# The size-structured (length-based) stock assessment methodology

# applied to west coast rock lobster

S.J. Johnston and D.S. Butterworth

MARAM
Department of Mathematics and Applied Mathematics
University of Cape Town
Rondebosch
Cape Town

## A The population dynamics model

The model of the west coast rock lobster dynamics is described by a size-structured model which operates at a 1mm interval scale, from $l = 1$mm to $l = l_{max}$, where $l$ refers to the carapace length (which for brevity will also be referred to as "size") in mm. The largest size class in the model, $l_{max}$, is equal to 200mm. Male and female lobsters are modelled separately, with $m$ referring to males and $f$ referring to females. In this report, the superscript $s$ refers to either the male or female sex, where the two sexes are treated separately but similarly. Although the fishing season starts towards the end of year $t$ and runs into year $t+1$, it is denoted as season $t$ (unless otherwise specified). Where data are collected and represented in 5mm size class intervals, the size class $x$ refers to lobsters of sizes $x$, $x+1$, $x+2$ …$x+4$ mm.

### A.1 Basic dynamics
The basic model in vector-matrix notation is:

$$N^s(t+1) = N^s(t)H^s(t)S^sA^s(t) + R(t+1) \tag{A1.1}$$

$N^s(t)$      is the (1 x 200) vector of number of males/females in each of the 1mm size classes at the start of season $t$,

$A^s(t)$      is the (200 x 200) somatic growth matrix for males/females over season $t$ (models the proportion of lobsters moving from one size class to another as a result of somatic growth),

$H^s(t)$      is the (200 x 200) fishing survivorship matrix for males/females over season $t$,

$S^s$      is the (200 x 200) diagonal natural survivorship matrix for males/females (constant over time), and

$R(t+1)$      is the (1 x 200) recruitment vector for season $t+1$ (the same for males and for females).

Note that the structure of this discrete model is such that harvest is followed by natural mortality and then by somatic growth, i.e. fishing is approximated by a pulse at the start of the season.

### A.2 Length-weight conversions
The model operates on a numbers basis, but the number of lobsters in each size class is easily converted to biomass using the length-weight conversion equations as follows:

$$w_l^m = 0.6518l^{2.8990} \qquad \text{for males, and} \tag{A2.1}$$

$$w_l^f = 0.5869l^{2.9729} \qquad \text{for females} \tag{A2.2}$$

where the total body weight, $w$, is in grams, and length $l$, is the carapace length in centimetres.

### A.3 Equilibrium starting conditions
Assuming no harvesting (pristine conditions), the equilibrium populations numbers are calculated as follows:

$$N^{s*} = N^{s*}S^sA^s + R \tag{A3.1}$$

and therefore

$$N^{s*} = R(I - S^sA^s)^{-1} \tag{A3.2}$$

$N^{s*}$ represents the male/female pristine equilibrium population vector, and $I$ is the unit matrix.

Equation (A3.2) is used to set up the initial pristine population size-structure in 1870, when catches are assumed to have commenced.

### A.4 Natural survivorship matrix S

The natural survivorship matrix $S^s$ models the proportion of lobsters in each size class at the beginning of the season which survive to the beginning of the next season in the absence of fishing. This matrix consists of zeros except for the diagonal entries which contain the $S_l^s$ values for size class $l$. The natural survivorship is assumed to be independent of size for lobsters greater than or equal to 60mm. However, the demography of immature lobsters is likely to be somewhat different from that of mature lobsters, so that smaller lobsters are assumed to have reduced survivorship. A linear increase in survivorship from 0.1 at size $l = 1$mm (an artificial size used purely for definition convenience) to $S^m$ (males) and $S^f$ (females) at size 60mm is assumed. The maximum male annual natural survivorship, $S^m$, is set at 0.90 for the Base Case. The maximum female natural survivorship, $S^f$, is an estimable parameter of the model. Natural survivorship is assumed to be constant over time. The specific entries on the diagonal of the natural survivorship matrix are calculated as follows.

Males:

$$S_l^m = (l-1)\left(\frac{S^m - 0.1}{59}\right) + 0.1 \qquad\qquad \text{for } 1 \le l < 60 \text{ mm} \qquad\qquad (A4.1)$$

$$S_l^m = S^m = 0.90 \qquad\qquad \text{for } l \ge 60 \text{ mm}$$

Females:

$$S_l^f = (l-1)\left(\frac{S^f - 0.1}{59}\right) + 0.1 \qquad\qquad \text{for } 1 \le l < 60 \text{ mm} \qquad\qquad (A4.2)$$

$$S_l^f = S^f \qquad\qquad \text{for } l \ge 60 \text{ mm}$$

### A.5 Gear selectivity

Different sexes and sizes of lobsters have different probabilities of being caught by various fishing gear, and each gear type operates in different parts of the overall habitat area where different sizes or sexes may aggregate at different times. The fishing selectivity functions model these different probabilities of being caught by the gear. In this model, selectivity is thus really a combination of the physical gear selectivity as well as availability.

For the Trap, Hoopnet and FIMS selectivity functions (for both male and female), the function for each gear (except FIMS female) is as follows:

$$b_l^{s,sel} = \frac{P.e^{-\mu l}}{1 + e^{-\delta(l-l^*)}} \qquad\qquad (A5.1)$$

$b_l^{s,sel}$ is the selectivity of a lobster carapace length $l$ and of sex $s$ ($m$ or $f$) for gear selectivity type $sel$.

$\mu, \delta$ and $l^*$ are three estimable parameters that control the shape of the function.

where,

    $\mu$ - controls the magnitude of the slope (up or down),

    $\delta$ - controls the steepness of the left-hand side,

    $l^*$ - shifts the function to the left or right (is the length at which 50% of $P$ occurs for $\mu = 0$), and

    $P$ - is a scalar, which for males is fixed at a value such that $b_{90}^{m,sel} = 1.00$, whereas for females it is a further estimable

       parameter.

In order to improve the fits of the model to the various catch-at-size data, some further modifications of the selectivity functions above are made, to increase the flexibility of these functions to be able to fit to the data for smaller size classes in particular. The following modifications apply:

**Trap Male:**

$$b_l^{m,trap} = 0 \qquad \text{for } l \leq 45 \ mm$$

Three further selectivity parameters are estimated: $b_{60}^{m,trap}$, $b_{70}^{m,trap}$, and $b_{80}^{m,trap}$. Linearity is assumed between $b_{45}^{m,trap}$, $b_{60}^{m,trap}$, $b_{70}^{m,trap}$, $b_{80}^{m,trap}$ and $b_{90}^{m,trap}$, for example,

$$b_l^{m,trap} = l\left(\frac{b_{60}^{m,trap} - b_{45}^{m,trap}}{60 - 45}\right) + b_{45}^{m,trap} - 45\left(\frac{b_{60}^{m,trap} - b_{45}^{m,trap}}{60 - 45}\right) \qquad \text{for } 45 > l \leq 60 \ mm \qquad (A5.2)$$

**Trap female:**
The $P$ (scalar) parameter for the female Trap and Hoopnet functions is modelled as follows:

$$P = P_1 \qquad \text{for } t \leq 1976$$

$$P = t\left(\frac{P_2 - P_1}{1990 - 1976}\right) + P_1 - 1976\left(\frac{P_2 - P_1}{1990 - 1976}\right) \qquad \text{for } 1976 < t < 1990 \qquad (A5.3)$$

$$P = P_2 \qquad \text{for } t \geq 1990$$

Also,

$$b_l^{f,trap} = 0 \qquad \text{for } l \leq 45 \ mm$$

Two further selectivity parameters are now estimated: $b_{55}^{f,trap}$, and $b_{65}^{f,trap}$. Linearity is assumed between $b_{45}^{f,trap}$, $b_{55}^{f,trap}$, $b_{65}^{f,trap}$, and $b_{75}^{f,trap}$.

**Hoop male:**

$$b_l^{m,hoop} = 0 \qquad \text{for } l \leq 45 \ mm$$

Also, assume linearity between $b_{45}^{m,hoop}$ and $b_{70}^{m,hoop}$.

**Hoop female:**
As for trap females,

$$P = P_1 \qquad \text{for } t \leq 1976$$

$$P = t\left(\frac{P_2 - P_1}{1990 - 1976}\right) + P_1 - 1976\left(\frac{P_2 - P_1}{1990 - 1976}\right) \qquad \text{for } 1976 < t < 1990 \qquad (A5.4)$$

$$P = P_2 \qquad \text{for } t \geq 1990$$

Also,

$$b_l^{f,hoop} = 0 \qquad \text{for } l \leq 45 \ mm$$

Also, assume linearity between $b_{45}^{f,hoop}$ and $b_{70}^{f,hoop}$.

**FIMS male:**
No further modifications needed.

**FIMS female:**
It was found that the logistic selectivity function was not flexible enough to produce a really good fit to the FIMS female catch-at-size data. Five selectivity parameters are thus fitted: $b_{57}^{f,FIMS}$, $b_{62}^{f,FIMS}$, $b_{67}^{f,FIMS}$, $b_{72}^{f,FIMS}$, and $b_{77}^{f,FIMS}$, to replace the logistic function.

We also assume,

$$b_l^{f,FIMS} = 0 \qquad\qquad \text{for } l \le 45 \ mm \text{, and}$$

$$b_{100}^{f,FIMS} = 0 \qquad\qquad \text{for } l \ge 100 \ mm.$$

The selectivity function is then modeled by assuming linearity between $b_{45}^{f,FIMS}$, $b_{57}^{f,FIMS}$, $b_{62}^{f,FIMS}$, $b_{67}^{f,FIMS}$, $b_{72}^{f,FIMS}$, $b_{77}^{f,FIMS}$ and $b_{100}^{f,FIMS}$.

### A.6 Fishing survivorship matrix H

The fishing survivorship matrix $H$ consists of diagonal entries $[1 - \{p_l^{s,trap}(t) + p_l^{s,hoop}(t) + p_l^{s,poach}(t) + p_l^{s,recr}(t)\}]$

with zeros everywhere else, where the $p_l^{s,c}(t)$ terms, the proportion of lobsters in size class $l$ caught during season $t$ for the specified catch

type , are calculated as follows:

$$p_l^{s,c}(t) = b_l^{s,sel}(t) \, d_l(t) \, F^c(t) \qquad\qquad \text{(A6.1)}$$

where $c$ refers to either the commercial trap or hoop, poaching or recreational catch.

For trap and hoops:

$$d_l(t) = d \qquad\qquad \text{for } l < l_{\min}(t)$$

$$d_l(t) = 1 \qquad\qquad \text{for } l \ge l_{\min}(t)$$

where $l_{\min}(t)$ is the commercial legal minimum size limit in season $t$.

For poaching:

$$d_l(t) = 1 \qquad\qquad \text{for all } l$$

For recreational:

$$d_l(t) = d \qquad\qquad \text{for } l < l_{\min}r(t)$$

$$d_l(t) = 1 \qquad\qquad \text{for } l \ge l_{\min}r(t)$$

where $l_{\min}r$ is the recreational minimum size limit

The parameter $d$ refers to the **discard mortality**, and is the proportion of undersized lobsters that are caught and returned to the sea which subsequently die as a result of the original capture-and-return process. A value of $d = 0.10$ is used.

The factor $F^c(t)$ in equation A6.1 is the fishing mortality for fully selected animals, which links the model-generated catches to the observed catches by weight (catches are only recorded by weight, not number), and is calculated as follows:

$$F^c(t) = \frac{C_t^c}{\sum\limits_{l \ge l_{\min(t)}} \left[ N_l^m(t) b_l^{m,sel}(t) w_l^m + N_l^f(t) b_l^{f,sel}(t) w_l^f \right]}, \qquad\qquad \text{(A6.2)}$$

where

$C_t^c$ is the observed catch (trap, hoop, poaching or recreational) by weight in season $t$, and

$N_l^m(t)$ and $N_l^f(t)$ are the numbers-at-length for season $t$.

Note: for poaching, the $F^c(t)$ is summed over all sizes, not just above the minimum size.

### A.7 The 1991 fishing season

The 1991 season in the model receives special treatment as the minimum size was altered during this season and only males were harvested. After this alteration, both males and females were however subject to discard mortality. The model thus calculates the $p_l^{m,c}(1991)$ and $p_l^{f,c}(1991)$ in two steps. The first refers to the initial part of the season when the minimum size limit was 89mm (denoted by superscript 89), while the second refers to the subsequent part when the minimum size limit was 75mm (denoted by superscript 75). Therefore, for the initial part:

$$p_l^{m,c,89} = b_l^{m,sel}(1991) \, d_l^{89}(1991) F^{c,89}(1991) \qquad\qquad \text{(A7.1)}$$

where

$$d_l^{89}(1991) = d \qquad\qquad \text{for } l < 89\text{mm for males}$$

$$d_l^{89}(1991) = d \qquad\qquad \text{for females of all sizes}$$

$$d_l^{89}(1991) = 1 \qquad\qquad \text{for } l \geq 89\text{mm for males}$$

and

$$p_l^{f,c,89} = b_l^{f,sel}(1991)d_l^{89}(1991)F^{c,89}(1991)$$

The fully-selected fishing mortality $F^{c,89}(1991)$ above is calculated as follows:

$$F^{c,89}(1991) = \frac{C_{1991}^{c,89}}{\sum\limits_{l \geq 89mm}\left[N_l^m(1991)b_l^{m,sel}(1991)w_l^m\right]} \qquad\qquad (A7.2)$$

where $C_{1991}^{c,89}$ refers to the catch (by weight) taken in the initial part of the season when the minimum size limit was 89mm. Before proceeding with the calculations for the second part of the season, the numbers-at-size are updated to the end of the first part of the season taking into account the losses due to fishing mortality only, i.e.:

$$N_l^{m,89}(1991) = N_l^m(1991)\left(1 - p_l^{m,89}(1991)\right), \quad \text{and also therefore} \qquad (A7.3)$$

$$N_l^{f,89}(1991) = N_l^f(1991)\left(1 - p_l^{f,89}(1991)\right)$$

where

$$p_l^{m,89} = \sum\limits_c p_l^{m,c,89}, \text{ and}$$

$$p_l^{f,89} = \sum\limits_c p_l^{f,c,89}$$

For the subsequent part of the season:

$$p_l^{m,c,75} = b_l^{m,sel}(1991)d_l^{75}(1991)F^{c,75}(1991) \qquad\qquad (A7.4)$$

where

$$d_l^{75}(1991) = d \qquad\qquad \text{for } l < 75\text{mm for males}$$

$$d_l^{75}(1991) = d \qquad\qquad \text{for females of all sizes}$$

$$d_l^{75}(1991) = 1 \qquad\qquad \text{for } l \geq 75\text{mm for males}$$

and

$$p_l^{f,c,75} = b_l^{f,sel}(1991)d_l^{75}(1991)F^{c,75}(1991)$$

where, further:

$$F^{c,75}(1991) = \frac{C_{1991}^{c,75}}{\sum\limits_{l \geq 75mm}\left[N_l^{m,89}(1991)b_l^{m,sel}(1991)w_l^m\right]} \qquad\qquad (A7.5)$$

and $C_{1991}^{c,75}$ is the catch (by weight) taken in the subsequent part of the season to which a minimum size limit of 75mm applied,

$$p_l^{m,75} = \sum\limits_c p_l^{m,c,75}, \text{ and}$$

$$p_l^{f,75} = \sum\limits_c p_l^{f,c,75}$$

We therefore have $\qquad p_l^m(1991) = 1 - (1 - p_l^{m,89})(1 - p_l^{m,75})$, and $\qquad\qquad (A7.6)$

$$p_l^f(1991) = 1 - (1 - p_l^{f,89})(1 - p_l^{f,75}).$$

### A.8 Somatic growth projection matrix A

The somatic growth projection matrix models the proportion of lobsters in one size class that move into another size class between seasons as a result of somatic growth. The entry $a_t^s(i, j)$ represents the proportion of lobsters in size class $i$ that move into size class $j$ during the moult at the end of season $t$. Moult increments of lobsters have been shown to be highly variable. This variance in somatic growth is modelled using a normal probability distribution truncated at $\pm$ 3 standard deviations. In order to calculate the $a_t^s(i, j)$ values using this probability distribution, the mean annual moult increment value, $g_{l,t}^s$, must be evaluated as well as an estimate of standard deviation around this mean moult increment, $\sigma_{l,t}^s$, for each length $l$ and year $t$. The calculation of the $g_{l,t}^s$ and $\sigma_{l,t}^s$ values are reported in detail in the Appendix. Note that this distribution allows for the possibility that lobsters shrink during the moult.

The calculation of the $a_t^s(i, j)$ values are described by the following steps for any given season $t$ and sex $s$ - the $t$ and $s$ indices have been omitted below for less cluttered notation.

[Note that because $i$ represents the length class ($i$-1, $i$) mm, e.g. $i$ = 75 means animals of size $l$ where $74 < l \leq 75$ mm, it is convenient to use the midpoint $i$-0.5 of this interval in the following calculations.]

Step 1: For any given $i$, initialise $a(i,j)$ to zero for $j = 1, 2 ,...200$, and calculate the mean length after moult ($m_i$) for length $l_i$ (where $l_i = i - 0.5$):

$$m_i = l_i + g_i \tag{A8.1}$$

where $g_i$ is the mean moult increment for length $i$.

Step 2: Calculate the two endpoints of the growth distribution:

$$m_i(1) = m_i - 3\sigma_i \qquad \text{(lower point)} \tag{A8.2}$$

$$m_i(2) = m_i + 3\sigma_i \qquad \text{(upper point)}$$

where $\sigma_i$ is the standard deviation for the mean moult increment $g_i$.

If $\sigma_i = 0$, then set $a(i,j) = 1$ [where $j$ is such that $j - 1 < m_i \leq j$] and go back to step 1, incrementing $i$ by 1.

Step 3: Find the midpoints $mp(1)$ and $mp(2)$ of the length classes into which $m_i(1)$ and $m_i(2)$ respectively fall.

Step 4: Define $mp_j$ as the midpoint corresponding to length class ($j$-1, $j$); then for
$mp_j = mp(1), mp(1)+1, ..., mp(2)$ do the following.

A:    If $mp_j = mp(1)$,     set $y(1) = m_i(1)$                                          (A8.3)
                            set $y(2) = mp_j + 0.5$                                      (A8.4)

    If $mp_j = mp(2)$,     set $y(1) = mp_j$ - 0.5                                      (A8.5)
                            set $y(2) = m_i(2)$                                          (A8.6)

    For all other cases,     set $y(1) = mp_j$ - 0.5                                    (A8.7)
                            set $y(2) = mp_j + 0.5$                                      (A8.8)

B: Calculate the $z$ values for $y(1)$ and $y(2)$ (to convert to a normal distribution N(0;1)):

$$z(1) = \frac{y(1) - m_i}{\sigma_i} \quad \text{and} \tag{A8.9}$$

$$z(2) = \frac{y(2) - m_i}{\sigma_i} \tag{A8.10}$$

C: Calculate the area under the normal curve for each of these z values:
$A(1)$ = area from $z(1)$ to 0
$A(2)$ = area from $z(2)$ to 0

D: Calculate the proportion of lobsters moving from $i$ to $j$ (equal to the area under the normal curve from $z(1)$ to $z(2)$).
If        $1 < mp_j < 199$, do the following:

$$j = mp_j + 0.5$$
$$a(i, j) = A(1) + A(2) \qquad \text{if } y(1) < m_i < y(2) \tag{A8.11}$$
$$\text{otherwise}$$
$$a(i, j) = \left| A(1) - A(2) \right|$$

If $\quad mp_j < 1$, do the following:
$$j = 1$$
$$a(i, j) = \left| A(1) - A(2) \right| \tag{A8.12}$$

If $\quad mp_j > 199$, do the following:
$$j = 200$$
$$a(i, j) = \left| A(1) - A(2) \right| \tag{A8.13}$$

Step 5: Normalise $a(i,j)$ so that $\sum\limits_{j=1}^{200} a(i, j) = 1$

This is to correct for the fact that normal distributions are used at Step 4B, whereas Step 2 explicitly limits growth to $\pm$ three standard deviations from the mean.

Step 6: Go back to Step 1, incrementing $i$ by 1.

### *A.9 Recruitment R*
Recruitment is assumed to contribute to the first 15 size classes only. The value of recruitment for each of these size classes in season $t$ is $R(t)$. Recruitment is assumed equal for male and female lobsters, and hence:

$$R_l^m(t) = R_l^f(t) = R(t) \qquad \text{for } l \leq 15\,\text{mm} \tag{A9.1}$$

$$R_l^m(t) = R_l^f(t) = 0 \qquad \text{for } l > 15\,\text{mm}$$

Recruitment is assumed to be linear between $R(1870)$, $R(1920)$, $R(1950)$, $R(1970)$, $R(1975)$, $R(1980)$, $R(1985)$, $R(1990)$ and $R(1995)$. $R(1995)$ is calculated as the geometric mean of the four values $R(1975)$-$R(1990)$.

### *A.10 Catch*

The model is constrained so that the model-predicted catch $\hat{C}_t$ exactly equals the "observed" catch $C_t^{obs}$. The "observed" catch consists of three contributions in each season t:

    i)      the recorded commercial catch, $C_t^{comm} = C_t^{trap} + C_t^{hoop}$

    ii)      an estimate of poaching, $C_t^{poach}$, and

    iii)      an estimate of the public recreational catch, $C_t^{rec}$.

Thus:
$$C_t^{obs} = C_t^{comm} + C_t^{poach} + C_t^{rec} \tag{A10.1}$$

The model-predicted catch is calculated by:
$$\hat{C}_t = \sum_c \sum_{l \geq l_{\min}(t)} \left[ F^c(t) b_l^{m,sel}(t) N_l^m(t) w_l^m + F^c(t) b_l^{f,sel}(t) N_l^f(t) w_l^t \right] \tag{A10.2}$$

*A10.1 Commercial catches*
Various references (Pollock 1986, for example) indicate that commercial harvesting began around 1870. The commercial catch record is reported in RLWS/DEC05/ASS/7/1/1.

*A.10.2 Poaching and recreational catches*

Poaching:
Poaching (at a non-negligible level) is assumed to start in 1951 and increase linearly to a value $P_{max}$ in the year 1990 (these assumptions are based on anecdotal reports from the industry). Poaching is assumed to remain at the $P_{max}$ level after 1990. The estimates of $C_t^{poach}$ used in the model are thus:

$$C_t^{poach} = 0 \qquad\qquad \text{for } t < 1950 \qquad\qquad (A10.3)$$

$$C_t^{poach} = P_{max} \left( \frac{t - 1950}{1990 - 1950} \right) \qquad\qquad \text{for } 1950 \leq t \leq 1990$$

$$C_t^{poach} = P\,max \qquad\qquad \text{for } t > 1990$$

The reference case assessment assumes $P_{max}$ = 500 MT based upon rough suppositions by SFRI personnel for the area-aggregated model.

Recreational Catch:
Recreational catches (at a non-negligible level) are assumed to have started in 1959 (this assumption was based on discussions held by the RLWG in conjunction with the industry). From 1959 to 1992 the recreational catches are assumed to have increased linearly from zero to 469 MT (as estimated in the telephone survey for that year). Telephone survey estimated values are used for seasons 1993-1998. The estimates of $C_t^{rec}$ (in tons) used in the model are reported in RLWS/DEC05/ASS/7/1/1.

**_A.11 Biomass_**
Since the model maintains information on numbers-at-size by sex, the biomass of any desired component of the resource is readily computed. These biomass estimates can refer to the male portion of the stock, $B^m$, the female portion of the stock, $B^f$, or to the combined total of males and females $B$. Furthermore, a biomass estimate can be calculated for any given size range. For example, the biomass estimate for male and female lobsters above size $l$ = 75mm in season $t$ is calculated as follows:

$$B_{75}(t) = \sum_{l \geq 75} (N_l^m(t) w_l^m + N_l^f(t) w_l^f) \qquad\qquad (A11.1)$$

All biomass estimates quoted in this paper have units metric tons (MT) or '000 MT.

**_A.12 MSY Calculations_**
The maximum sustainable yield (MSY) can be calculated for a number of scenarios. The following must be defined:
  i)     somatic growth rate,
  ii)    recruitment level,
  iii)   gear selectivity, and
  iv)   minimum legal size.
Equations A.3.4 and A.3.5 are modified to include the fishing survivorship matrix, so that an equilibrium size structure can be calculated for any level of harvesting. For a fishing mortality level $F$, which defines a fishing survival matrix $H$:

$$N^{s*}(F) = N^{s*}(F) H^s S^s A^s + R \qquad\qquad (A12.1)$$

so that

$$N^{s*}(F) = R(I - H^s S^s A^s)^{-1} \qquad\qquad (A12.2)$$

The sustainable yield for any $F$ can be calculated as follows:

$$\text{SY}(F) = \sum_{l \geq min} ( b_l^{s,sel} d_l F \ (N_l^{m*}(F) w_l^m + N_l^{f*}(F) w_l^f)) \qquad\qquad (A12.3)$$

where the $N_l^{s*}(F)$ are the number of male/female lobsters in the equilibrium population at size $l$ and at fishing mortality level $F$. At some level of fishing mortality, the sustainable yield reaches a maximum - the maximum sustainable yield (MSY). The $F$ value corresponding to MSY is denoted $F_{MSY}$.

The biomass level (above the legal minimum size) at which MSY occurs, $B_{MSY}$, is calculated as follows:

$$B_{MSY} = \sum_{l \geq min} (N_l^{m*}(F_{MSY}) w_l^m + N_l^{f*}(F_{MSY}) w_l^f) \qquad\qquad (A12.4)$$

### A.13 The Likelihood function

The estimable parameters of the size-structured model are estimated by fitting the model to a number of data series. This is achieved by minimising the negative of the log of the likelihood function for these data.

A.13.1 Data included in the likelihood
The negative log likelihood function incorporates three types of data.

*i) CPUE*
Three sources of CPUE data for the resource are used.
  a)  Commercial trap CPUE data for the 1981-2004 period.
  b)  Commercial hoopnet CPUE data for the 1981-2004 period.
  c)  FIMS CPUE data for the 1992-2004 period.
  (Remember, 1995 refers to the 1995/96 season, for example.)
These CPUE values are assumed to be log-normally distributed about their model-based expectations.

*ii) Catch-at-size data*
Four sources of catch-at-size data are used.
  a)  Data obtained from commercial trap samples for 1976-2004.
  b)  Data obtained from commercial hoopnet samples for 1976-2004.
  c)  Data obtained from the FIMS samples for 1992-2004.
  d)  Data obtained from trap samples which cover the full size range including the sub-legal size classes for 1994-1998.
Males and females are treated separately. Data are reported by 5mm size classes, i.e. the $n^{th}$ size class incorporates lobsters of size $n$ mm, $n+1$ mm ... $n+4$ mm. Data for size classes equal or larger than the minimum legal size at the time are used from the commercial trap and hoopnet data. Data for the full size range are used for both the FIMS and the trap sub-legal data. Females are omitted for the 1991 season as there was a ban on catching females for that season. Due to there being very few observations for some size classes, plus- and minus-groups have been formed (indicated in bold) where necessary. This ensures that no observed size-class has a percentage less than 1%.

*iii) Percent females in the catch ($F^{\%}$)*
Three sources of data are used.
  a)  Data from the commercial trap samples used for calculating catch-at-size data for 1976-2004.
  b)  Data from the commercial hoopnet samples used for calculating catch-at-size data for 1976-2004.
  c)  Data from the FIMS samples for 1992-2004.
Percent females ($F^{\%}$) values are the percentage by **number** of females in the samples. Once again the season 1991 is omitted as there was a ban on catching females then.

A.13.2 The Likelihood function
The superscript $s$ is sometimes used below. It refers to either of the two sexes, where males and females are treated separately but identically. After removal of constants from the negative log likelihood, the function to be minimised is:

$$-\ln L = \sum_{A=1,2,3} ( \, n_{cpue}^A \ln \sigma_{cpue}^A + \frac{1}{2\sigma_{cpue}^{A}{}^2} \sum_t \left\{ \ln CPUE_t^A - \ln C\hat{P}UE_t^A \right\}^2 )$$

$$0.1 \Big[ \sum_{A=1...4} \Big[ \sum_t \sum_l \sum_s \left\{ \ln \left( \sigma_{size}^A / \sqrt{\hat{p}_{t,l}^{A,s}} \right) + \hat{p}_{t,l}^{A,s} \left( \ln p_{t,l}^{A,s} - \ln \hat{p}_{t,l}^{A,s} \right)^2 / 2\sigma_{size}^{A}{}^2 \right\} \Big] \Big] \qquad (A13.1)$$

$$+ -\ln L \,(F\%)$$

[Note that the catch-at-size data are downweighted by a factor of 0.1.]
where

| | |
|---|---|
| $A$ | is the data series identifier: |
| | where $A = 1$ refers to the commercial trap data |
| | $A = 2$ refers to the commercial hoopnet data |
| | $A = 3$ refers to the FIMS data |
| | $A = 4$ refers to the trap sublegal catch-at-size data |
| $t$ | is the season |
| $l$ | is the length class in 5 mm intervals |
| $n_{cpue}^A$ | is the numbers of seasons of CPUE data for series $A$ |
| $CPUE_t^A$ | is the  observed CPUE value in season $t$ for series $A$ |
| $C\hat{P}UE_t^A$ | is the model-estimated CPUE value in season $t$ for series $A$ |
| $p_{t,l}^{A,s}$ | is the observed catch-at-size proportion for male/female lobsters of size $l$ in season $t$ for  series $A$ |

$\hat{p}_{t,l}^{A,s}$ is the model-estimated catch-at-size proportion for male/female lobsters of size $l$ in season $t$ for series $A$

$F_t^{\%,A}$ is the observed percentage females in the catch in season $t$ for series $A$

$\hat{F}_t^{\%,A}$ is the model-estimated percentage females in the catch in season $t$ for series $A$

Also:

$$\hat{F}_t^{\%,A} = \frac{\sum\limits_{l \geq l_{min}} (b_l^{f,sel}(t)N_l^f(t))}{\sum\limits_{l \geq l_{min}} (b_l^{m,sel}(t)N_l^m(t) + b_l^{f,sel}(t)N_l^f(t))} \cdot 100 \tag{A13.2}$$

$$\hat{p}_{t,l}^{A,s} = \frac{b_l^{s,sel}(t)N_l^s(t)}{\sum\limits_{l \geq l_{min}} (b_l^{s,sel}(t)N_l^s(t))} \tag{A13.3}$$

$$\sigma_{cpue}^A = \sqrt{\frac{1}{n_{cpue}^A} \sum\limits_t (\ln CPUE_t^A - \ln CP\hat{U}E_t^A)^2} \tag{A13.4}$$

$$\sigma_{size}^A = \sqrt{\sum\limits_t \sum\limits_l \sum\limits_s [\hat{p}_{t,l}^{A,s}(\ln p_{t,l}^{A,s} - \ln \hat{p}_{t,l}^{A,s})/1]} \tag{A13.5}$$

For series $A = 1$ and $A = 2$: (trap and hoopnet data)

$$CP\hat{U}E_t^A = q^A \sum\limits_{l \geq l_{min}} (w_l^m b_l^{m,sel}(t)N_l^m(t) + w_l^f b_l^{f,sel}(t)N_l^f(t)) \tag{A13.6}$$

$$\ln q^A = \frac{1}{n_{cpue}^A} \sum\limits_t \left( \ln CPUE_t^A - \ln\{ \sum\limits_{l \geq l_{min}} w_l^m b_l^{m,sel}(t)N_l^m(t) + w_l^f b_l^{f,sel}(t)N_l^f(t)\} \right) \tag{A13.7}$$

For series $A = 3$ : (FIMS data)

$$CP\hat{U}E_t^A = q^A \sum\limits_{l \geq 60} (N_l^m(t) + N_l^f(t)) \tag{A13.8}$$

$$\ln q^A = \frac{1}{n_{cpue}^A} \sum\limits_t \left( \ln CPUE_t^A - \ln\{ \sum\limits_{l \geq 60} N_l^m(t) + N_l^f(t)\} \right) \tag{A13.9}$$

**F% likelihood function**

The method of including the F% (percentage females in the catch) data in the likelihood function was changed when the assessments were expanded to include area-disaggregated approaches. This change was necessary, as previously all the area-aggregated observed F% data were greater than zero. Moving to the super-area level, it was found that there were some instances where the true observed F% for a given year was in fact zero. The then current likelihood function required taking logarithms of these observed values, which produced a mathematical problem (one can't take a logarithm of zero!). It was thus decided to move to a binomial probability distributional assumption for the F% likelihood function calculation. The F% likelihood contribution is now calculated as follows:

$$- \ln L(F\%) = -N\sum\limits_y \left[ F_y^{\%,obs} \ln \hat{F}_y^\% - F_y^{\%,obs} \ln F_y^{\%,obs} + (1 - F_y^{\%,obs}) \ln(1 - \hat{F}_y^\%) - (1 - F_y^{\%,obs})(\ln(1 - F_y^{\%,obs})) \right]$$

where

$N$ is the sample size

$F_y^{\%,obs}$ is the observed female **proportion** by number in catch in year $y$

$\hat{F}_y^\%$ is the model estimated female **proportion by number** in catch in year $y$

To ensure one does not have a computational problem with taking logarithms of a zero value, the following also applies (based on the fact that the limit as $x \to 0$ of $x \ln x$ is 0):

If $F_y^{\%,obs} = 0$

   X=0

else

   $X = \ln F_y^{\%,obs}$

and

If $F_y^{\%,obs} = 1$

   Y=0

else

   $Y = \ln(1 - F_y^{\%,obs})$

where

$$-\ln L(F\%) = -N\sum_y \left[ F_y^{\%,obs} \ln \hat{F}_y^{\%} - F_y^{\%,obs} X + (1 - F_y^{\%,obs}) \ln(1 - \hat{F}_y^{\%}) - (1 - F_y^{\%,obs})Y \right] \tag{A13.10}$$

**Calculation of the sample size $N$**

We do not have the various sample sizes, but we require an estimate of $N$ for the above $-\ln L$ equation. What is done is to calculate an approximation to the effective sample size based on the model fit as follows:

For each F% series calculate $N$, so that $\quad N = \dfrac{\sum_y \hat{F}_y^{\%}(1 - \hat{F}_y^{\%})}{\sum_y (F_y^{\%,obs} - \hat{F}_y^{\%})^2}$ . $\qquad\qquad$ (A13.11)

This was done separately for each super-area (and area-aggregated scenario) and for both RC1 and RC2. The $N$ values for each area were then averaged over RC1 and RC2.

**F% $\sigma$ values**

The F% $\sigma$ values are calculated as follows:

$$\sigma_{F\%} = \sqrt{\dfrac{\sum_y (F_y^{\%,obs} - \hat{F}_y^{\%})^2}{n}} \tag{A13.12}$$

**RC2 Stock-recruit function penalty**

For RC2, a penalty ($SR_{pen}$) is added to the likelihood function as follows:

The recruitment following a Beverton-Holt stock-recruit curve is calculated for the following years (years in which the recruitment parameters are estimated): 1920, 1950, 1970, 1975, 1980, 1985, 1990 and 1995.

Beverton-Holt curve:

$$R_y^{BH} = (\alpha Egg_y)/(\beta + Egg_y) \tag{A13.13}$$

where
$h = 0.8$

$\sigma_R = 0.45$

$\alpha = (4hR_{1870})/(5h - 1)$

$\beta = (Egg_{1870}(1 - h))/(5h - 1)$

The penalty is:

$$SR_{pen} = \sum_y [\ln \sigma_R + (\ln R_y^{BH} - \ln \hat{R}_y)^2 / 2\sigma_R^2 \qquad (A13.14)$$

where $y$ is summed over the years mentioned above and $\sigma_R$ is set equal to 0.45.

# Appendix: Somatic growth model

In order to compute the entries of the projection matrix, in the size-structured population model (see section A.8), estimates of the mean annual growth (moult) increment values for each size class are needed. Values are calculated separately for males ( $g_{l,t}^{m}$ ) and females ( $g_{l,t}^{f}$ ). Adult lobsters (70mm carapace length and larger) are assumed to moult once a year. Juveniles are known to moult several times a year, so that both the moult increment and moult frequency need to be taken into account when calculating the annual moult increment values for juveniles.

## *App1.1 Mature males*
The annual moult increment for a male lobster length *l* for year *t* $g_{l,t}^{m}$, is modelled:

$$g_{l,t}^{m} = \beta_t^{m} + \rho l + \varepsilon \qquad \qquad \text{(App1.1)}$$

See RLWS/DEC05/DAT/6/1/3/3 and RLWS/DEC05/DAT/6/1/3/4 for details on the GLM. The GLM assumes that the slope parameter ($\rho$) remains constant over time and that all growth variation can be modelled by a parallel shift of the growth curve each year, thus changing the growth axis-intercept value $\beta$. Linear interpolation is used ofr years where data are missing.

## *App1.2 Mature females*
Moult increment data for female lobsters are scarce, and limited to two periods only: 1969-1971 (Newman and Pollock 1977) and 1992 (SFRI data, unpublished). For 1969, data are available from Dassen Island, Cape Point and Robben Island. For 1992, data are available for Cape Point and Hout Bay. As for males, a GLM approach was used to calculate a $\beta_t^{f}$ value for each of the two data periods. The slope parameter is estimated to be $\rho = -0.0206$. As with males:

$$g_{l,t}^{f} = \beta_t^{f} + \rho l + \varepsilon \qquad \qquad \text{(App1.2.1)}$$

In order to calculate the $\beta_t^{f}$ values for the years for which no data are available, linear interpolation is used where the female $\beta_t^{f}$ values are assumed to follow the same interannual fluctuating pattern as calculated for the male $\beta_t^{m}$ values.

## *App1.3 Juveniles*
Juvenile annual moult increments refer to lobsters smaller than 60mm. Male and female lobsters are assumed to have equal juvenile moult increments. Unlike mature lobsters which are assumed to moult only once a year, juvenile lobsters moult several times a year, with the moult frequency declining gradually with increasing size. The annual moult increment calculation for a juvenile lobster must therefore take the moult frequency or inter-moult period, as well as the inter-moult growth increment, into account.

### *App1.3.1. The juvenile inter-moult period*
Only limited juvenile moult frequency data are available (Pollock 1973). A linear model is fitted to this data:

$$p_l = \alpha + \beta l \qquad \qquad \text{(App1.3.1.1)}$$

where $p_l$ is the inter-moult period in days for length *l* (mm), when fitted to these data yields

$\alpha = 73.70$ days, and
$\beta = 1.36$ days.mm$^{-1}$.

### *App1.3.2. The juvenile moult increment model*
Data are available for three periods: 1973, 1993 and 1995. The quadratic growth model for juveniles for each period is:

$$\Delta g_{l,1973}^{juv} = \alpha + \beta l + \gamma l^2 + \varepsilon \qquad \qquad \text{for 1973} \quad l \leq 60$$

(App1.3.2.1)

$$\Delta g_{l,1993}^{juv} = \alpha + \beta l + \gamma l^2 + \varepsilon \qquad \qquad \text{for 1993} \quad l < 15mm$$

$$\Delta g_{l,1993}^{juv} = \delta_1^{1993} (\alpha + \beta l + \gamma l^2 + \varepsilon) \qquad \qquad \text{for 1993} \quad 15 \leq l \leq 30 \ mm$$

$$\Delta g_{l,1993}^{juv} = \delta_2^{1993} (\alpha + \beta l + \gamma l^2 + \varepsilon) \qquad \qquad \text{for 1993} \quad 30 < l \leq 60 \ mm$$

$$\Delta g_{l,1995}^{juv} = \alpha + \beta l + \gamma l^2 + \varepsilon \qquad \qquad \text{for 1995} \quad l < 15mm$$

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$$\Delta g_{l,1995}^{juv} = \delta_1^{1995} (\alpha + \beta l + \gamma l^2 + \varepsilon) \qquad\qquad \text{for 1995} \quad 15 \le l \le 30 \text{ mm}$$

$$\Delta g_{l,1995}^{juv} = \delta_2^{1995} (\alpha + \beta l + \gamma l^2 + \varepsilon) \qquad\qquad \text{for 1995} \quad 30 < l \le 60 \text{ mm}$$

where

$l$ = the mean length before moult (mm)

$\Delta g_{l,t}^{juv}$ = the mean moult increment (mm) for a juvenile lobster in season $t$.

$\varepsilon = N(0, \sigma_l^2)$, and where $\sigma_l = \sigma_0 e^{\mu l}$ .

The model above was fitted to all three sets of juvenile moult increment data – see Table 1 for estimated parameter values. The net **annual** growth for juvenile lobsters is thus:

$$g_{l,t}^{juv} = \frac{\Delta g_{l,t}^{juv} .365}{p_l} \qquad\qquad \text{for 5mm} < l \le 60\text{mm} \qquad\qquad \text{(App1.3.2.2)}$$

Further, to avoid negative growth for very small lobsters, it is assumed that $g_{l,1970}^{juv} = g_{l,1994}^{juv}$ for $l \le 5$ mm.

*App1.3.3 Interpolation between adults and juveniles*

A straight line is fitted to the corresponding sets of relative 70mm male moult increments and the $\delta_1^{1993}$ and $\delta_1^{1995}$ , and the $\delta_2^{1993}$ and $\delta_2^{1995}$ values to calculate both $\delta_1^t$ and $\delta_2^t$ respectively in years for which there is no data, and hence no estimates of $\delta_1^t$ and $\delta_2^t$ .

The equations to be used for calculating $\delta_1^t$ and $\delta_2^t$ for all years are:

for $\delta_1^t$ :

$$\delta_1^t = mX^t + c \qquad\qquad\qquad \text{(App1.3.3.1)}$$

where

$X^t$ is the relative 70mm male moult increment in year $t$ = $\dfrac{g_{70}^t}{g_{70}^{1973}}$

$m = \dfrac{(1 - 0.734)}{(1 - 0.625)}$

$c = 1 - m$ ,

and similarly, for $\delta_2^t$ :

$$\delta_2^t = mX^t + c \qquad\qquad\qquad \text{(App1.3.3.2)}$$

where

$m = \dfrac{(1 - 0.715)}{(1 - 0.625)}$

$c = 1 - m$ .

**App1.4.Moult increment variance estimates**

The projection matrix calculations of require not only estimates of the mean annual moult increment for each length, but also estimates of the associated variances around these values. Standard deviations for moult increments have been calculated from the moult increment data. Moult increment variance models for male, female and juvenile lobsters are as follows. The variances for both adult male and female moult increments are considered to vary with the moult increment value.

*App1.4.1 Mature males*

$$\sigma_{l,g}^m = a_0 + a_1 (g - 2) + a_2 (l - 110) \qquad\qquad \text{for } l \le 110 \text{ mm} \qquad\qquad \text{(App1.4.1.1)}$$

$$\sigma_{l,g}^m = a_0 + a_1 (g - 2) + a_3 (l - 110) \qquad\qquad \text{for } l > 110 \text{ mm} \qquad\qquad \text{(App1.4.1.2)}$$

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where,

$g$         is the moult increment value

$l$          is the length before moult

$a_0 = 1.802$

$a_1 = 0.137$

$a_2 = 0.00589$

$a_3 = -0.02241$

(See Butterworth *et a.l* 1999 for details).

*App1.4.2 Mature females*

$$\sigma_{l,g}^{f} = \left(\frac{0.351}{X}\right)\left(a_0 + a_1(g-2)\right)$$       (App1.4.2.1)

where,

$a_0 = 1.802$ mm,

$a_1 = 0.138$

and

$X$       the male average $\sigma(l,g)$ over the 21 years of growth data (1968-1998) calculated to be 1.88mm.

The female growth variances follow a similar but damped trend as for males for varying values of *g*. The *l* dependence for males is lesser, and marked only for larger lengths ($\geq 110$mm) seldom reached by females, so is neglected in this proposal.

*App1.4.3 Juveniles*
There is a standard deviation associated with
          i) the inter-moult period *SD(p)*, and
          ii) with the inter-moult growth increment *SD(Δg)*.

The $SD(p) = \sigma_p$ (a constant) was estimated from the linear model fit to Pollock's (1973) data. The $SD(\Delta g) = \sigma_0 e^{\mu l}$ was estimated in the process of fitting the quadratic function to the available data. The estimated values are as follows:

$\sigma_p = 6.57$ days

$\sigma_0 = 0.265$ mm

$\mu = 0.0044$ mm$^{-1}$

If equation (App1.3.2.2) is rewritten with simplified notation:

$$g = \frac{\Delta g . 365}{p}$$

then, using a delta method of approximation, the coefficient of variation (CV) of *g* is given by:

$$CV(g) = \sqrt{CV^2(\Delta g) + CV^2(p)}$$       (App1.4.3.1)

where

$$CV(\Delta g) = \frac{SD(\Delta g)}{\Delta g}$$

and

$$CV(p) = \frac{SD(p)}{p}$$

so that

$$\sigma_g = gCV(g) \qquad = \frac{\Delta g.365}{p}\sqrt{\left(\frac{\sigma_0 e^{\mu l}}{\Delta g}\right)^2 + \left(\frac{\sigma_p}{p}\right)^2}$$       (App1.4.3.2)

Note: Unlike the variance for adult lobster growth, the juvenile variance is time-dependent (as Δ*g* is time-dependent).

*App1.4.4 Interpolation between juveniles and adults*
As with the annual moult increment values, linear interpolation is used to provide the standard deviation estimates between 60mm (juvenile) and 70mm (adult) for male lobsters, and between 50mm (juvenile) and 70mm (adult) for females.

***References***

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Table 1: Best fit parameter values for the juvenile moult increment model.

| Parameter | Best fit value |
|---|---|
| $\alpha$ | -0.439 mm |
| $\beta$ | 0.2345 |
| $\gamma$ | -0.0027 mm$^{-1}$ |
| $\sigma_0$ | 0.265 mm |
| $\mu$ | 0.0044 mm$^{-1}$ |
| $\delta_1^{1993}$ | 0.845 |
| $\delta_2^{1993}$ | 0.785 |
| $\delta_1^{1995}$ | 0.714 |
| $\delta_2^{1995}$ | 0.705 |