The Age-Structured Production Model for the south coast rock lobster population

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The south coast rock lobster resource is modelled using an age-structured-production-model (ASPM).

1. The population model

The resource dynamics are modelled by the equations:

$$N_{y+1,0} = R_{y+1} \tag{1}$$

$$N_{y+1,a+1} = N_{y,a} e^{-(M_a + S_a F_y)} = N_{y,a} e^{-Z_{y,a}} \tag{2}$$

$$N_{y+1,m} = N_{y,m-1} e^{-(M_{m-1} + S_{m-1} F_y)} + N_{y,m} e^{-(M_m + S_m F_y)} \tag{3}$$

where

$N_{y,a}$ is the number of lobsters of age a at the start of year y,

M_a denotes the natural mortality rate on lobsters of age a,

S_a is the age-specific selectivity,

F_y is the fully selected fishing mortality in year y, and

m is the maximum age considered (taken to be a plus-group).

The number of recruits at the start of year y is related to the spawner stock size by a stock-recruitment relationship:

$$R_y = \frac{\alpha B_y^{sp}}{\beta + (B_y^{sp})^\gamma} e^{\varsigma_y} \tag{4}$$

where

α, β and γ are spawner biomass-recruitment parameters ($\gamma = 1$ for a Beverton-Holt relationship),

ς_y reflects fluctuation about the expected (median) recruitment for year y, and

B_y^{sp} is the spawner biomass at the start of year y, given by:

$$B_y^{sp} = \sum_{a=1}^{m} f_a w_a N_{y,a} \tag{5}$$

where w_a is the begin-year mass of fish at age a and f_a is the proportion of fish of age a that are mature.

In order to work with estimable parameters that are more meaningful biologically, the stock-recruit relationship is re-parameterised in terms of the pre-exploitation

equilibrium spawning biomass, K^{sp}, and the "steepness" of the stock-recruit relationship (recruitment at $B^{sp} = 0.2K^{sp}$ as a fraction of recruitment at $B^{sp} = K^{sp}$):

$$\alpha = \frac{\left(5 - 0.2^{\gamma-1}\right)hR_1\left(K^{sp}\right)^{\gamma-1}}{5h - 1} \tag{6}$$

and

$$\beta = \frac{\left(K^{sp}\right)^{\gamma}\left(1 - 0.2h\right)^{\gamma-1}}{5h - 1} \tag{7}$$

where

$$R_1 = K^{sp} / \left[\sum_{a=1}^{m-1} f_a w_a e^{-\sum_{a'=0}^{a-1} M_{a'}} + f_m w_m \frac{e^{-\sum_{a'=0}^{m-1} M_{a'}}}{1 - e^{-M_m}} \right] \tag{8}$$

The total catch by mass in year y is given by:

$$C_y = \sum_{a=0}^{m} w_{a+\frac{1}{2}} N_{y,a} \frac{S_a F_y}{Z_{y,a}} (1 - e^{-Z_{y,a}}) \tag{9}$$

where $w_{a+\frac{1}{2}}$ denotes the mid-year mass of a lobster at age a.

The model estimate of mid-year exploitable biomass is given by:

$$\hat{B}_y = \sum_{a=0}^{m} w_{a+\frac{1}{2}} S_a N_{y,a} e^{-(Z_{y,a})/2} \tag{10}$$

where

$\hat{B}_y$ is the model estimate of exploitable biomass for year y, and

S_a is the fishing selectivity-at-age for age a.

Models that do not allow for the possibility of fluctuations about the stock-recruitment relationship (i.e. those which set $\varsigma_y = 0$ in equation 4) assume that the resource is at the deterministic equilibrium that corresponds to an absence of harvesting at the start of the initial year ($B_{1973}^{sp} = K^{sp}$). For models that allow for that possibility, this assumption together with that of the associated equilibrium age-structure is made for 1973, with the biomass and age-structure thereafter potentially impacted by such fluctuations.

Commercial selectivity-at-age: The following time-invariant logistic curve is assumed for the commercial selectivity:

$$S_a = \frac{1}{1 + e^{(-\ln(19)(a - a_{50})/(\Delta))}} \tag{11}$$

where

a_{50} is the age-at-50% selectivity which is estimated,

$\Delta = a_{95} - a_{50}$, which is estimated, and where

a_{95} is the age-at-95% selectivity.

Time-varying selectivity-at-age:

In some models the selectivity function (which depends on age) is allowed to vary over the time period for which catch-at-age data are available (1994-2003). To effect this, the form of the selectivity function is generalised to:

$$S_{y,a} = \frac{1}{1 + e^{-K(a-(a50+\delta_y))}} \quad \text{where} \quad K = \frac{\ln 19}{\Delta} \tag{12}$$

The estimable parameters are thus: $a50$, Δ and δ_y for y = 1994-2003 (excluding 1999 as there are no catch-at-age data for 1999).

It is also assumed that for $y < 1994$, 1999, and 2004+ the $\delta_y = 0$.

An extra term is added to the likelihood function in order to smooth the extent of change in the selectivity, as follows:

$$-\ln L = -\ln L + \sum_{y=1994}^{y=2003} \left(\frac{\delta_y}{\sigma_{sel}}\right)^2 \text{ (sum excludes 1999)} \tag{13}$$

where the σ_{sel} is input (a value of 0.75 was found to provide reasonable performance).

Another issue is that for equation (1), if δ_y decreases, this means that selectivity is increasing on younger lobsters, while given that the model fitting procedure assumes that

$$C\hat{P}UE_y = q\sum_a w_a S_{y,a} N_{y,a} \tag{14}$$

this situation seems implausible, in that an enhanced CPUE would result even if there was no increase in abundance.

Presumably enhanced catches of younger animals are achieved by spatially redistributing effort on a scale finer than captured by the GLM standardisation of the CPUE. A standard method to adjust for this, while maintaining a constant catchability coefficient q, is to renormalise the selectivity function in some way:

$$S_{y,a}^* = S_{y,a} / X_y \tag{15}$$

where here as a simple initial approach we have chosen:

$$X_y = \sum_{a1}^{a2} \frac{S_{y,a}}{a2-a1+1} \tag{16}$$

i.e., normalising selectivity by its average over a certain age range, so that now if δ_y decreases, the $S_{y,a}^*$ will decrease for large a to compensate for the effort spread to locations where younger animals are found associated with the increase for smaller a.

The authors experimented with choices for $a1$ and $a2$. A choice of $a1=8$ and $a2=12$ as a standard gave reasonable performance. The table below shows the effect of increasing $a2$ from 12 to 16.

	$\sigma_{sel} = 0.75$	$\sigma_{sel} = 0.75$	$\sigma_{sel} = 0.75$
$a1$	8	8	8
$a2$	**12**	**13**	**16**
σ CPUE	0.146	0.153	0.177
-lnL CPUE	-38.43	-37.16	-33.01
σ catch-at-age	0.057	0.057	0.056
-lnL catch-at-age	-113	-113	-114
-lnL sel	5.25	5.39	4.47
-lnL (TOTAL)	-144.47	-143.84	-141.52
MSY (MT)	423	435	426
B_{04}^{exp}/K	0.162	0.203	0.264
MSYL	0.164	0.153	0.160

What was found was that for values of $a2$ larger than 12, the fit to the observed CPUE deteriorated quite considerably, and the model was not able to reproduce the recent upward trend in CPUE.

2. The likelihood function

The model is fitted to CPUE and catch-at-age or catch-at-length data to estimate model parameters. Contributions by each of these to the negative log-likelihood (-ln*L*) are as follows:

2.1 Relative abundance data (CPUE)

The likelihood is calculated assuming that the observed abundance index is log-normally distributed about its expected (median) value:

$$CPUE_y = qB_y e^{\varepsilon_y} \text{ or } \varepsilon_y = \ln(CPUE_y) - \ln(qB_y) \qquad (17)$$

where

   $CPUE_y$ is the CPUE abundance index for year y,

   B_y is the model estimate of mid-year exploitable biomass for year y given by equation 10,

   q is the constant of proportionality (catchability coefficient), and

   ε_y from $N(0, \sigma^2)$.

The contribution of the abundance data to the negative of the log-likelihood function (after removal of constants) is given by:

$$-\ln L = \sum_y \left[(\varepsilon_y)^2 / 2\sigma^2 + \ln\sigma \right] \qquad (18)$$

where

   σ is the residual standard deviation estimated in the fitting procedure by its maximum likelihood value:

$$\hat{\sigma} = \sqrt{1/n \sum_y \left(\ln CPUE_y - \ln \hat{q}\hat{B}_y \right)^2} \qquad (19)$$

where

   n is the number of data points in the CPUE series, and

   q is the catchability coefficient, estimated by its maximum likelihood value:

$$\ln \hat{q} = 1/n \sum_{y} \left(\ln CPUE_{y} - \ln \hat{B}_{y} \right) \qquad (20)$$

2.2 Catches-at-age

The contribution of the catch-at-age data to the negative of the log-likelihood function when assuming a log-normal error distribution and when making an adjustment to effectively weight in proportion to sample size is given by:

$$-\ln L = \sum_{y} \sum_{a} \left[\ln(\sigma_{age} / \sqrt{p_{y,a}}) + p_{y,a} \left(\ln p_{y,a} - \ln \hat{p}_{y,a} \right)^{2} / 2 \left(\sigma_{age} \right)^{2} \right] \qquad (21)$$

where

$p_{y,a} = C_{y,a} / \sum_{a'} C_{y,a'}$ is the observed proportion of fish caught in year y that

are of age a,

$\hat{p}_{y,a} = \hat{C}_{y,a} / \sum_{a'} \hat{C}_{y,a'}$ is the model predicted proportion of fish caught in year y

that are of age a, where:

$$\hat{C}_{y,a} = N_{y,a} \frac{S_{y,a} F_{y}}{Z_{y,a}} (1 - e^{-Z_{y,a}}) \qquad (22)$$

and σ_{age} is the standard deviation associated with the catch-at-age data, estimated in the fitting procedure by:

$$\hat{\sigma}_{age} = \sqrt{\left[\sum_{y} \sum_{a} p_{y,a} (\ln p_{y,a} - \ln \hat{p}_{y,a})^{2} / \sum_{y} \sum_{a} 1 \right]} \qquad (23)$$

Note that allowance is made for a "minus" group (lobsters age 8 and younger) in the catch-at-age contribution to the likelihood function, as well as for a "plus" group (lobsters aged 20 and over).

2.3 Catches-at-length (from Rademeyer 2003)

The predicted proportions-at-age ($\hat{p}_{y,a}$) are converted into proportions-at-length using the von Bertalanffy growth equation, assuming that the length-at-age distribution remains constant over time:

$$\hat{p}_{y,l} = \sum_{a} \hat{p}_{y,a} A_{a,l} \qquad (24)$$

where $A_{a,l}$ is the proportion of fish of age a that fall in the length group l (thus $\sum_{l} A_{a,l} = 1$ for all ages a).

The matrix A is calculated under the assumption that length-at-age is normally distributed about a mean given by the Von Bertalanffy equation (Brandão *et al.*, 2002), i.e.:

$$L_{a} \sim N^{*} \left[L_{\infty} \left(1 - e^{-\kappa(a-t_0)} \right); \theta_{a}^{2} \right] \qquad (25)$$

where

N^{*} is the normal distribution truncated at ± 3 standard deviations, and

θ_{a} is the standard deviation of length-at-age a, which is modelled to be proportional to the expected length-at-age a, i.e.:

$$\theta_{a} = \beta L_{\infty} \left(1 - e^{-\kappa(a-t_0)} \right) \qquad (26)$$

with β a parameter estimated in the model fitting process.

In this analysis, the growth curve and the extent of variability about it have been assumed to be constant over time.

Note that since the model of the population's dynamics is based upon a one-year time step, the value of β and hence the θ_a's estimated will reflect the real variability of the length-at-age as well as the 'spread' that arises from the fact that fish in the same annual cohort are not all spawned at exactly the same time, and that catching takes place throughout the year so that there are differences in the age (in terms of fractions of a year) of fish allocated to the same cohort.

The following term is then added to the negative log-likelihood:

$$- \ell n\, L^{\text{length}} = w_{len} \sum_y \sum_l \left[\ln\!\left(\sigma_{\text{len}} / \sqrt{p_{y,l}} \right) + p_{y,l} \left(\ln p_{y,l} - \ln \hat{p}_{y,l} \right)^2 / 2\left(\sigma_{len} \right)^2 \right] \quad (27)$$

where

$p_{y,l}$ is the observed proportion (by number) in length group l in the catch in year y, and

σ_{len} is the standard deviation associated with the length-at-age data, which is estimated in the fitting procedure by:

$$\hat{\sigma}_{len} = \sqrt{ \sum_y \sum_l p_{y,l} \left(\ln p_{y,l} - \ln \hat{p}_{y,l} \right)^2 / \sum_y \sum_l 1 } \quad (28)$$

Equation (27) makes the assumption that proportion-at-length data are log-normally distributed about their model-predicted values. The associated variance is taken to be inversely proportional to $p_{y,l}$ to downweight contributions from observed small proportions which will correspond to small predicted sample sizes.

The w_{len} weighting factor may be set at a value less than 1 to downweight the contribution of the catch-at-length data to the overall negative log-likelihood compared to that of the CPUE and survey data. The reason that this factor is introduced is that the $p_{y,l}^f$ data for a given year show evidence of strong positive correlation, and so are not as informative as the independence assumption underlying the form of equation 27 would otherwise suggest.

Selectivity-at-length function

Fitting to the catch-at-length data requires a selectivity-at-length function. This function is identical to that for the selectivity-at-age, except that parameters refer to length not age, i.e. for the time-invariant selectivity-at-length function

$$S_l = \frac{1}{1 + e^{(-\ln(19)(l - l_{50})/(\Delta))}} \quad (29)$$

where

     l_{50} is the length-at-50% selectivity which is estimated,

     $\Delta = l_{95} - l_{50}$, which is estimated, and where

     l_{95} is the length-at-95% selectivity.

2.4 Stock-recruitment function residuals

The assumption that these residuals are log-normally distributed and could be serially correlated defines a corresponding joint prior distribution. This can be equivalently regarded as a penalty function added to the log-likelihood, which for fixed serial correlation ρ is given by:

$$-\ln L = \sum_{y=y1}^{y2}\left[\frac{\varsigma_y - \rho\varsigma_{y-1}}{\sqrt{1-\rho^2}}\right]^2 / 2\sigma_R^2 \qquad (30)$$

where

$\varsigma_y = \rho\tau_{y-1} + \sqrt{1-\rho^2}\,\varepsilon_y$ is the recruitment residual for year y (see equation 4), which is estimated for years $y1$ to $y2$ if $\rho = 0$, or $y1+1$ to $y2$ if $\rho > 0$,

$\varepsilon_y \sim N(0,\sigma_R^2)$,

σ_R is the standard deviation of the log-residuals, which is input, and

ρ is their serial correlation coefficient, which is input.

Note that for the Reference Case assessment, ρ is set equal to zero, i.e. the recruitment residuals are assumed uncorrelated, and σ_R is set equal to 0.4. Because of the absence of informative age data for a longer period, recruitment residuals are estimated for years 1974 to 1995 only.

2.5 "Effort saturation"

When the possibility of "effort saturation" is taken into account, the CPUE abundance relationship of equation 11 is modified as follows:

$$CPUE_y = q_y B_y e^{\varepsilon_y} \text{ or } \varepsilon_y = \ln(CPUE_y) - \ln(q_y B_y) \qquad (31)$$

where

$$q_y = q'\left/\left[1+\left(\frac{E_y - E'}{E^* - E'}\right)^{n^*}\right]\right. \qquad \text{if } E_y > E' \qquad (32)$$

$$q_y = q' \qquad\qquad\qquad\qquad \text{if } E_y \le E'$$

where

$CPUE_y$ is the GLM standardised CPUE data,

E_y is the estimated effort given by $\dfrac{C_y}{CPUE_y}$,

$$q' = e^{\left(\sum_{y,E_y<E'}\left(\ln(CPUE_y)-\ln B_y\right)+\sum_{y,E_y\ge E'}\left(\ln\left(CPUE_y\left[1+\left(\frac{E_y-E'}{E^*-E'}\right)^{n^*}\right]\right)-\ln B_y\right)\right)/n}$$

E^* quantifies the extent of "effort saturation",

E' is the threshold effort above which "effort saturation" sets in, and

n^* allows for flexibility in the "effort saturation" relationship.

For this scenario, equation (17) is modified by replacing q with the q_y as defined above.

The effort saturation model includes fitting to the 1998 Effort Saturation Experiment data (Groeneveld *et al.* 1999). Considering the "full effort" exerted in Dec-Jan of the 1998 experiment as the standard, the extent of effort reduction (λ) and the associated relative change in CPUE (GLM-standardised to adjust for normal monthly trends),

$f^{obs}(\lambda)$, were as follows for the four area-period combinations considered in the experiment:

Area-period	λ	$f^{obs}(\lambda)$
1	0.93	1.25
2	1.24	1.30
3	1.15	1.04
4	0.60	0.71

When fitting directly to the data, the following penalty term needs to be added to the likelihood function:

$pen = 4\ln\sigma_E + 2$

where $\sigma_E = \sqrt{SS(E^*)/4}$,

where σ_E is the standard deviation of the residuals, and

$SS(E^*)$ is given by equation A2 in Butterworth (2000).

For the effort saturation model, parameters E' and n^* are fixed at 2500 and 1.0 respectively. Thus the extent of effort saturation is determined soley by E^*.

3. Further Model parameters

Natural mortality: Natural mortality, M_a, is assumed to be the same (M) for all age classes.

Age-at-maturity: The proportion of lobsters of age a that are mature is approximated by $f_a = 1$ for $a > 9$ years (i.e. $f_a = 0$ for a = 0, …,9).

Minimum age: Age 8 it taken to be a minus group.

Maximum age: $m = 20$, and is taken as a plus-group.

Mass-at-age: The mass w of a lobster at age a is given by:

$$w = \alpha\left[l_\infty\left(1 - e^{-\kappa(a-t_0)}\right)\right]^\beta \qquad (33)$$

where the values assumed for the growth parameters are shown in Table 1.

Stock-recruitment relationship: The shape parameter, γ, is fixed to 1, corresponding to a Beverton-Holt form.

4. The Bayesian approach

The Bayesian method entails updating prior distributions for model parameters according to the respective likelihoods of the associated population model fits to the CPUE, catch-at-age and tag-recapture data, to provide posterior distribution for these parameters and other model quantities. Note that tag-recapture data were used for

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earlier assessments, but discarded when it became apparent that they had little impact on results.

In the case of an age-structured production model, the Bayesian computations require integration over the following priors:

- K^{sp} - the pristine spawning biomass in the first year (1973)
- The "steepness" of the stock-recruit relationship (h), and
- Natural mortality (M_a), assumed independent of age.

In addition, we integrate over the two parameters defining the shape of the selectivity-at-age curve (a_{50} and a_{95}).

Furthermore, priors for the parameters characterising the postulated "effort saturation" effects (E^*, E' and n^*) of equation 32 are also required. Unfortunately, due to lack of informative priors (e.g. as derived from the effort saturation experiment results), uninformative prior distributions (i.e. no external information) have had to be assumed. The prior for E^* is uniform U[2500; 15 000].

The catchability coefficient (q) and the standard deviations associated with the CPUE and catch-at-age data (σ and σ_{age}) are estimated in the fitting procedure by their maximum likelihood values, rather than integrating over these three parameters as well. This is adequately accurate given reasonably large sample sizes (Walters and Ludwig 1994, Geromont and Butterworth 1995).

Modes of posteriors, obtained by finding the maximum of the product of the likelihood and the priors, are then estimated rather than performing a full Bayesian integration, due to the time intensiveness of the latter.

4.1 Priors
The following prior distributions are assumed:

$\ln K$ U[7.6, 9.9] corresponding to values for K of 1998 and 19930.

h: N(0.95, SD^2) with SD=0.2, where the normal distribution is truncated at $h = 1$.

M: U[0.1, 0.25]

a_{50}:   U[1, 19]

Δ U[0, 10 [remember $a_{95} = a_{50} + \Delta$]

SR residuals ς_y $N(0, \sigma_R^2)$ where σ_R =04., bounded by [-2, 2]

Time varying δ_y $N(0, \sigma_{sel}^2)$ where σ_{sel} =0.75, bounded by [-5, 5]   (used for the scenario where selectivity varies with time i.e. Model 2)

4. Projections

The population is projected forwards for a ten-year period till the start of 2016. Thus the future catches are for 2006-2015. The equations used to update the population each year in the projection period are the catch equations of Popes' approximation, rather than the Baranov equations used for the pre-2006 period. Appendix 1 provides justification for this simplification which eases computation.

Projections of Numbers-at-age

$$N_{y+1,0} = R_{y+1} \tag{34}$$

$$N_{y+1,a+1} = \left(N_{y,a}\, e^{-M_a/2} - C_{y,a}\right) e^{-M/2} \qquad \text{for } 0 \le a \le m-2 \tag{35}$$

$$N_{y+1,m} = \left(N_{y,m-1}\, e^{-M/2} - C_{y,m-1}\right) e^{-M/2} + \left(N_{y,m}\, e^{-M/2} - C_{y,m}\right) e^{-M/2} \tag{36}$$

where

$N_{y,a}$ is the number of lobsters of age a at the start of year y,

R_y is the recruitment (number of 0-year-old lobsters) at the start of year y,

M denotes the natural mortality rate ,

$C_{y,a}$ is the number of lobsters of age a caught in year y, and

m is the maximum age considered (taken to be a plus-group).

Total catch and catches-at-age

The catch by mass in year y is given by:

$$C_y = \sum_{a=0}^{m} w_{a+1/2}\, C_{y,a} = \sum_{a=0}^{m} w_{a+1/2}\, N_{y,a}\, e^{-M/2}\, S_{y,a}\, F_y \tag{37}$$

where

$w_{a+1/2}$ denotes the mid-year mass of fish of age a,

$C_{y,a}$ is the catch-at-age, i.e. the number of lobsters of age a, caught in year y,

$S_{y,a}$ is the commercial selectivity (i.e. vulnerability to fishing gear) at age a for year y; when $S_{y,a} = 1$, the age-class a is said to be fully selected, and

F_y is the fished proportion for a fully selected age class a.

The model estimate of the mid-year exploitable ("available") component of biomass for each fleet is calculated by converting the numbers-at-age into mid-year mass-at-age (using the mid-year individual weights) and applying natural and fishing mortality for half the year:

$$B_y^{ex} = \sum_{a=0}^{m} w_{a+1/2} S_{y,a} N_{y,a} e^{-M/2} (1 - S_{y,a} F_y / 2) \tag{38}$$

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Table 1: Somatic growth parameters as detailed in Glazer and Groeneveld (1999).

α (w in gm)	0.0007
β	2.846
l_∞ (mm CL)	111.9
κ (year^{-1})	0.08
t_0 (years)	0.0

Appendix: Baranov *vs* Pope's Approximation

The south coast rock lobster resource is modelled using an age-structured-production-model (ASPM) using Baranov catch equations (Baranov 1918). These equations take catches into account in the form of continuous fishing mortality. Using the Baranov catch equations requires estimation of the annual fishing mortalities when fitting the model to data. This clearly greatly increases the number of estimable parameters and speed of computations. Whilst this is not too much of a problem for the model estimation procedure, the use of the Baranov catch equations when projecting into the future becomes more problematic, particularly when developing a feedback-type OMP. The authors thus use Pope's approximation (Pope 1984) to the catch equations for the projection period. Pope's approximation equations assume that the catches are taken as a pulse in the middle of the year. As long as fishing mortality rates are not very high, the differences between the Baranov and Pope's equations will be minimal. In order to illustrate this, the authors compare the reference case assessment of the south coast rock lobster resource using both types of equations (see Table A1). As expected, the results are almost identical.

Table A1: Stock assessment results for the current Reference Case using either the Baranov catch equations (the current norm) or Pope's approximation equations. Units of mass-related quantities (e.g. *MSY*) are tons.

	Baranov	Pope's approximation
K^{sp}	8299	8301
h	0.857	0.850
M	0.107	0.106
a_{50}	10.08	10.01
a_{95}	12.49	12.42
σ	0.184	0.184
σ_{age}	0.070	0.070
σ_{length}	-	-
-lnL CPUE	-32.21	-32.18
-lnL age	-88.77	-88.76
-lnL S-R	3.20	3.33
-lnL(total)	**-118.27**	**-118.18**
MSY	365	364
$MSYL^{exp}/K$	0.218	0.222
B_{05}^{sp}	2545	2533
B_{05}^{exp}	2261	2277
B_{2004}^{exp}/K^{exp}	0.298	0.298
$B_{2004}^{exp}/B_{msy}^{exp}$	1.358	1.343
B_{2004}^{sp}/K^{sp}	0.322	0.322