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The South African horse mackerel assessment using an age-structured production model, with future biomass projections

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1. Introduction

The South African horse mackerel (*Trachurus trachurus capensis*) fishery began in 1950. It currently consists of a demersal/midwater trawl fleet (concentrated on the South coast) and a pelagic purse-seine fishery (concentrated on the West Coast). Adult horse mackerel are taken as a by-catch by the demersal trawl fleet and as a targeted catch by the mid-water trawl fleet. Juvenile horse mackerel are taken as a by-catch by the pelagic purse-seine fleet.

Previous stock assessments for this fishery include a surplus production model (Punt 1989, 1992), and a Beverton-Holt yield-per-recruit approach (Butterworth and Raubenheimer 1992; Butterworth and Clarke 1996).

For convenience, the rest of this paper uses "demersal" to imply both midwater and demersal operations.

2. Methods

An age structured production model (ASPM) is used to model the South African horse mackerel resource. The model assumes one combined stock (West Coast plus South Coast). This model has been applied previously by Horsten (1999a, 1999b) and OLRAC (2001) for assessments of this resource. The work presented here does however incorporate updated catch and survey biomass data which previous assessments have not had available to them. The age-structured production model is described in full in the Appendix, along with the details of the likelihood function used for fitting the model to the data.

The model is deterministic and fits only one parameter, K^{sp}. Both h (the steepness parameter of the stock-recruit curve) and q_2 (the catchability coefficient corresponding to survey 2) are parameters set externally. Two values of h are considered (0.6 and 0.9) and two values of q_2 are considered (0.5 and 1.0). These provide for four possible combinations of h and q_2.

The reason for fixing values of steepness h externally is that, as well become evident from the results below, the available data do not possess the information content to clearly distinguish widely different values for h. The horse mackerel swept areas surveys are known to provide negatively biased estimates of abundance in absolute terms, but the extent of this bias in unknown. Results are presented for externally fixed values of q_2 because, again, the data do not have much power to distinguish these values.

The model assumes the population is at an unexploited equilibrium in 1950.

3. Input Data and Model assumptions

a) Historic catch

The historic catch record for both the demersal (strictly demersal + midwater) and pelagic fisheries for 1950-2002 are reported in Table 1. BEN/DEC04/HM/SA/1b provides a more detailed breakdown of the historic catch.

b) Survey biomass estimates

The survey biomass estimates (demersal swept area surveys) and their associated CVs are reported in Table 2. For the Spring survey (Survey 1 – on South Coast), data for 1987, 1989-2000 are available. For the Autumn survey (Survey 2 – estimates from South and West coasts added), data for 1987-2000 are available. BEN/DEC04/HM/SA/1b provides further details of these survey estimates.

c) Natural Mortality

Natural mortality is assumed to constant for all ages. The base case value used here for $M = 0.3$.

Previous South African horse mackerel assessments (Punt and Leslie (1989), Butterworth *et al.* (1990) – for Namibian stock, Punt (1990), Butterworth and Raubenheimer (1992), Horsten 1999b, and Kinloch *et al*. (1986)) have used a value of M of 0.4 as a matter of convention. Kinloch *et al*. (1986) quote Pauly (1980) for the derivation of $M = 0.4$, following his relationship between natural mortality, growth rate, asymptotic length and average sea temperatures.

Horsten 1999a used three values of M (0.2, 0.3 and 0.4) in an age-structured production model for horse mackerel. Horsten 1997 explored the sensitivity of the Butterworth and Clarke (1996) model to different values of natural mortality, and concluded that that model output was very sensitive to the value of M and that it would be very valuable to obtain a more reliable value for M. Horsten (1999c) goes on to report sensitivity of an ASPM for horse mackerel to values of M, and concludes that the ASPM model appears less sensitivity to the natural mortality assumption, and that changing the value of M had little relative effect on the negative log likelihood.

Here, the choice of the base case $M = 0.3$ is somewhat arbitrary, although sensitivity to alternate assumptions regarding M are reported.

d) Selectivity

Selectivity at age values used (from Horsten 1999a, b) are reported in Table 3. Note that there are three selectivity vectors for the pelagic fishery associated with three different periods. Essentially there is a different selectivity function for the pre-1963 period and a different selectivity function for the 1968+ period, with the average of these two selectivity functions used for the period in between (1963-1967). The reason for this change in selectivity is due to the change in fishing gear that occurred in the pelagic fishery. In 1968, anchovy gill nets were widely introduced to the purse-seine industry. These nets had 11mm wide mesh, compared to the previous 32mm nets. This led to the horse mackerel pelagic fleet targeting much smaller horse mackerel (generally ages 0-2), as opposed to the earlier years when juveniles were mostly avoided, and older fish aged 2-6 years were caught.

To quantify this change in pelagic selectivity, length distributions were collected spanning the history of the fishery. Van der Westhuizen (pers. Commn) provided the purse-seine size-frequencies at the time. At this time, length distributions for the demersal fishery (Punt and Leslie 1990) were also examined to produce a suitable demersal selectivity function. The selectivity curves were developed, based on the catch proportions-by-age extracted from the length frequency distributions, using Kerstan's 1999 (pers commn.) growth parameters.

e) Weight-at-age

The weight-at-age values are reported in Table 3 and are based upon a von Bertalanfy growth curve with parameters: $l_\infty = 54.56$ (cm), t_0 = -0.654 (yr), $\kappa = 0.183$ (yr^{-1}), and a weight-length relationship $w = 0.0078 l^{3.0}$ (g). BEN/DEC04/HM/SA/3a provides further information regarding these these functions.

f) Age at maturity

Age-at-maturity is assumed to be the age corresponding to 100% sexual maturity, which is assumed here to be described by a knife-edge function of age. For South African horse mackerel, the age-at-maturity is assumed to be 3 years (R.W. Leslie pers. Commn in Butterworth and Clarke 1996).

Note: Reliable CPUE data series for this fishery are not available. The main reason is that most horse mackerel are caught as a by-catch, making "effort" spent on catching horse mackerel very difficult to quantify. The Japanese fleet (which specifically targeted horse mackerel) was able to provide a consistent CPUE series during the 1980s, but this is for the 1976-1988 period only.

4. Model variants

Four assessment model variants corresponding to four combinations of the model parameters q_2 and h are considered. They are:

- Model 1: $q_2 = 1.0$; $h = 0.6$
- Model 2: $q_2 = 1.0$; $h = 0.9$
- Model 3: $q_2 = 0.5$; $h = 0.6$
- Model 4: $q_2 = 0.5$; $h = 0.9$

These four models are selected as they seem likely to contain the most probable q_2 and h value combinations of the original nine models explored in Johnston and Butterworth (2001). Note that q_2 is the bias of the survey estimates: a value of 0.50 for example, means that the biomass is actually twice as large as the survey estimates. The h parameter is some measure of the productivity of the resource: the higher the h, the more productive the resource is.

Sensitivity analyses
Sensitivity to assumptions regarding natural mortality are presented. The base case model assumes natural mortality is constant for all ages and is equal to 0.3. The following sensitivity analyses are reported for Model 3 (q_2 = 0.5; h = 0.6).

- M = 0.2
- M = 0.4
- M is age-dependent (M = 0.6 for a = 0; M = 0.5 for a = 1; M = 0.4 for a = 2; and M = 0.3 for a = 3+).

5. Output statistics

The following output statistics are reported.

K^{sp}	the spawning biomass level in 1950 (the estimable parameter)
q_1 , q_2	the catchability coefficients corresponding to the two survey series
h	the steepness parameter of the stock-recruit curve
-*lnL* total	the total –*lnL* value which is minimised
MSY	the demeral *MSY* (when assuming the pelagic catch is zero, for simplicity)
B_{MSY}	the spawning biomass level that will result in *MSY*
B(1950)	the demersal exploitable biomass (mid-year) for 1950
B(2001)	the demersal exploitable biomass (mid-year) for 2001
B_{MSY}/K^{sp}	the ratio of B_{MSY} to K^{sp}.

6. Projections

The model is used to project the resource biomass ahead for the period 2002-2020. A number of alternate future demersal and pelagic catch scenarios are considered as follows:

Future demersal catch scenarios

- 34000 MT for all future years (2002-2020)
- 44000 MT for 2006-2020, with a linear increase from 34000 MT in 2001 to 44000 MT in 2005
- 60000 MT for 2006-2020, with a linear increase from 34000 MT in 2001 to 44000 MT in 2005

(These options were considered because at the time computations were carried out, management's particular interest was in steadily increasing the demersal catch over a four year (2002-2005) period of allocated fishing rights.)

Future pelagic catch scenarios [for 2002-2020]

- 0 MT
- 5000 MT
- 10000 MT
- 15000 MT

7. Results

Table 4a reports the various model estimates for each of the four models considered. The *MSY* estimates reported correspond to the assumption that all catch is demersal. Table 4b compares results for Model 3 (q_2 = 0.5; h = 0.6) for different assumptions regarding natural mortality.

Tables 5a-d report the spawing biomass relative to K^{sp} values for the four assessment models considered. Results are presented for all combinations of the future demersal and pelagic scenarios considered.

Figures 1a and 1b illustrate the four assessment models' estimated spawning biomass relative to K^{sp} trends for 1950-2001. Figures 2a-c illustrate the projected spawning biomass relative to K^{sp} values for the different future catch scenarios.

Juvenile Biomass Estimates

Table 6 compares the assessment model estimated mid-year juvenile (ages 0-2) biomass values (MT) for the start of 2001, as well as the acoustic recruitment biomass (MT) survey estimate for 2001 (Coetzee *pers. commn.*).

Figure 3 compares the assessment model (h = 0.6, q_2 = 0.5) estimated mid-year juvenile (ages 0-2) biomass and results from acoustic recruitment biomass surveys (Coetzee *pers. commn*) for the period 1987-2001. Acoustic survey results are shown with $\pm$1se.

8. Discussion

Table 4a shows that the best of the four fits to the data is provided by Model 3 (q_2 = 0.5; h = 0.6). The MSY estimate for Model 3 is some 65 500 t. Model 3 estimates the 2002 exploitable biomass (some 675 000 t) to be 62% of carrying capacity. The B_{msy}/K^{sp} is estimated to be 0.35. Model 3 indicates that this resource is currently under-exploited. Only Model 1 (the most pessimistic model) estimates the 2002 exploitable biomass level to be below 50% *K*.

The model appears to be fairly robust to assumptions regarding natural mortality (Table 4b).

Examination of the projections reveal that models 1 and 2 (q_2 = 1.0) are clearly more pessimistic than models 3 and 4 (q_2 = 0.5). The option of increasing the demersal catch to 60 000 tons is clearly problematic for q_2 = 1.0, and also for q_2 = 0.5 for pelagic catches exceeding 5000 tons.

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Table 1: Demersal and pelagic horse mackerel catch (MT) – values for last two years shown are preliminary/estimated.

Year	Demersal	Pelagic
1950	129	49900
1951	200	98900
1952	117	102600
1953	49	85200
1954	72	118100
1955	193	78800
1956	328	45800
1957	190	84600
1958	237	56400
1959	439	17700
1960	429	62900
1961	453	38900
1962	554	66700
1963	521	23300
1964	8371	24400
1965	5829	55000
1966	6124	26300
1967	4893	8800
1968	8807	1400
1969	10870	26800
1970	14272	7900
1971	27242	2200
1972	18237	1300
1973	24708	1600
1974	29567	2500
1975	50611	1600
1976	39495	400
1977	93132	1900
1978	34001	3600
1979	45509	4300
1980	36330	400
1981	33880	6100
1982	30238	1100
1983	35522	2100
1984	33402	2800
1985	25589	700
1986	29528	500
1987	31736	2800
1988	31831	6300
1989	28147	25500
1990	44976	7134
1991	37301	548
1992	33714	1968
1993	20725	11646
1994	10064	8210
1995	7273	1991
1996	9261	18980
1997	22922	12700
1998	27942	26661
1999	20400	2050
2000	18430	4800
2001	26682	5000

Table 2: Survey biomass estimates (MT) for the spring (Survey 1) and autumn (Survey 2) biomass series.

Year	Survey 1	CV	Survey 2	CV
1987	308300	0.15	308816	0.15
1988	0	0	203625	0.23
1989	501100	0.23	510281	0.24
1990	579900	0.18	431275	0.19
1991	467000	0.24	518211	0.19
1992	320200	0.18	529152	0.19
1993	373500	0.23	422911	0.23
1994	279400	0.23	241648	0.28
1995	0	0	320342	0.71
1996	0	0	290338	0.24
1997	0	0	220849	0.24
1998	0	0	0	0
1999	0	0	327409	0.25
2000	0	0	321512	0.33

Table 3. Selectivity and weight-at-age vectors.

a	S_a^p 1950-1962	S_a^p 1963-1967	S_a^p 1968+	S_a^d 1950+	w_a (g)*
0	0.00	0.14	0.28	0.00	1.81
1	0.00	0.50	1.00	0.33	22.57
2	0.30	0.40	0.50	0.67	72.14
3	1.00	0.50	0.00	1.00	146.88
4	0.50	0.25	0.00	1.00	238.71
5	0.50	0.25	0.00	1.00	339.40
6	0.25	0.13	0.00	1.00	442.17
7	0	0.00	0.00	1.00	542.11
8	0	0.00	0.00	1.00	636.01
9	0	0.00	0.00	1.00	722.00
10+	0	0.00	0.00	1.00	799.27

Table 4a: Base Case horse mackerel stock assessment results when fitting to data in Tables 1 - 2. *B* refers to the mid-year exploitable biomass for the demersal fishery.

q_2	h	K^{sp}	q_1	-lnL total	MSY	Bmsy (sp)	B(1950)	B(2002)	$\frac{B(2002)}{B(1950)}$	B_{msy}/K^{sp}
1.0	0.6	818651	1.07	-7.58	51093	285076	846489	356344	0.421	0.348
1.0	0.9	687817	1.02	-7.20	60713	174248	711205	371746	0.523	0.253
0.5	0.6	1049620	0.54	-9.21	65508	365503	1085310	675761	0.623	0.348
0.5	0.9	959633	0.51	-8.92	84706	234168	992265	664675	0.670	0.253

Table 4b: Comparison of horse mackerel stock assessment results for different assumptions regarding natural mortality. Results are for Model 3 (q_2 = 0.5; h = 0.6).

M	K^{sp}	q_1	-lnL total	MSY	Bmsy (sp)	B(1950)	B(2002)	$\frac{B(2002)}{B(1950)}$	B_{msy}/K^{sp}
0.2	1353680	0.54	-10.09	60630	479331	1391940	641147	0.461	0.354
0.3 (BC)	**1049620**	**0.54**	**-9.21**	**65508**	**365503**	**1085310**	**675761**	**0.623**	**0.348**
0.4	919896	0.54	-8.77	75345	312574	964323	700346	0.726	0.340
M age dependent	1024930	0.54	-9.19	64784	354401	1066160	692832	0.650	0.354

Table 5a: Values of future spawning biomass relative to K^{sp} for four different future pelagic catch scenarios (0 MT, 5000 MT, 10000 MT and 15000 MT). Future demersal catches are assumed to be either 34000 MT, 44000 MT (2006+) or 60000 MT (2006+). Results are presented for the $\boldsymbol{q_2}$ **= 1.0;** $\boldsymbol{h}$ **= 0.6** scenario.

Future demersal catch (MT)	**Year**	**Future pelagic catch (MT)**			
		0	**5000**	**10000**	**15000**
34000	2002	0.40	0.40	0.40	0.40
	2010	0.51	0.42	0.31	0.21
	2020	0.62	0.44	0.19	0
44000	2002	0.40	0.40	0.40	0.40
	2010	0.45	0.35	0.25	0.14
	2020	0.50	0.28	0	0
60000	2002	0.40	0.40	0.40	0.40
	2010	0.37	0.28	0.18	0.07
	2020	0.28	0	0	0

Table 5b: As for Table 1a but for the $\boldsymbol{q_2}$ **= 1.0;** $\boldsymbol{h}$ **= 0.9** scenario.

Future demersal catch (MT)	**Year**	**Future pelagic catch (MT)**			
		0	**5000**	**10000**	**15000**
34000	2002	0.46	0.46	0.46	0.46
	2010	0.60	0.49	0.38	0.26
	2020	0.67	0.51	0.32	0.06
44000	2002	0.46	0.46	0.46	0.46
	2010	0.52	0.41	0.30	0.18
	2020	0.56	0.38	0.16	0
60000	2002	0.46	0.46	0.46	0.46
	2010	0.44	0.33	0.21	0.10
	2020	0.36	0.13	0	0

Table 5c: As for Table 1a but for the q_2 **= 0.5;** ***h*** **= 0.6** scenario.

Future demersal catch (MT)	**Year**	**Future pelagic catch (MT)**			
		0	**5000**	**10000**	**15000**
34000	2002	0.60	0.60	0.60	0.60
	2010	0.69	0.62	0.55	0.47
	2020	0.75	0.63	0.50	0.35
44000	2002	0.60	0.60	0.60	0.60
	2010	0.64	0.57	0.50	0.42
	2020	0.68	0.55	0.41	0.25
60000	2002	0.60	0.60	0.60	0.60
	2010	0.59	0.45	0.45	0.38
	2020	0.56	0.27	0.27	0.07

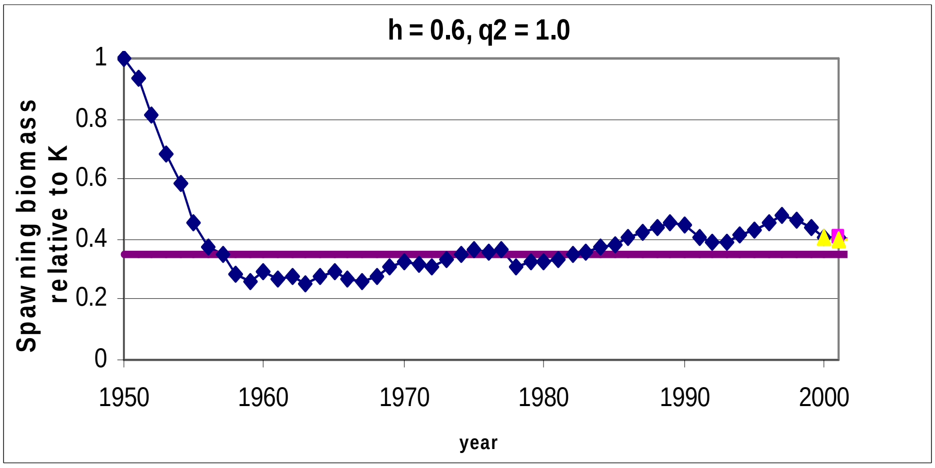
Table 5d: As for Table 1a but for the q_2 **= 0.5;** ***h*** **= 0.9** scenario.

Future demersal catch (MT)	**Year**	**Future pelagic catch (MT)**			
		0	**5000**	**10000**	**15000**
34000	2002	0.64	0.64	0.64	0.64
	2010	0.74	0.67	0.59	0.51
	2020	0.79	0.68	0.57	0.45
44000	2002	0.64	0.64	0.64	0.64
	2010	0.70	0.62	0.54	0.47
	2020	0.72	0.61	0.50	0.37
60000	2002	0.64	0.64	0.64	0.64
	2010	0.64	0.56	0.49	0.41
	2020	0.61	0.50	0.37	0.24

Table 6: Model estimated juvenile (ages 0-2) biomass values (MT) for the start of 2001, as well as the acoustic recruitment biomass (MT) survey estimate for 2001 (Coetzee *pers. commn.*).

q_2	h	**Model juvenile biomass estimate (MT)**	**Accoustic recruitment biomass survey estimate (MT)**
1.0	0.6	102226	96769
1.0	0.9	99228	
0.5	0.6	153604	
0.5	0.9	150478	

Figure 1a: Spawning biomass relative to K^{sp} trends. The B_{msy}/K level is shown as a solid line.



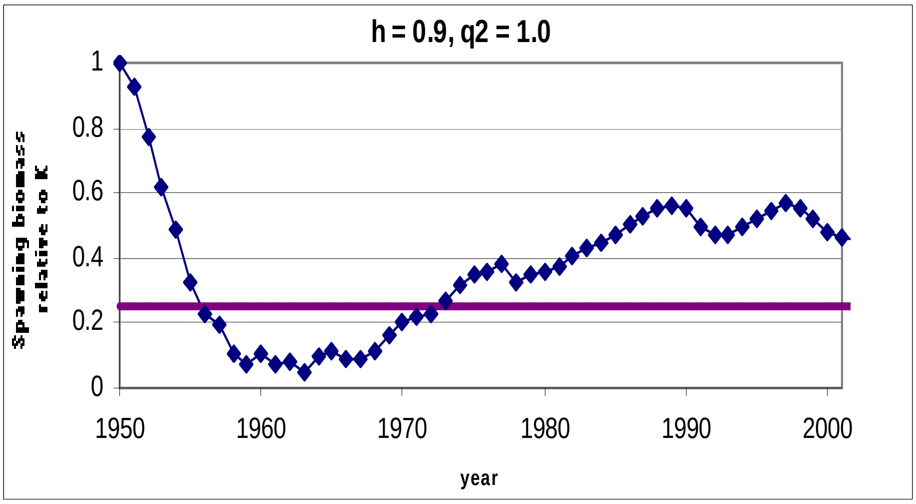
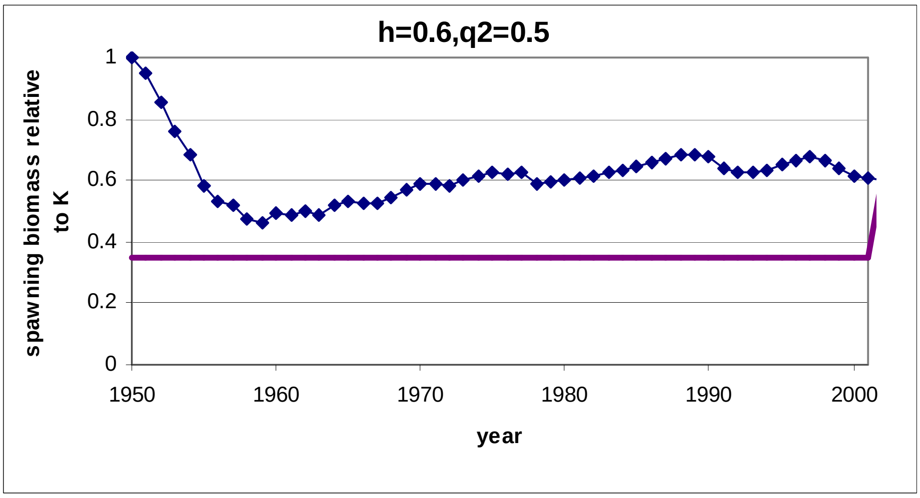


Figure 1b: Spawning biomass relative to K^{sp} trends. The B_{msy}/K level is shown as a solid line.



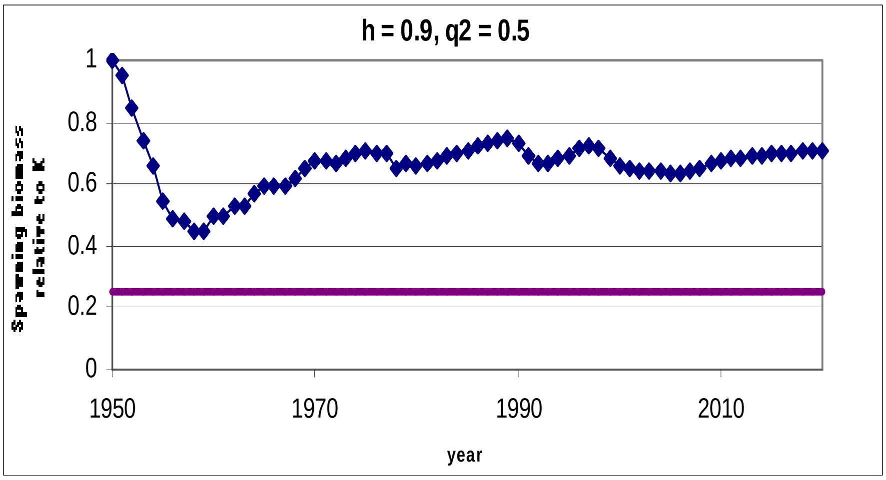


Figure 2a: Trajectories of spawning biomass relative to K^{sp}. Projections are shown for four different future pelagic catch scenarios (0 MT, 5000 MT, 10000 MT and 15000 MT), as well as for a future demersal catch of **34000** MT.

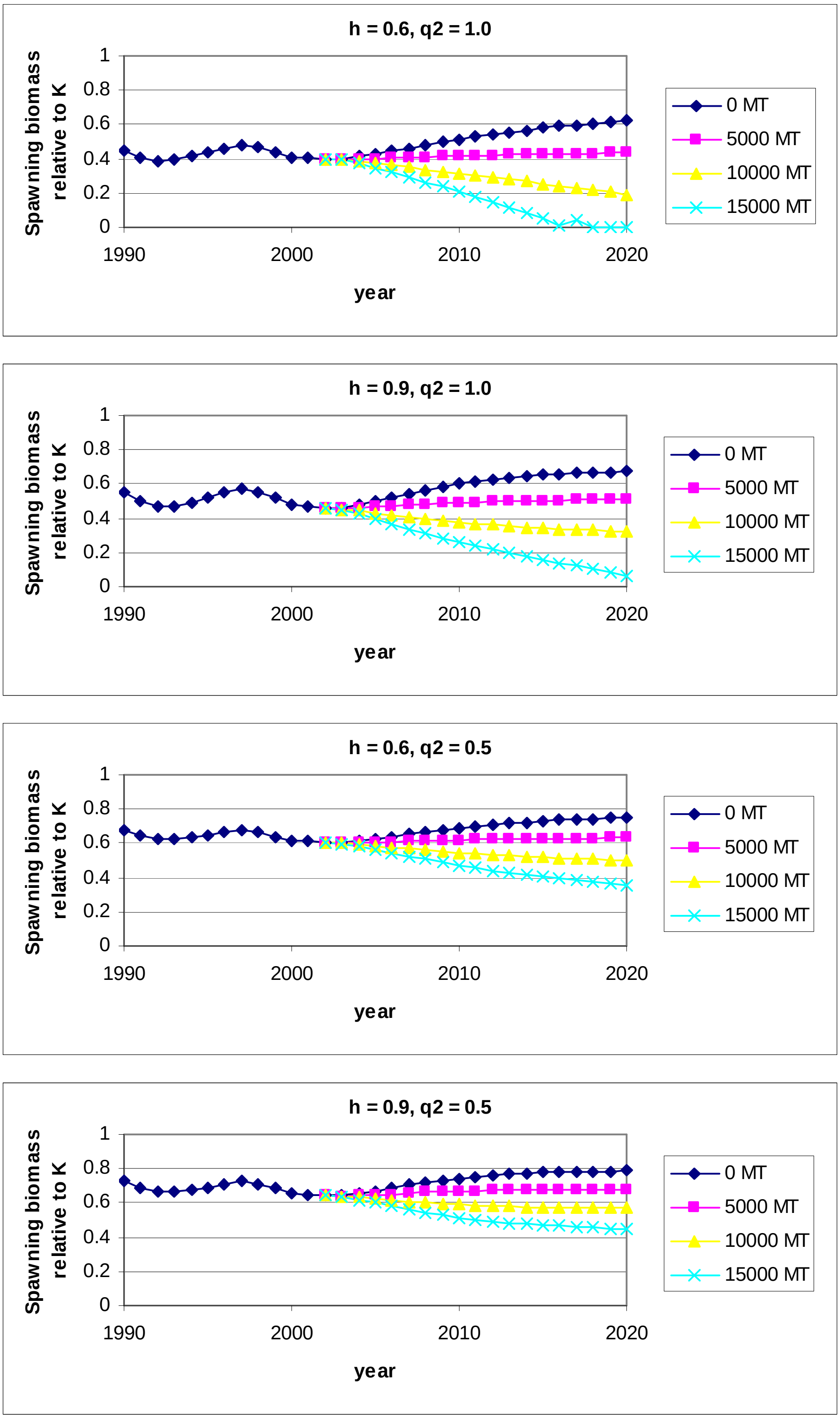
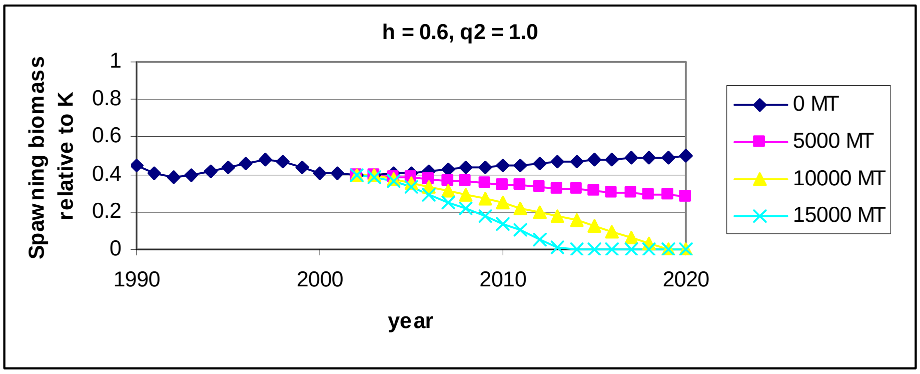
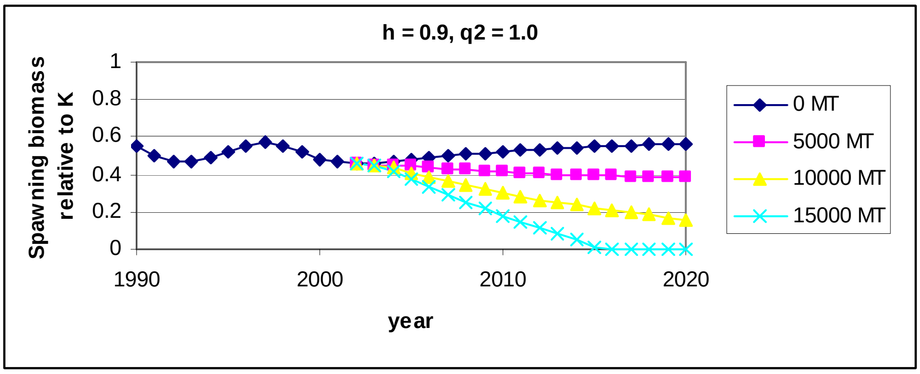
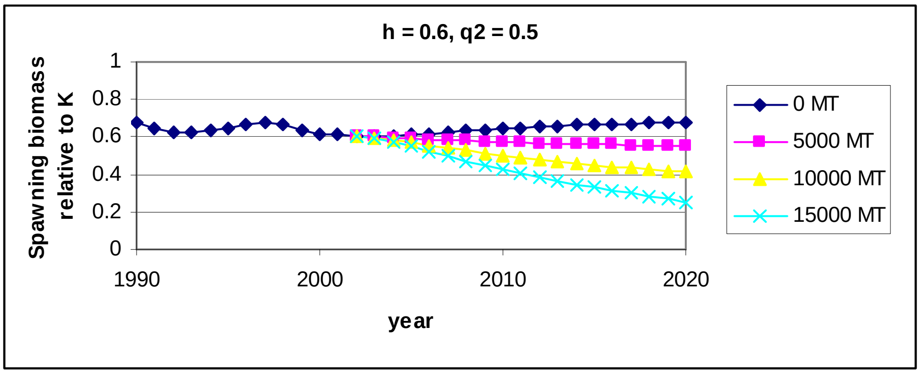


Figure 2b: As for Figure 2a, but assuming a future (2006+) demersal catch of **44000** MT.







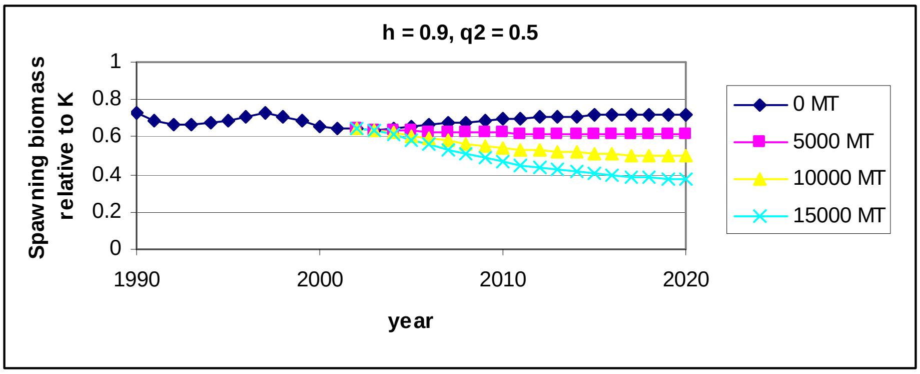


Figure 2c: As for Figure 2a, but assuming a future (2006+) demersal catch of **60000** MT.

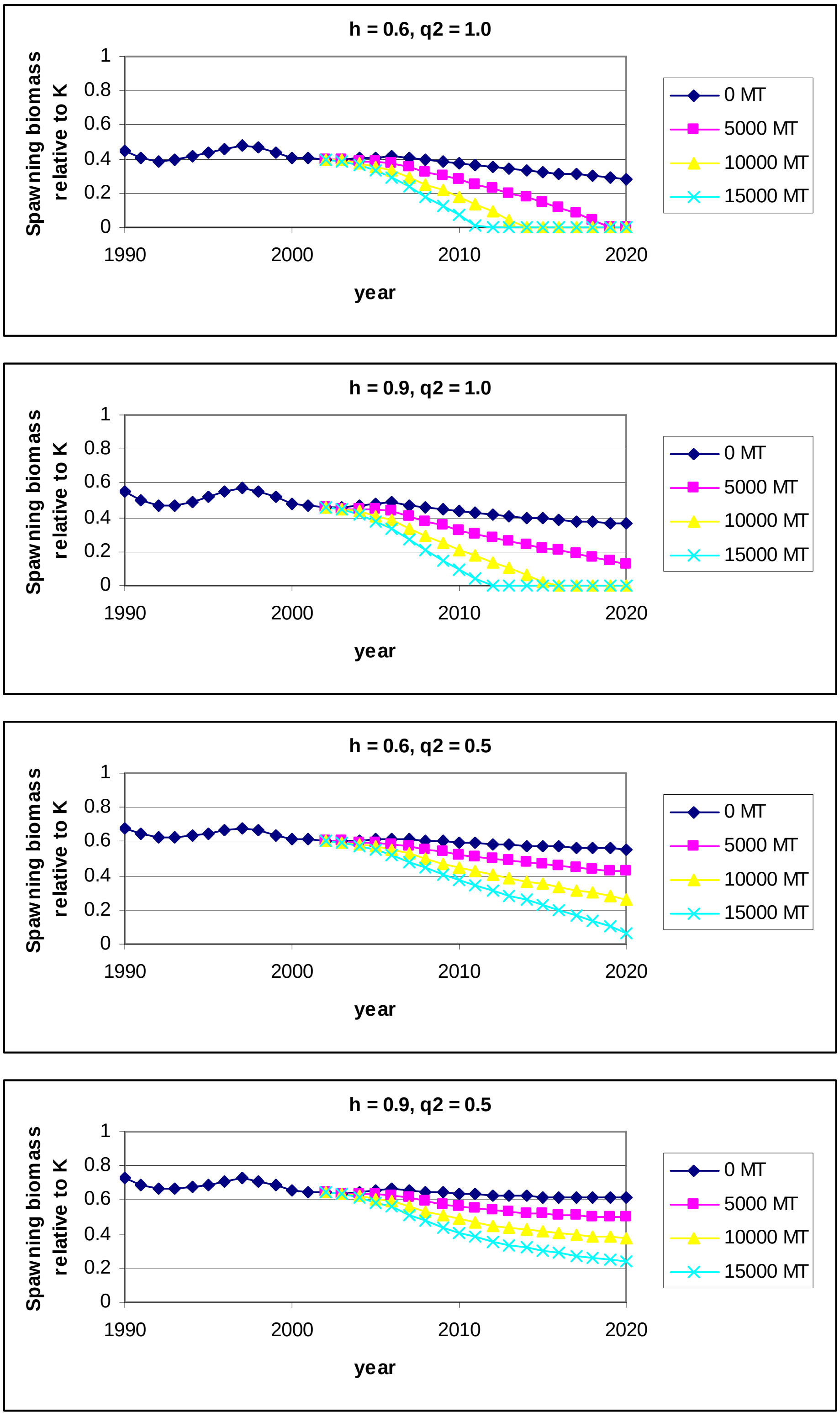
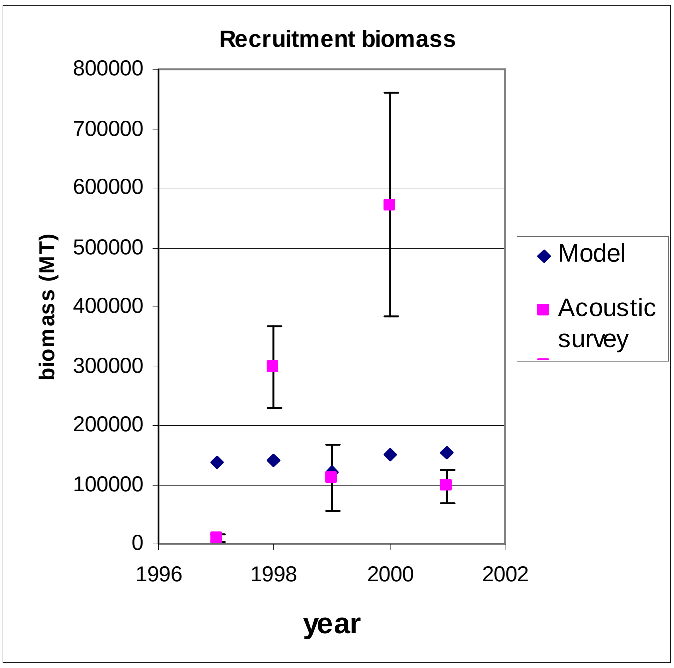


Figure 3: Comparison between model 3 (h = 0.6, q_2 = 0.5) estimated juvenile (ages 0-2) biomass and results from acoustic recruitment biomass surveys (Coetzee *pers. commn*). Acoustic survey results are shown with 1 SD.



Appendix

Mathematical details of the age-structured production model (ASPM) applied

Dynamics

The dynamics of the population are described using the following deterministic equations:

$$N_{y+1,0} = R(B^{sp}_{y+1}) \tag{A.1}$$

$$N_{y+1,a+1} = (N_{y,a}e^{-\frac{M_a}{2}} - C_{y,a})e^{-\frac{M_a}{2}} \qquad 0 \le a \le m\text{-}2 \tag{A.2}$$

$$N_{y+1,m} = (N_{y,m}e^{-\frac{M_m}{2}} - C_{y,m})e^{-\frac{M_m}{2}} + (N_{y,m-1}e^{-\frac{M_{m-1}}{2}} - C_{y,m-1})e^{-\frac{M_{m-1}}{2}} \tag{A.3}$$

where $N_{y,a}$ is the number of horse mackerel of age a at the start of year y,

$C_{y,a}$ is the total number of horse mackerel of age a taken by the fishery, i.e. by the pelagic and demersal (plus midwater) fleets combined, in year y,

$R(B^{sp})$ is the recruitment vs spawner biomass relationship assumed (see below),

M_a is the natural mortality rate for fish of age a, and

m is the largest age considered (and corresponds to a "plus group" and has a value of 10 here).

The approximation of the fishery as a pulse catch in the middle of the season is considered of sufficient accuracy for present purposes.

The total number of horse mackerel of age a caught each year ($C_{y,a}$) is given by:

$$C_{y,a} = \sum_f C^f_{y,a} \tag{A.4}$$

where f indicates the fishery/fleet concerned (pelagic or demersal).

The annual catch by mass (C^f_y) for fleet f is given by:

$$C^f_y = \sum_{a=0}^{m} w_{a+½} C^f_{y,a}$$

$$= \sum_{a=0}^{m} w_{a+\frac{1}{2}} S_a^f F_y^f N_{y,a} e^{-M_a/2} \qquad \text{(A.6)}$$

where S_a^f is the fishing selectivity-at-age for fleet $f = p$ (pelagic) or $f = d$ (demersal). [Note that the pelagic selectivity is assumed to change over time – see Table 3]. F_y^f is the fleet-specific fishing "mortality" (i.e. maximum of proportional catch over age classes) in year *y*, and $w_{a+\frac{1}{2}}$ denotes the mid-year mass of a horse mackerel of age *a*, assumed equal to the average of the begin-year and end-of-year mass.

The fleet-specific exploitable ("available") component of abundance is computed in terms of exploitable biomass at mid-year:

$$B_y^f = \sum_{a=0}^{m} w_{a+\frac{1}{2}} S_a^f N_{y,a} e^{-M_a/2} \qquad \text{(A.6)}$$

or numbers:

$$N_y^f = \sum_{a=0}^{m} S_a^f N_{y,a} e^{-M_a/2} \qquad \text{(A.7)}$$

The proportion of the resource harvested each year (F_y^f) by fleet *f* is therefore given by:

$$F_y^f = C_y^f / B_y^f \qquad \text{(A.8)}$$

and

$$C_{y,a}^f = S_a^f F_y^f N_{y,a} e^{-M_a/2} \qquad \text{(A.9)}$$

[Note: In some runs of this model for a high value of q_2, individual cohorts can become negative for early years in the fishery, even though biomass as a whole remains positive. This possibility has not been excluded, as essentially it indicates that selectivity assumptions for the early years of the fishery need some changes, but such would not affect overall results greatly.]

Spawning biomass - recruitment relationship

The spawning biomass in year *y* is given by:

$$B_y^{sp} = \sum_{a=a_m}^{m} w_a N_{y,a} \qquad \text{(A.10)}$$

where a_m is the age corresponding to 100% sexual maturity, which is assumed here to be described by a knife-edge function of age. For horse mackerel we assume a_m =3 years.

The number of recruits at the start of fishing year y is related to the spawner stock size by a stock-recruitment relationship. A Beverton-Holt form is assumed, i.e. :

$$R\left(B_y^{sp}\right)=\frac{\alpha\, B_y^{sp}}{\beta + B_y^{sp}} \tag{A.11}$$

In order to work with estimable parameters that are more meaningful biologically, the stock-recruit relationship is re-parameterised in terms of the pre-exploitation equilibrium spawning biomass, K^{sp}, and the "steepness" of the stock-recruit relationship, where "steepness" is the fraction of pristine recruitment (R_0) that results when spawning biomass drops to 20% of its pristine level, i.e.:

$$hR_0 = R\left(0.2K^{sp}\right) \tag{A.12}$$

from which it follows that:

$$h = 0.2\left[\beta + K^{sp}\right]/\left[\beta + 0.2K^{sp}\right] \tag{A.13}$$

and hence:

$$\alpha = \frac{4hR_0}{5h-1} \tag{A.14}$$

and:

$$\beta = \frac{K^{sp}(1-h)}{5h-1} \tag{A.15}$$

Given a value for the pre-exploitation spawning biomass K^{sp} of horse mackerel, together with the assumption of an initial equilibrium age structure, the following can be solved for R_0 :

$$K^{sp} = R_0[\sum_{a=1}^{m-1} f_a w_a e^{-\sum_{a'=0}^{a-1} M_{a'}} + f_m w_m e^{-\sum_{a'=0}^{m-1} M_{a'}} /(1-e^{-M_m})] \tag{A.16}$$

where a_m = 3 is fixed in the model, so that f_a, which is the proportion of fish of age a that are mature, is 0 for a < 3 and 1 thereafter, corresponding to the knife-edge relationship assumed.

Numbers-at-age for subsequent years are then computed by means of equations (A.1)-(A.11).

The likelihood function

In order to estimate K^{sp}, the model is fitted to two series of survey biomass data [see Table 2] by maximising an associated likelihood function.

The likelihood is calculated assuming that the observed abundance index is log-normally distributed about its expected value:

$$I_y^s = \hat{I}_y^s e^{\varepsilon_y^s} \qquad \text{or} \qquad \varepsilon_y^s = \ell n(I_y^s) - \ell n(\hat{I}_y^s) \tag{A.17}$$

where I_y^s is the survey biomass data for year *y* for survey *s* (*s* = 1 (spring) or 2 (autumn)),

$\hat{I}_y^s = q_s B_y^f$ is the corresponding model estimated value, where B_y^f is the model value for demersal exploitable resource biomass at mid-year corresponding to the demersal fleet, given by equation (A.6), and

q_s is a constant of proportionality (the demersal catchability coefficient).

The negative of the log-likelihood function (after removal of constants) is given then by:

$$-\ell \text{n}L = \sum_s \sum_y \left[\ell\text{n}\sigma_y^s + \left(\varepsilon_y^s\right)^2 / 2\left(\sigma_y^s\right)^2\right] \tag{A.18}$$

The standard deviations are calculated from the CVs reported in Table 2 by the following formula:

$$\sigma_y^s = \sqrt{\ln(1 + CV_{s,y}^2)} \tag{A.19}$$

Under this assumption, the maximum likelihood estimate of q^1 is given by:

$$\hat{q}_1 = \exp\left[\sum_y \left\{\ell\text{n}\, I_y^1 - \ell\text{n}\, \text{B}_\text{y}^\text{f}\right\} / n\right] \tag{A.20}$$

The value of q_2 is set externally.