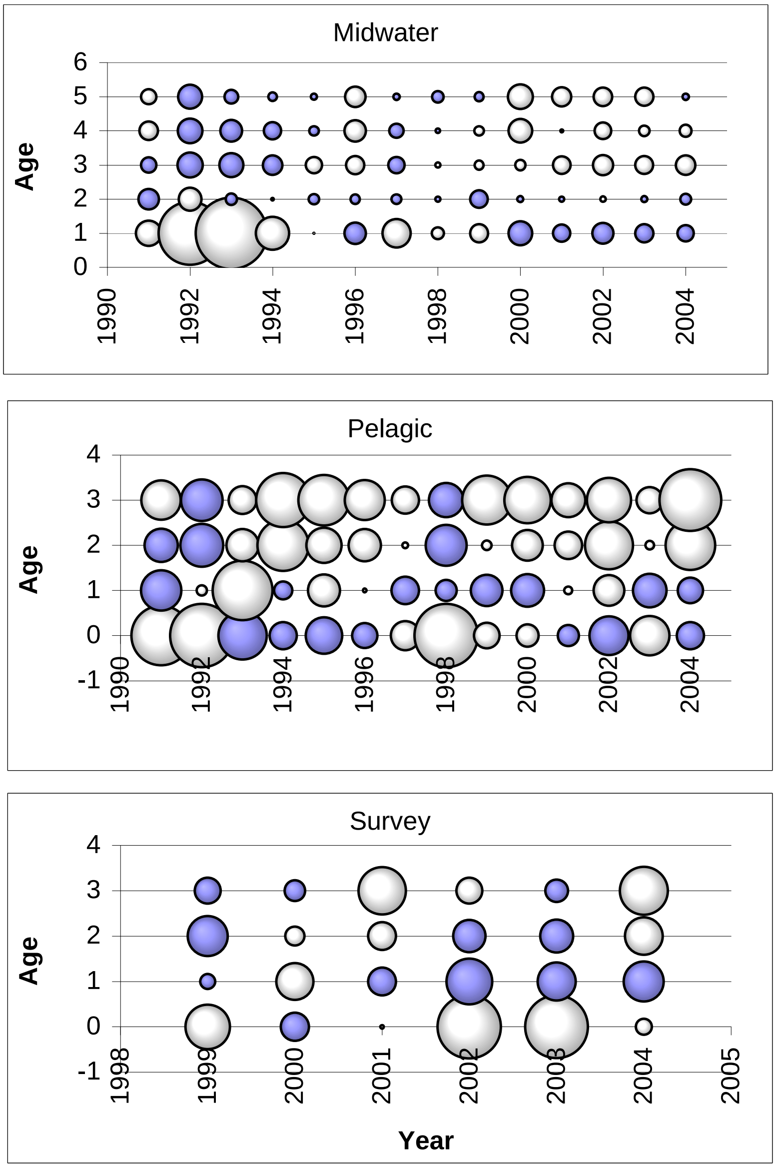


Figure 6: *Residuals of annual by age estimated and observed proportion of fish caught for the different fleets as well as the surveys.*

The observed catch-at-age matrices are used to estimate the various selectivities (Figure 7), which are then used for the projections. The purse-seine fleet does not catch the larger sized fish, as is reflected in the selectivities. It is evident from Figure 5 that the survey underestimates the proportion of the older fish, which is caught by the midwater fleet, and this is reflected in the estimated selectivity pattern.

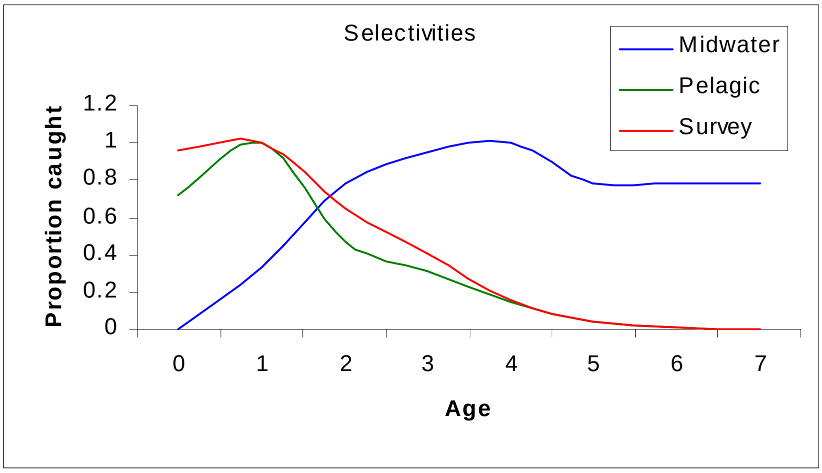


Figure 7: *Estimated selectivities used for further predictions*

Recruitment residuals have been estimated in the models for the years 1991-2004 and these are represented in Figure 8.

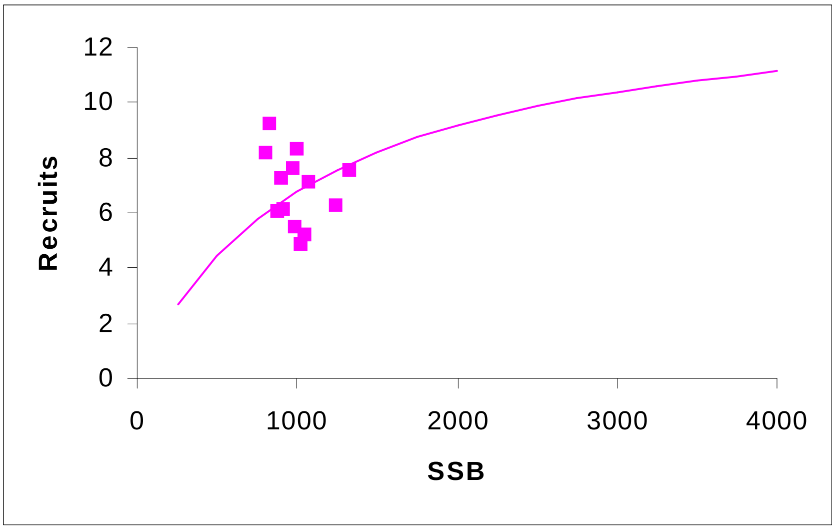


Figure 8: *Estimated recruitment function with the recruitment residuals*

A matrix (Table 2) is shown for all the different combinations of catchability, natural mortality and steepness parameters. The model obtains its best fit by decreasing the observed acoustic biomass by about half. This suggests that the biomass is overestimated by 50%-100% (q=1.5 – 2.0). The observed acoustic biomass series is very short and not much information can be deduced from this. The assessment results do not change extensively, irrespective of whether q = 1.5 or 2 is used in the model (Table 2). The model is sensitive to natural mortality (*M*) and the steepness parameter (*h*). The productivity of the stock depends on both these parameters. A high *M* and *h* would indicate a fairly productive stock. Horse mackerel would be classified as a medium productive stock, and therefore intermediate values for these parameters would be expected from the results. The best fit to the model is obtained when *h* is as low as 0.4 (Table 2) and *M* is 0.3 per year. However, this fit produced unrealistic selectivity curves and was therefore not chosen as a base case.

Table 2: *Stock assessment results using various combinations of the parameters M, h and q-survey. Maximum sustainable yield (MSY) for the midwater fleet, current depletion, current biomass in thousand tonnes and the negative log-likelihood are tabulated. The highlighted case has been used as a base case.*

q=2 M=0.4	MSY	Bsp2005/ Bsp1961	B2005	-lnL
h=0.4	304	0.21	1523	-76.96
h=0.6	341	0.17	951	-76.98
h=0.8	657	0.54	3324	-69.6
q=2 M=0.3				
h=0.4	121	0.23	2046	-78.12
h=0.6	331	0.16	1099	-78.46
h=0.8	348	0.14	786	-78.75
q=2 M=0.5				
h=0.4	314	0.21	1289	-75.59
h=0.6	325	0.23	1127	-71.82
h=0.8	639	0.57	2973	-67.7
q=1.5 M=0.4				
h=0.4	306	0.23	1660	-76.82
h=0.6	341	0.18	970	-76.33
h=0.8	721	0.6	4103	-70.9
q=1.5 M=0.3				
h=0.4	132	0.25	2285	-77.55
h=0.6	332	0.17	1128	-77.33
h=0.8	349	0.14	791	-77.66
q=1.5 M=0.5				
h=0.4	315	0.22	1383	-75.83
h=0.6	325	0.24	1174	-71.67
h=0.8	695	0.62	3444	-67.51
q=1 M=0.4				
h=0.4	324	0.26	1925	-75.52
h=0.6	342	0.18	973	-74.36
h=0.8	847	0.69	5677	-72.18
q=1 M=0.3				
h=0.4	135	0.31	2923	-75.73
h=0.6	713	0.63	7330	-73.13
h=0.8	350	0.14	786	-75.27
q=1 M=0.5				
h=0.4	316	0.24	1529	-75.05
h=0.6	326	0.26	1247	-70.3
h=0.8	804	0.69	4461	-71.29

Further, in Figure 9, the depletion values for 2005, the biomass for 2005 and the MSY are plotted versus the log-likelihood values for the different combination of the parameter values. In the absence of more data and more informed knowledge about natural mortality, steepness and catchability, this illustration is used to describe the state of the stock. It is shown that, for all the combinations of M, h and q values considered, the depletion value is low (between 0.14 – 0.31 per year) for the highest log-likelihood. This means that the best fit of the model to the available data, clearly shows a fairly depleted stock, irrespective of the combinations of the parameter values used in the assessment. Further it is shown that for the highest log likelihood the current biomass is probably less than a million tonnes. For these assessments the MSY could possibly be between 300 to 350 thousand tonnes.

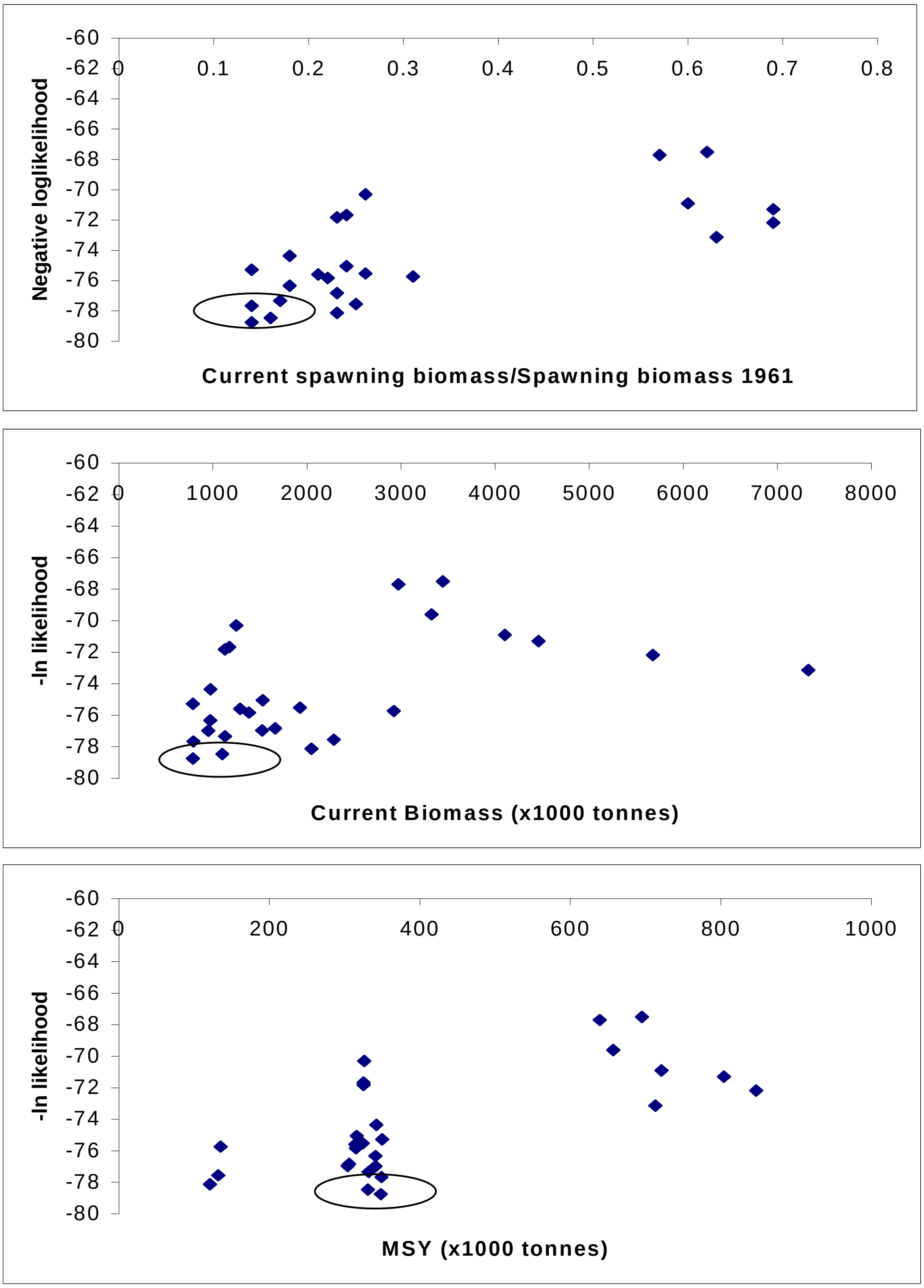


Figure 8: *Depletion (2005) levels are plotted versus log-likelihood values for all cases considered in Table 2. The circled values are the parameter combinations that produced the best fit to the available data.*

Two management quantities were estimated with 90 percentiles using the posterior distributions of the mcmc function in ADMB. Natural mortality and steepness parameter was fixed at 0.3 and 0.6 respectively. The current total biomass was estimated to be between 0.63 and 1.2 million tonnes, with the best estimate being 0.86 million tonnes. The depletion level was best estimated at 16%, but the 90 percentiles were between 12% and 20% (Table 3).

Table 3: *Estimated management quantities with their respective 90 percentiles.*

(M = 0.3, h = 0.6)		
Management quantities	**Best estimate**	**90 percentiles**
Current biomass (B_{cur})	856	632 - 1157
Current depletion level	16	12 – 20%

The current fishing mortality is estimated at 0.29 per year for the midwater fleet and 0.07 per year for the pelagic fleet. The fishing mortality at MSY was estimated to be 0.16 per year for the midwater fleet. According to these results, the midwater fishing mortality is currently too high. The maximum sustainable yield level (MSYL) was estimated at approximately 35%, which is much higher than the current level of around 16%. Once the MSYL of 35% is reached the stock can support annual catches of approximately 350 000 tonnes for the midwater and pelagic horse mackerel fishery. Even though the confidence intervals are wide, the indications are that the stock is currently below its MSYL.

Discussion

It is recommended that a management plan is considered to rebuild the stock to at least the maximum sustainable "fishing down" level of about 35 to 40%. No CPUE or survey data before 1990 has been taken into account, only catches. A serious attempt should be made to read the available otholiths from 1991-2004, as at the moment only one age-length key is used and this is not a recommended way of assessing a stock.

Appendix 1: Input data

Table I: Midwater and pelagic horse mackerel catches, 1961 – 2002, Namibia

Year	Midwater	Pelagic	Total	TAC
1961	47	0	47	
1962	23	0	23	
1963	21	0	21	
1964	71	0	71	
1965	126	0	126	
1966	100	0	100	
1967	72	0	72	
1968	69	0	69	
1969	47	0	47	
1970	51	0	51	
1971	77	14	91	
1972	51	22	73	
1973	250	12	262	
1974	154	31	185	
1975	255	14	269	
1976	484	24	508	
1977	281	82	363	
1978	538	10	548	
1979	388	33	421	
1980	507	39	546	500
1981	586	4	590	500
1982	592	68	660	500
1983	493	107	600	641
1984	519	88	607	630
1985	438	22	460	630
1986	416	84	500	485
1987	514	34	548	440
1988	393	17	410	472
1989	381	32	413	497
1990	342	85	427	410
1991	351	83	434	400
1992	310	116	426	450

1993	401	74	475	450
1994	331	33	364	500
1995	259	51	310	400
1996	229	91	320	400
1997	212	88	300	350
1998	286	25	311	375
1999	294	27	321	375
2000	336	21	357	410
2001	299	23	322	350
2002	297	61	356#	350
2003	308	52	369#	350
2004	320*	41		

*****_This is an estimate_

#These include bycatch and experimental fisheries catch

Table II: *Proportions of catch-at-age for the midwater fleet*

	Age							
Year	0	1	2	3	4	5	6	7
1991	0	0.135	0.399	0.288	0.124	0.041	0.008	0.004
1992	0	0.017	0.234	0.353	0.252	0.107	0.027	0.01
1993	0	0.009	0.338	0.338	0.219	0.077	0.013	0.006
1994	0.004	0.105	0.343	0.293	0.18	0.059	0.012	0.004
1995	0.006	0.257	0.343	0.195	0.137	0.042	0.015	0.004
1996	0.013	0.348	0.363	0.165	0.083	0.022	0.005	0.002
1997	0.001	0.141	0.382	0.277	0.142	0.046	0.007	0.004
1998	0.004	0.141	0.35	0.276	0.157	0.057	0.009	0.005
1999	0.01	0.156	0.324	0.257	0.17	0.062	0.014	0.006
2000	0.046	0.284	0.333	0.184	0.109	0.034	0.006	0.003
2001	0.02	0.334	0.335	0.17	0.101	0.031	0.006	0.003
2002	0.018	0.389	0.365	0.138	0.071	0.015	0.003	0.001
2003	0.003	0.301	0.416	0.18	0.08	0.016	0.003	0.001
2004	0.002	0.233	0.389	0.226	0.109	0.033	0.004	0.003

Table III: *Proportion for catch-at-age for the pelagic fleet*

	Age							
Year	0	1	2	3	4	5	6	7
1991	0.184	0.596	0.2	0.001	0.018	0	0	0
1992	0.169	0.397	0.271	0.109	0.048	0.006	0	0
1993	0.708	0.187	0.076	0.019	0.01	0	0	0
1994	0.549	0.41	0.036	0.002	0.003	0	0	0
1995	0.639	0.304	0.052	0	0.005	0	0	0
1996	0.486	0.435	0.068	0.002	0.008	0	0	0
1997	0.265	0.558	0.145	0.013	0.019	0	0	0
1998	0.165	0.401	0.295	0.078	0.05	0.008	0.002	0
1999	0.371	0.515	0.101	0	0.013	0	0	0
2000	0.444	0.483	0.067	0	0.006	0	0	0
2001	0.551	0.38	0.058	0.006	0.005	0	0	0
2002	0.59	0.369	0.036	0.002	0.003	0	0	0
2003	0.239	0.588	0.145	0.007	0.02	0	0	0
2004	0.487	0.453	0.055	0	0.006	0	0	0

Table IV: *Proportions of catch-at-age of survey samples*

	Age							
Year	0	1	2	3	4	5	6	7
1994	0.613	0.293	0.068	0.016	0.01	0	0	0
1995	0.553	0.274	0.1	0.032	0.031	0.008	0.002	0.001
1996	0.69	0.19	0.097	0.011	0.01	0.002	0.001	0
1997	0.25	0.428	0.227	0.054	0.036	0.004	0	0
1998	0.66	0.172	0.087	0.042	0.027	0.009	0.002	0.001
1999	0.333	0.343	0.21	0.061	0.038	0.009	0.002	0.001
2000	0.601	0.221	0.109	0.042	0.02	0.005	0.001	0
2001	0.54	0.37	0.077	0.002	0.01	0	0	0
2002	0.226	0.564	0.181	0.011	0.017	0	0	0
2003	0.188	0.491	0.243	0.048	0.028	0.001	0	0
2004	0.416	0.435	0.116	0.016	0.016	0.001	0	0

Table V: *Proportion mature fish-at-age based on J. Krakstad (pers. commn)*

Age	Proportion mature
0	0.001
1	0.093
2	0.799
3	0.989
4	0.999
5	1
6	1
7	1

Table VI: *Commercial catch-per-unit effort data*

Year	**CPUE (tonnes/hour)**
1990	6.05
1991	7.75
1992	8.24
1993	8.24
1994	7.16
1995	5.65
1996	5.30
1997	5.43
1998	5.86
1999	5.83
2000	5.08
2001	4.83
2002	5.02
2003	5.65
2004	4.83

Table VII: *Acoustic survey biomass estimates (with CV) is given in 1000 tonnes.*

Year	estimate	CV
1999	1810	0.24
2000	1457	0.3
2001	863	0.21
2002	805	0.36
2003	1061	0.18
2004	1377	0.14

Appendix 2: Model used for the assessment

Introduction

This model makes use of all the available data on Namibian horse mackerel and tries to reflect the history of the stock since its exploitation. It should, therefore, be noted that the results of the model are consequences of these available input data. Resource projections are based on the values of the parameters that are estimated by fitting the model to the data.

The model used is called an age-structured production model. With this model the year prior to exploitation is the starting point and assumes a population age structure corresponding to deterministic unexploited equilibrium. From this initial year, the resource is projected forward by allowing for natural mortality and removing catches off the various age classes. The final output of the model is a matrix of numbers of fish for each age for each year. The matrix is changed continuously, by varying the values of certain parameters (the estimable parameters), until the smallest difference between the estimated and observed values (data collected) is obtained. In this assessment, the estimable parameters were the spawning biomass before exploitation (B_0^{sp}), the constant of proportionality (*q*) for both indices of abundance (CPUE and acoustic biomass), the selectivities of both fleets involved as well as the survey.

The age-structured production model (ASPM)

The resource dynamics of the Namibian Horse mackerel are modelled by a deterministic age-structured model with the following set of population dynamics equations:

Dynamics

$$N_{y+1,0} = R_{y+1} \tag{A.1}$$

$$N_{y+1,a+1} = \left(N_{y,a}\, e^{-M/2} - \sum C^f_{y,a} \right) e^{-M/2} \quad \text{for } 0 \le a < m-2 \tag{A.2}$$

$$N_{y+1,m} = \left(N_{y,m-1}\, e^{-M/2} - \sum_f C^f_{y,m-1} \right) e^{-M/2} + \left(N_{y,m} e^{-M/2} - \sum C^f_{y,m} \right) e^{-M/2} \tag{A.3}$$

where $N_{y,a}$ is the number of fish of age *a* at the start of year *y*,

R_y is the recruitment in year *y*,

M denotes constant natural mortality rate on fish of all ages,

$C^f_{y,a}$ is the number of fish of age *a* caught in year *y* by fleet *f*, and

m is the maximum age considered (taken to be a plus-group).

These equations reflect Pope's approximation to the more customary Baranov catch equations.

Total catch and catches-at-age

The number of fish of age a caught in year y is given by:

$$C^f_{y,a} = N_{y,a} \cdot e^{-M/2} \cdot S^f_a \cdot F^f_y \qquad \text{(A.4)}$$

Where

S^f_a is the fleet-disaggregated age-specific commercial selectivity (assumed to be constant over the years),

F^f_y is the fleet-disaggregated fully selected fishing mortality in year y, given by:

$$F_y = \frac{Y^f_y}{\sum_{a=0}^{m} N_{y,a} \cdot e^{-M/2} \cdot S^f_a \cdot w_{a+1/2}} \qquad \text{(A.5)}$$

where

Y^m_y is the total fleet-disaggregated observed catch (yield) by mass in year y, and

$w_{a+1/2}$ is the mid-year mass of a fish of age a+1/2.

The estimated fleet-disaggregated catch (yield) by mass in year *y* is given by:

$$C_y^f = \sum_{a=0}^{m} w_{a+1/2} N_{y,a} e^{-M/2} S_{y,a}^f F_y^f \tag{A.6}$$

The model estimate of the mid-year exploitable ("available") component of biomass for each fleet is calculated by converting the numbers-at-age into mid-year masses-at-age (using the midyear individual weights) and applying natural mortality and fishing mortality for half the year.

$$B_y^f = (\sum_{a=0}^{m} w_{a+1/2} S_a^f N_{y,a} e^{-M/2}) \tag{A.7}$$

whereas the survey estimates of biomass at the start of the year (summer):

$$B_y^{sur} = \sum w_a S_a^{surv} N_{y,a} \tag{A.8}$$

where S_a^{surv} is the survey selectivity.

Spawner-biomass recruitment relationship

The number of recruits at the start of year *y* is related deterministically to the spawning stock size by the Beverton-Holt stock-recruitment relationship:

$$R_y = \frac{\alpha B_y^{sp}}{\beta + B_y^{sp}} \tag{A.9}$$

where

α and β are spawning biomass-recruitment relationship parameters, and

B_y^{sp} is the spawning biomass at the start of year *y*,

given by:

$$B_y^{sp} = \sum_{a=0}^{m} p_a w_a N_{y,a} \tag{A.10}$$

where w_a is the begin-year mass of fish of age *a* and p_a is the proportion of fish of age *a* that are mature.

In order to work with estimable parameters that are more meaningful biologically, the stock-recruitment relationship is re-parametrised in terms of the pre-exploitation equilibrium spawning biomass, K^{sp}, and the "steepness", *h*, of the stock-recruitment relationship, where "steepness" is the fraction of pristine recruitment that results when spawning biomass drops to 20% of its pristine level, i.e.

$$h \cdot R_0 = R(0.2 \cdot B_0^{sp}) \tag{A.11}$$

from which follows that:

$$h = \frac{0.2 \cdot \left[\beta + B_0^{sp}\right]}{\left[\beta + 0.2 \cdot B_0^{sp}\right]} \tag{A.12}$$

and hence:

$$\alpha = \frac{5hR_0}{5h-1} \tag{A.13}$$

and:

$$\beta = \frac{K^{sp}(1-h)}{5h-1} \tag{A.14}$$

Biomass trajectories

Given a value for the pre-exploitation spawning biomass B_0^{sp} of Namibian Horse mackerel, together with the assumption of an initial equilibrium age structure, we have:

$$R_0 = \frac{B_0^{sp}}{\left(\sum_{a=0}^{m-1} p_a \cdot w_a \cdot e^{-M \cdot a}\right) + p_m \cdot w_m \cdot e^{-M \cdot m} \Big/ \left(1 - e^{-M}\right)} \tag{A.13}$$

The initial numbers at age for the projections, corresponding to the deterministic equilibrium, are:

$$N_{0,a} = R_0 \cdot e^{-M \cdot a} \qquad 0 \le a \le m-1 \tag{A.14}$$

$$N_{0,m} = R_0 \cdot e^{-M \cdot m} / \left(1 - e^{-M}\right) \qquad \text{(A.15)}$$

The likelihood function

The model is fitted to CPUE and survey abundance data, commercial and survey catch-at-age data, as well as stock-recruitment residuals to estimate model parameters. Contributions by each of these to the negative of the log-likelihood ($-\ell n\ L$) are as follows.

CPUE Abundance data

The likelihood is calculated assuming that the observed abundance index is log-normally distributed about its expected value:

$$I_y^i = \hat{I}_y^i \exp\left(\varepsilon_y^i\right) \quad \text{or} \quad \varepsilon_y^i = \ell n\left(I_y^i\right) - \ell n\left(\hat{I}_y^i\right) \qquad \text{(A.16)}$$

where I_y^i is the abundance index for year *y* and series *i*,

$\hat{I}_y^i = \hat{q}^i \hat{B}_y^i$ is the corresponding model estimate, where B_y^i is the model estimate of biomass, given by equation A.7,

$\hat{q}^i$ is the constant of proportionality for abundance series *i* (effectively the multiplicative bias if the series reflects abundance in absolute terms, as for the surveys discussed below), and

ε_y^i from $N\left(0, \left(\sigma_y^i\right)^2\right)$.

The contribution of the abundance data to the negative of the log-likelihood function (after removal of constants) is given by:

$$-\ell n\, L = \sum_i \left[\sum_y \ell n\, \sigma_y^i + \left(\varepsilon_y^i\right)^2 / 2\left(\sigma_y^i\right)^2 \right] \qquad \text{(A.17)}$$

Homoscedasticity of residuals is assumed, so that $\sigma_y^i = \sigma^i$, the standard deviation of the residuals for the logarithms of abundance index *i* is estimated in the fitting procedure by its maximum likelihood value:

$$\hat{\sigma}^i = \sqrt{1/n^i \sum_y \left(\ell n\, I_y^i - \ell n\, q^i B_y^i\right)^2} \qquad \text{(A.18)}$$

where n^i is the number of data points for abundance series *i*.

q^i is estimated by its maximum likelihood value:

$$\ell n\, \hat{q}^i = 1/n^i \sum_y \left(\ell n\, I_y^i - \ell n\, \hat{B}_y^i\right) \qquad \text{(A.19)}$$

Survey abundance data

For the surveys, an estimate of the sampling variance is available for each survey. The associated σ_y is taken to be given by the corresponding survey CV (A20) or it is estimated using equation A21.

$$\left(\sigma_y\right)^2 = ln(1 + \left(CV\right)_y{}^2) \qquad \text{(A.20)}$$

$$\hat{\sigma} = \sqrt{1/n \sum_y \left(\ell n\, I_y^{sur} - \ell n\, q\;\; B_y^{sur}\right)^2} \qquad \text{(A.21)}$$

$$-lnL = \sum_y \left[\left(\varepsilon_y\right)^2 \big/ 2\left(\sigma_y\right)^2 + ln\sigma_y\right] \qquad \text{(A.22)}$$

where:

CV_y is the coefficient of variation of the survey estimate for year *y*

σ_y is the (sampling) standard error of the estimate for the survey in year *y*

$$\varepsilon_y = \ln\left(I_y^s\right) \text{-} \ln(q\;\; \hat{B}_y^{sur}) \qquad \text{(A23)}$$

for log-normally distributed errors, where:

I_y^{sur} is the observed survey estimate for year *y*

B_y^{sur} is the estimated survey biomass, and

q is the multiplicative bias given as input

Survey catches-at-age

The contribution of the survey catch-at-age data to the log-likelihood function when assuming an “adjusted” log-normal error distribution is given by:

$$-\ln L = \sum_y \sum_a \left[\ln\left(\sigma_{sur} / \sqrt{\hat{p}_{y,a}}\right) + \hat{p}_{y,a}\left(\ln p_{y,a} - \ln \hat{p}_{y,a}\right)^2 / 2\left(\sigma_{sur}\right)^2 \right] \qquad \text{(A.24)}$$

where

$p_{y,a} = C_{y,a}^{sur} / \sum_{a'=0}^{m} C_{y,a'}^{sur}$ is the observed proportion of fish of age a from the survey in year *y*

$\hat{p}_{y,a}$ is the expected proportion of fish of age *a* in year *y*, given by:

$$\hat{p}_{y,a} = \frac{S_a^{sur} N_{y,a}}{\sum_{a'=0}^{m} S_a^{sur} N_{y,a}} \qquad \text{(A.25)}$$

σ_{sur} is the standard deviation associated with the catch-at-age data for the survey, which is estimated in the fitting procedure by:

$$\sigma_{sur} = \sqrt{\sum_y \sum_a \hat{p}_{y,a}\left(\ln \hat{p}_{y,a} - \ln \hat{p}_{y,a}\right)^2 / \sum_y \sum_a 1} \qquad \text{(A.26)}$$

Commercial catch-at-age

The contribution of the midwater and pelagic fleet catch-at-age data to the log-likelihood function when assuming an “adjusted” log-normal error distribution is given by:

$$-\ln L = \sum_f \sum_y \sum_a \left[\ln\left(\sigma_{com}^f / \sqrt{\hat{p}_{y,a}^f}\right) + \hat{p}_{y,a}^f\left(\ln p_{y,a}^f - \ln \hat{p}_{y,a}^f\right)^2 / 2\left(\sigma_{com}^f\right)^2 \right] \qquad \text{(A.27)}$$

where

$p_{y,a}^{f} = C_{y,a}^{f} / \sum_{a'=0}^{m} C_{y,a'}^{f}$ is the observed proportion of fish of age a from the commercial fleets in year *y*

$\hat{p}_{y,a}^{f}$ is the expected proportion of fish for each commercial fleet of age *a* in year *y*, given by:

$$\hat{p}_{y,a}^{f} = \frac{S_a^f N_{y,a}}{\sum_{a'=0}^{m} S_a^f N_{y,a}} \qquad \text{(A.28)}$$

σ_{com}^{f} is the standard deviation associated with the catch-at-age data for the different commercial fleets, which is estimated in the fitting procedure by:

$$\sigma_{com}^{f} = \sqrt{\sum_y \sum_a \hat{p}_{y,a}^{f} \left(\ln \hat{p}_{y,a}^{f} - \ln \hat{p}_{y,a}^{f}\right)^2 / \sum_y \sum_a 1} \qquad \text{(A.29)}$$

The log-normal error distribution underlying equation A.24 and A.27 is chosen on the grounds that (assuming no ageing error) variability is likely dominated by a combination of interannual variation in the distribution of fishing effort, and fluctuations (partly as a consequence of such variations) in selectivity-at-age, which suggests that the assumption of a constant coefficient of variation is appropriate. However, for ages poorly represented in the sample, sampling variability considerations must at some stage start to dominate the variance. To take this into account in a simple manner, motivated by binomial distribution properties, Punt (pers. commn) advocates weighting by the expected proportions (as in equation A.24 and A.27) so that undue importance is not attached to data based upon a few samples only.

Survey and commercial fleet catch-at-age are incorporated in the likelihood function using equations A.24 and A.27, for which the summation over age a is taken from age a_{minus} (considered as a minus group) to a_{plus} (a plus group). The ages for the minus –and plus-groups are chosen so that few fish (approximately less than 1% of the total sampled) fall outside this age range.

Stock-recruitment function residuals

The stock-recruitment residuals are assumed to be log-normally distributed and serially correlated. Thus, the contribution of the recruitment residuals to the negative of the log-likelihood function is given by:

$$-\ln L = \sum_{y=y_1+1}^{y} \left[\ln\sigma_R + \left(\frac{\varsigma_y - \rho\varsigma_{y-1}}{\sqrt{1-\rho^2}} \right)^2 \Big/ 2\sigma_R^2 \right] \tag{A.30}$$

where

$\varsigma_y = \rho\varsigma_{y-1} + \sqrt{1-\rho^2}\,\varepsilon_y$ is the recruitment residual for year *y*, which is estimated for years y_1 to y_2 (see equation A9).

ε_y from $N\left(0,\left(\sigma_y^i\right)^2\right)$

σ_R is the standard deviation of the log-residuals, which is input, and

ρ is the serial correlation coefficient, which is input.

In the interest of simplicity, equation A30 omits a term in ς_{y_1} for the case when serial correlation is assumed ($\rho \neq 0$), which is generally of little quantitative consequence to values estimated.

Fishing selectivity-at-age

The fishing selectivity-at-age, S_a^f, is estimated directly:

$$S_a^f = \begin{Bmatrix} \text{estimated separately for } a \le 1 \\ =1 \quad \text{for } a > 1 \end{Bmatrix}$$

Both the survey and commercial selectivities can be modified for $a > a_{slope}$ by:

$$S_a^f \rightarrow S_a^f \exp\left(-s^f\left(a - a_{slope}\right)\right)$$

where

s^f is called ‘slope’ and measures the rate of decrease in selectivity with age for fish older than a_{slope} for the fleet concerned.

Two values have to be set for the selectivities to be estimated: The age where selectivity is assumed to be one and the age where selectivity is starting to decrease at older ages: The following values where applied in this assessment

	Age at which selectivity is 1	Age at which selectivity starts to decline
Midwater	2	7
Pelagic	0	2
Survey	1	2