The 2007 age-structured production model assessments and projections for the South Coast rock lobster resource – routine update using model fitting to catch-at-age data

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Executive Summary

The assessment conducted in 2006 (WG/06/06/WCRL3) has been routinely extended, taking account of a further year's catch, CPUE and catch-at-age data.

The observed CPUE shows a slight decrease for 2005 (2005/06 season). The sustainable yield estimates are generally very similar to those for the 2006 assessment, although estimates of current biomass levels relative to *K* increase. The Reference Case (RC) scenario suggests that a TAC of about 330 MT or less would be appropriate to prevent biomass decline in the future. The other two scenarios reported suggest higher values than this. If the catch-at-age data are down-weighted, then this 360 MT level for the TAC is increased to 390 MT.

Introduction

The age-structured production model which fits to catch-at-age data, and which has been applied previously to South Coast rock lobster, has been used to update the assessment of the resource and to provide a range of projections into the future for a number of harvesting policies. The age-structured production model is unchanged from that initially described by Geromont (2000a) and used for the 2001-2006 assessments (Johnston and Butterworth 2001; 2002a; 2003a; 2003b, 2004, 2005, 2006). The age-structured model is reported in detail here in the Appendix. Note that his model is sex- and area-aggregated.

The Reference Case (RC) "Bayesian" ASPM assessment as considered for 2007 involves the following choices (essentially unchanged from 2003-2006 except for taking the extra year into account).

1. Standard priors for P, h[1], M, a_{50}, a_{95}.
2. Use of GLM-standardised CPUE for 1977-2005[2].

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[1] The prior for h is a truncated (at 1.0) normal distribution with mean of 0.95 and σ =0.2
[2] In this report the year "2000", for example, refers to the 2000/01 season

3. Use of scientific-sample-based catch-at-age data for 1994-2005, with an 8- and 20+ grouping. Note that the Working Group agreed that the 1999 scientific catch-at-age data should not be included in the RC assessment due to poor spatio-temporal coverage for that season that may render them unrepresentative.
4. A Beverton-Holt stock recruit relationship.
5. Deterministic recruitment, except for estimation of recruitment residuals from 1974-1997 (i.e. one more year included than last year) with zero serial correlation ($\rho = 0$) and CV (σ_R) of 0.4.

Data

The annual total catch (by mass) (C_y) and relative abundance index ($CPUE_y$) data used are reported in Table 1a. The relative abundance index corresponds to the standardised CPUE time series provided by Glazer (pers. commn). The commercial catches-at-age ($C_{y,a}$) derived from the updated scientific length data are given in Table 2 (Bergh pers. commn). Table 3 summarises the somatic growth curve parameter values (Glazer and Groeneveld 1999) used in this process.

Sensitivity analyses

In addition to the RC, results for the following sensitivity analyses are also reported in Table 4a.

1) Effort Saturation

This scenario examines the possibility that the proportional relationship between CPUE and biomass does not hold true at high levels of effort due to competition between units of effort – i.e. effort saturation occurs. This effort saturation effect is taken into account here by allowing the constant of proportionality between the GLM derived CPUE index and exploitable biomass, q, to become a declining function of fishing effort once effort exceeds a certain level (see the appendix equations 15 and 16 for details). This analysis also includes fitting to the 1998 Effort Saturation Experiment data (Groeneveld *et al.* 1999). For this application, parameters E' and n^* are fixed at 2500 and 1.0 respectively (see Model 5c of Geromont 2000b). Thus the extent of effort saturation is determined by the parameter E^* alone. In previous stock assessment scenarios that have taken effort saturation into account, the approach was formulated slightly differently (the observed CPUE series was "detrended" to take account of effort saturation), but the resultant computations are mathematically identical so yield the same results.

2) Catch-at-age down-weight

The catch-at-age data is down-weighted by a multiplicative factor of 0.10 in the likelihood function as an *ad hoc* approach to allow for positive correlations in these data.

Projections

The resource is projected ahead from 2007 to 2016 under a number of constant catch (CC) levels: 330 MT, 360 MT, 390 MT, 420 MT and 450 MT.

Results

Assessment results

The assessment results for the RC model and the two sensitivity analyses are presented in Table 4a, and correspond to Bayesian posterior modes. Table 4b compares the current results with those obtained from the 2006 assessment. Fits to CPUE data and catch-at-age data are illustrated in Figures 1 and 2 respectively. The RC and effort saturation (Sensitivity 1) fits to the CPUE data are shown in Figure 1a, and those for the catch-at-age down-weight (Sensitivity 2) scenarios in Figure 1b. Figures 3a and 3b show the estimated exploitable biomass and spawning biomass trends for the RC and effort saturation scenarios.

The estimated stock-recruit residuals for the RC, effort saturation and catch-at-age down-weight scenarios are illustrated in Figure 4.

Projections

Table 5 presents results of projected spawning biomass trends for the RC and the five sensitivity analyses for a range of future constant catches.

Discussion

The 2006 RC assessment of the south coast rock lobster resource estimated the resource at the start of 2005 to be 31% of carrying capacity for the exploitable portion of the stock, and 33% of capacity for the spawning biomass. The updated 2007 RC assessment estimates these values to now be 33% and 36% respectively (see Table 4b). Whilst these values are comparatively slightly higher than those estimates for the 2006 assessment, both the spawning biomass and exploitable biomass are now estimated to have declined slightly between the years 2005 and 2006. The MSY for the resource is estimated to be 363 MT for the RC model, and 404 and 440 for the two sensitivity analyses reported here.

The RC MSY estimate (363 MT) is almost identical to that estimated by the 2006 assessment (367 MT) – see Table 4b.

The effort saturation scenario results are more positive than those for the RC model. The ES model estimated CPUE is able to reproduce the observed CPUE trends, particularly in more recent years, to a better extent that the RC (Figure 1a).

Down-weighting the catch-at-age data once again results in a more optimistic appraisal of the resource. Through this down-weighting, this model is able to fit better the CPUE data (Figure 1b), in particular the recent upturn in CPUE. The fits to the catch-at-age data do however deteriorate substantially (see Figure 2), particularly for more recent years such as the 2000-2005 seasons for which there is appreciable overestimation of the proportion of small and underestimation of that of large lobsters. This once again points to the incompatibility of the CPUE and catch-at-age data within this model structure.

The projected spawning biomass trends estimated for the different future constant catch harvesting strategies, are rather different across the various scenarios (see Table 5 for the RC and five sensitivity scenarios). The RC predicts that catches of a little less than 330 MT will result in the spawning biomass remaining at its current (2006) level. The two sensitivity scenarios produce slightly more optimistic results indicating an appropriate TAC of around 360 MT and 380 MT.

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Table 1: Total annual catch scenarios (data from WG/06/04/SCRL1) and GLM standardised CPUE (Glazer 2006a) data for the South Coast rock lobster fishery.

	RC	**Sensitivity 1: Historic Catches= MCM records+ over-catches**	**Sensitivity 2: Over-catches 87-97 set=100 tons per year**	
Year	**Total Catch (MT tails)**	**Total Catch (MT tails)**	**Total Catch (MT tails)**	**CPUE (kg tails/trap)**
1973	372	372	372	
1974	973	973	973	
1975	551	551	551	
1976	712	712	712	
1977	667	667	667	0.2187
1978	461	461	461	0.2059
1979	122	122	122	0.1607
1980	176	176	176	0.2041
1981	348	348	348	0.1930
1982	407	407	407	0.1657
1983	524	524	524	0.1958
1984	450	450	450	0.1625
1985	450	450	450	0.1589
1986	450	450	450	0.2074
1987	452	452	552	0.1864
1988	452	452	552	0.2211
1989	452	452	552	0.2050
1990	477	477	577	0.1737
1991	524.54	524.54	577	0.1428
1992	529.96	529.96	577	0.1393
1993	524.27	524.27	577	0.1271
1994	507.89	507.89	552	0.1161
1995	504.89	472.99	527	0.1077
1996	442.69	428.39	515	0.0900
1997	416.39	384.09	502	0.0823
1998	516.03	460.73	516.03	0.0786
1999	512.16	514.86	512.16	0.0800
2000	423.4	378	423.4	0.0896
2001	288	288	288	0.0998
2002	340	325	340	0.1107
2003	350	350	350	0.1154
2004	382	382	382	0.1298
2005	382	382	382	0.1136
2006	382	382	382	

Table 2: Scientific sampling-based catches at-age (proportions) for the South Coast rock lobster. [Note that the 1999 values are omitted from the assessment because of poor sampling levels that season.]

AGE	1994	1995	1996	1997	1998	2000	2001	2002	2003	2004	2005
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0039	0.0000	0.0056	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
7	0.0003	0.0006	0.0140	0.0003	0.0201	0.0012	0.0001	0.0011	0.0009	0.0004	0.0004
8	0.0029	0.0093	0.0266	0.0066	0.0484	0.0069	0.0010	0.0190	0.0092	0.0075	0.0059
9	0.0215	0.0554	0.0478	0.0609	0.0834	0.0389	0.0105	0.0510	0.0218	0.0379	0.0223
10	0.0709	0.1265	0.0819	0.1467	0.1233	0.1166	0.0451	0.0767	0.0446	0.0690	0.0540
11	0.1441	0.1838	0.1202	0.2080	0.1429	0.2099	0.1119	0.0930	0.0816	0.0924	0.0989
12	0.1537	0.1369	0.1256	0.1373	0.0939	0.1648	0.1548	0.0986	0.1033	0.1106	0.1108
13	0.1493	0.1110	0.1184	0.1079	0.0844	0.1224	0.1552	0.1143	0.1278	0.1180	0.1186
14	0.1343	0.0829	0.1054	0.0775	0.0744	0.0782	0.1437	0.1242	0.1453	0.1196	0.1203
15	0.0677	0.0440	0.0603	0.0412	0.0462	0.0397	0.0762	0.0708	0.0868	0.0734	0.0733
16	0.0786	0.0548	0.0782	0.0498	0.0637	0.0461	0.0924	0.0927	0.1155	0.1003	0.1003
17	0.0386	0.0342	0.0419	0.0262	0.0361	0.0252	0.0459	0.0510	0.0564	0.0534	0.0557
18	0.0293	0.0319	0.0349	0.0215	0.0315	0.0213	0.0354	0.0434	0.0433	0.0443	0.0479
19	0.0238	0.0274	0.0296	0.0192	0.0271	0.0195	0.0290	0.0368	0.0372	0.03880	0.0419
20+	0.0849	0.1013	0.1113	0.0968	0.1192	0.1094	0.0990	0.1275	0.1266	0.1350	0.1498

Table 3: Somatic growth parameters as detailed in Glazer and Groeneveld (1999).

α (w in gm)	0.0007
β	2.846
l_∞ (mm CL)	111.9
κ (year^{-1})	0.08
t_0 (years)	0.0

Table 4a: Stock assessment results (Bayesian posterior modes) for the Reference Case and a number of sensitivity analyses. Units of mass-related quantities (e.g. *MSY*) are tons. Note that recruitment residuals from 1974 to 1997 are estimated in all instances.

	Reference Case	**Sensitivity 1: Effort saturation**	**Sensitivity 2: Catch-at-age log-likelihood down-weighted by 0.10 multiplier**
$\boldsymbol{K^{sp}}$	8466	7917	7327
h	0.884	0.868	0.924
M	0.099	0.123	0.134
a_{50}	10.11	10.08	11.25
a_{95}	12.52	12.44	13.75
n^*	-	1.0 fixed	-
E'	-	2500 fixed	-
E*	-	6829	-
σ	0.202	0.112	0.075
$\boldsymbol{\sigma_{age}}$	0.067	0.066	0.137
-*lnL* CPUE	-31.82	-48.87	-60.47
-*lnL* age	-115.86	-116.42	-12.16
-*lnL* S-R	4.44	6.22	5.72
-*lnL* effort expt	**-**	1.16	**-**
-*lnL*(total)	**-143.79**	**-160.79**	**-56.58**
MSY	363	404	440
$\boldsymbol{MSYL^{exp}/K}$	0.209	0.205	0.151
$B^{\exp}_{2006}/K^{\exp}$	0.311	0.351	0.359
$B^{\exp}_{2006}/B^{\exp}_{msy}$	1.489	1.712	2.386
B^{sp}_{2006}/K^{sp}	0.334	0.374	0.438
B^{sp}_{2016}/K^{sp} **CC=330 MT**	0.328	0.397	0.485
$B^{sp}_{2016}/B^{sp}_{06}$ **CC=330 MT**	0.989	1.067	1.109

Table 4b: Stock assessment results (Bayesian posterior modes) for the Reference Case analysis and three of the sensitivity analyses. Units of mass-related quantities (e.g. *MSY*) are tons. The results in parenthesis are those for the corresponding 2006 assessment (note that here all $B^{\exp}$ estimates refer to 2005 rather than 2006 as in Table 4a).

	Reference Case	**Sensitivity 1: Effort saturation**	**Sensitivity 2: Catch-at-age log-likelihood down-weighted by 0.10 multiplier**
h	0.884 (0.879)	0.868 (0.885)	0.924 (0.954)
$\boldsymbol{M}$	0.099 (0.102)	0.123 (0.127)	0.134 (0.138)
a_{50}	10.11 (10.07)	10.08 (10.04)	11.25 (11.26)
a_{95}	12.52 (12.47)	12.44 (12.36)	13.75 (13.74)
$\boldsymbol{E^*}$	-	6829 (6799)	-
$\boldsymbol{\sigma}$	0.202 (0.200)	0.112 (0.106)	0.075 (0.074)
$\boldsymbol{\sigma_{age}}$	0.067 (0.068)	0.066 (0.067)	0.137 (0.140)
$\boldsymbol{K^{sp}}$	8466 (8396)	7917 (7793)	7327 (7170)
MSY	363 (367)	404 (418)	440 (458)
$\boldsymbol{MSYL^{exp}/K}$	0.209 (0.210)	0.205 (0.194)	0.151 (0.134)
$B^{\exp}_{2005}/K^{\exp}$	0.331 (0.307)	0.378 (0.358)	0.359 (0.375)
$B^{\exp}_{2005}/B^{\exp}_{msy}$	1.582 (1.460)	1.843 (1.844)	2.384 (2.807)
B^{sp}_{2005}/K^{sp}	0.358 (0.333)	0.404 (0.384)	0.438 (0.466)

Table 5: Projected spawning biomass estimates for various harvesting strategies and models. Units of mass-related quantities (e.g. *RY*) are tons. [Shaded cells show a biomass reduction relative to 2006.]

Statistic	Strategy	Reference Case	Sensitivity1: Effort saturation	Sensitivity 2: Catch-at-age log-likelihood down-weighted by 0.10 multiplier
B^{sp}_{2006}/K^{sp}	ALL	0.334	0.374	0.438
B^{sp}_{2016}/K^{sp}				
	CC=450	0.219	0.288	0.373
	CC=420	0.245	0.315	0.401
	CC=390	0.273	0.342	0.429
	CC=360	0.300	0.370	0.457
	CC=330	0.328	0.397	0.485
$B^{sp}_{2016}/B^{sp}_{2006}$				
	CC=450	0.655	0.771	0.851
	CC=420	0.737	0.844	0.915
	CC=390	0.820	0.919	0.980
	CC=360	0.905	0.993	1.044
	CC=330	0.989	1.067	1.109

Figure 1a: Observed and estimated CPUE for the Reference Case (RC) and effort saturation (ES – Sensitivity 1) scenarios.

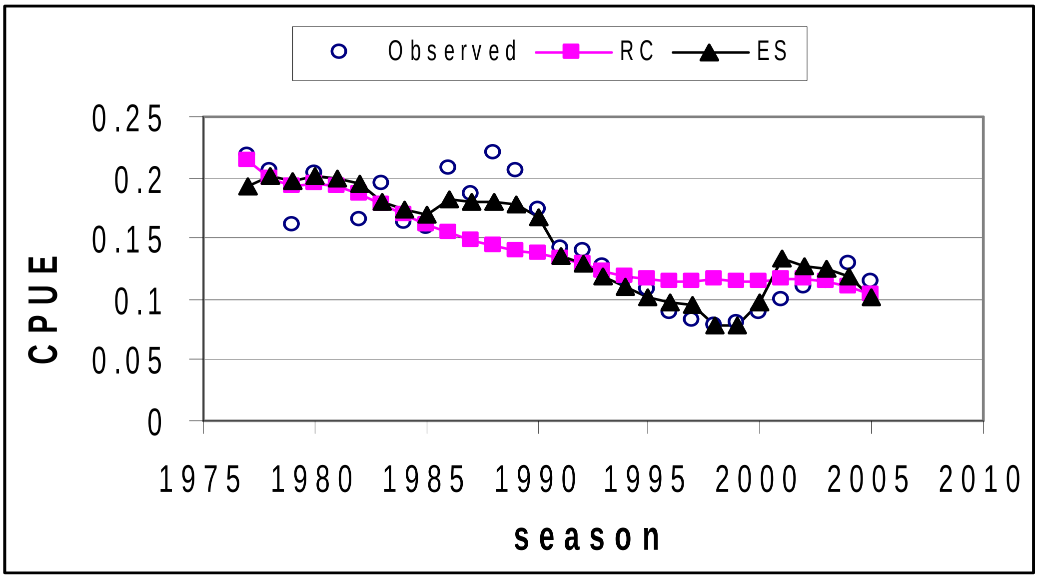


Figure 1b: Observed and estimated CPUE for the catch-at-age down-weight (cdw – Sensitivity 2) scenario.

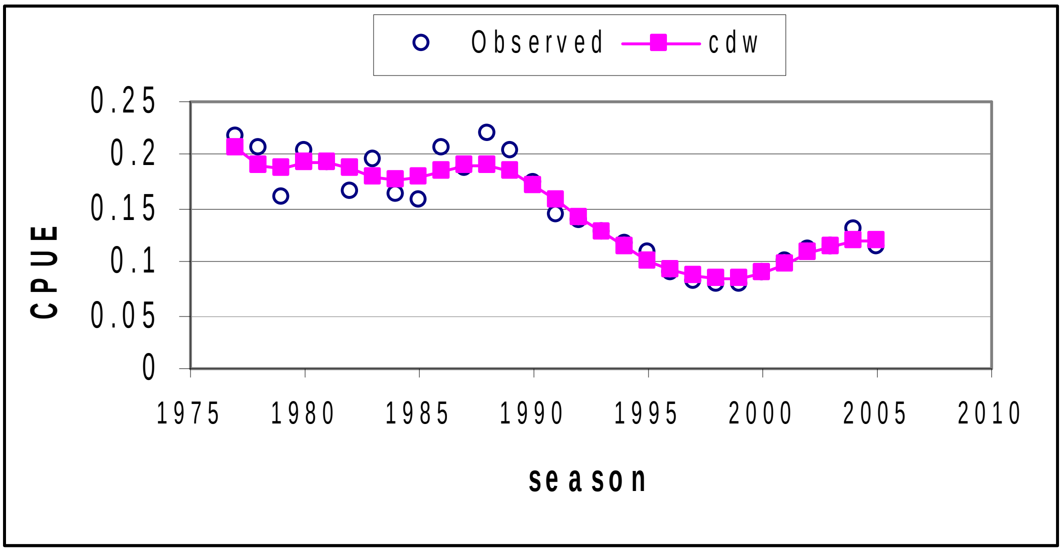


Figure 2: Observed and estimated catch-at-age proportions for the Reference Case (RC) and catch-at-age down-weight (cdw – Sensitivity 5) scenarios.

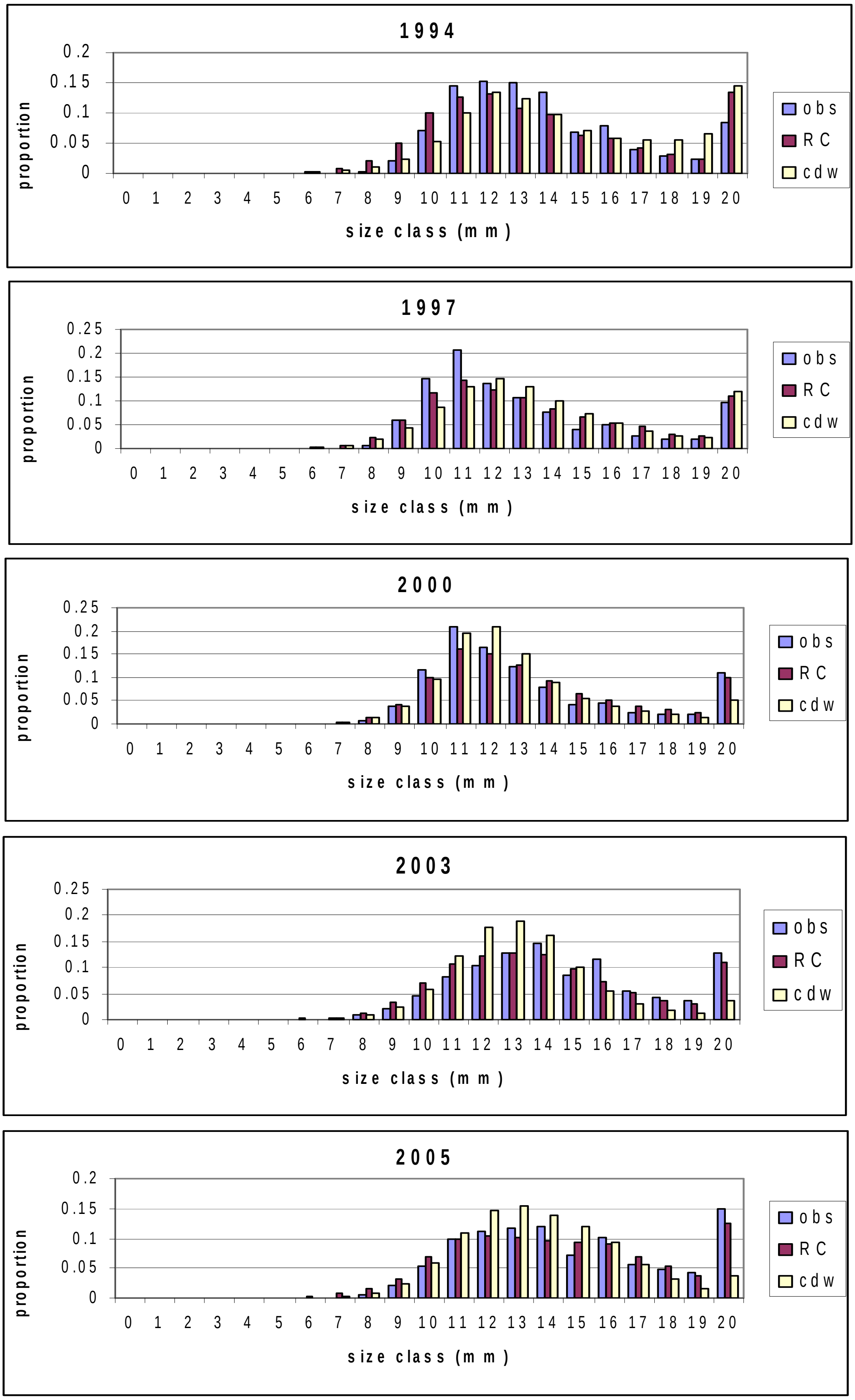


Figure 3a: Exploitable biomass trends for the Reference Case and effort saturation (Sensitivity 1) scenarios.

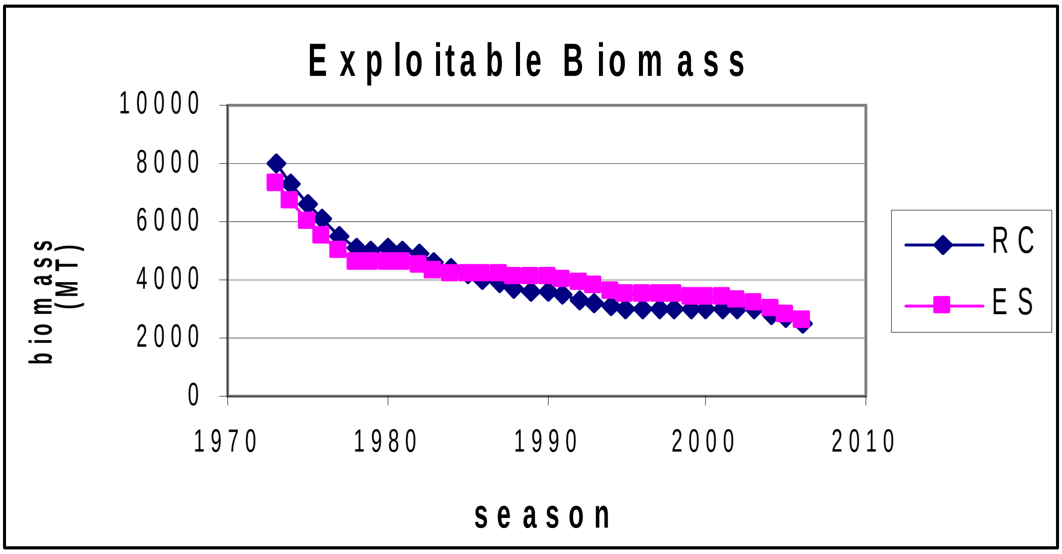


Figure 3b: Spawning biomass trends for the Reference Case and effort saturation (Sensitivity 1) scenario.

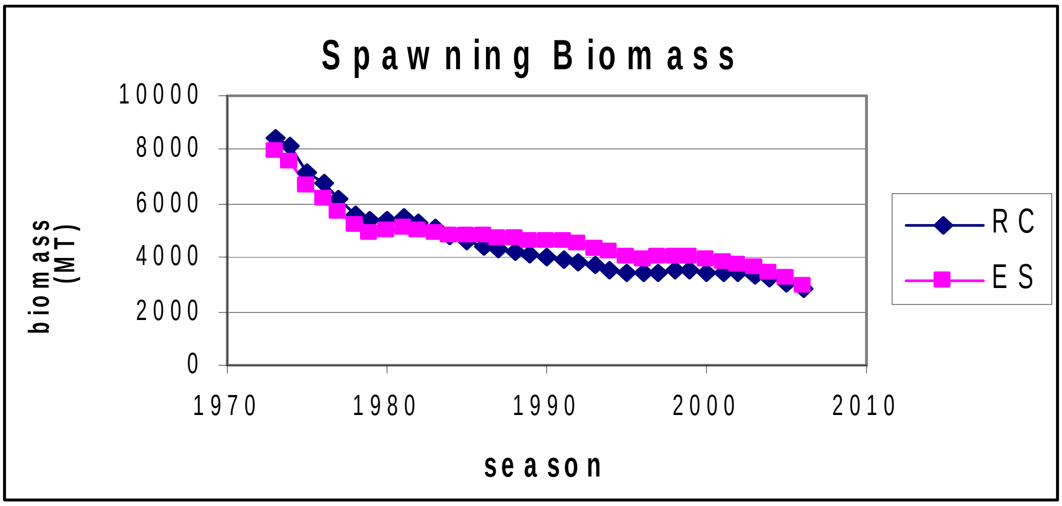
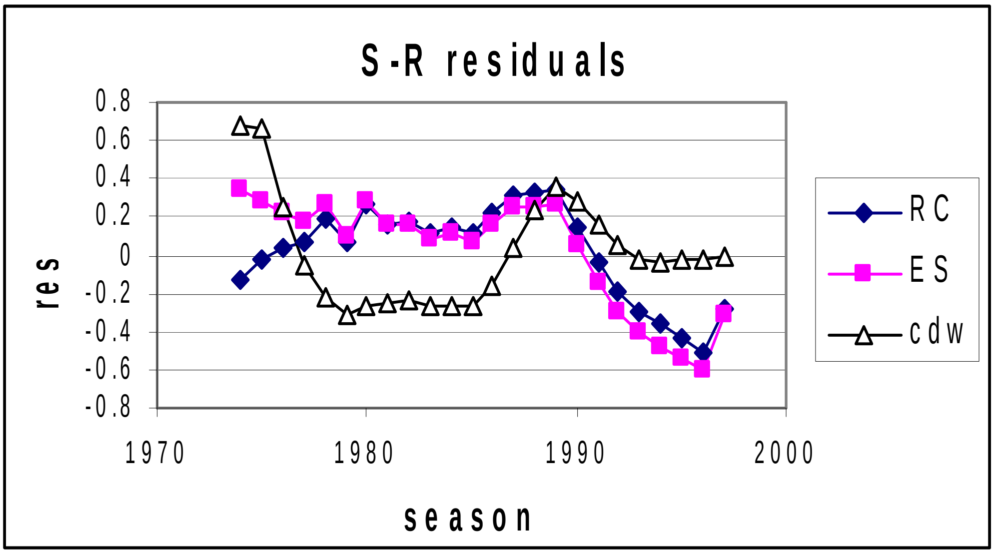


Figure 4: Stock-recruitment residuals for the Reference Case, effort saturation (Sensitivity 1) and catch-at-age down-weighting (Sensitivity 2) scenarios.



Appendix: The Age-structured production model for the South Coast rock lobster resource.

1. The population model:

The resource dynamics are modeled by the equations:

$$N_{y+1,0} = R_{y+1} \tag{1}$$

$$N_{y+1,a+1} = N_{y,a}e^{-(M_a+S_aF_y)} = N_{y,a}e^{-Z_{y,a}} \tag{2}$$

$$N_{y+1,m} = N_{y,m-1}e^{-(M_{m-1}+S_{m-1}F_y)} + N_{y,m}e^{-(M_m+S_mF_y)} \tag{3}$$

where

$N_{y,a}$ is the number of lobsters of age *a* at the start of year *y*,

M_a denotes the natural mortality rate on lobsters of age *a*,

S_a is the age-specific selectivity,

F_y is the fully selected fishing mortality in year *y*, and

m is the maximum age considered (taken to be a plus-group).

The number of recruits at the start of year *y* is related to the spawner stock size by a stock-recruitment relationship:

$$R_y = \frac{\alpha B_y^{sp}}{\beta + (B_y^{sp})^{\gamma}} e^{\varsigma_y} \tag{4}$$

where

α, β and γ are spawner biomass-recruitment parameters (γ=1 for a Beverton-Holt relationship),

ς_y reflects fluctuation about the expected recruitment for year *y*, and

B_y^{sp} is the spawner biomass at the start of year *y*, given by:

$$B_y^{sp} = \sum_{a=1}^{m} f_a w_a N_{y,a} \tag{5}$$

where w_a is the begin-year mass of fish at age *a* and f_a is the proportion of fish of age *a* that are mature.

In order to work with estimable parameters that are more meaningful biologically, the stock-recruit relationship is re-parameterised in terms of the pre-exploitation equilibrium spawning biomass, K^{sp}, and the "steepness" of the stock-recruit relationship (recruitment at $B^{sp} = 0.2K^{sp}$ as a fraction of recruitment at $B^{sp} = K^{sp}$):

$$\alpha = \frac{\left(5 - 0.2^{\gamma-1}\right)hR_1\left(K^{sp}\right)^{\gamma-1}}{5h-1} \tag{6}$$

and

$$\beta = \frac{\left(K^{sp}\right)^{\gamma}\left(1-0.2h\right)^{\gamma-1}}{5h-1} \tag{7}$$

where

$$R_1 = K^{sp} / \left[\sum_{a=1}^{m-1} f_a w_a e^{-\sum_{a'=0}^{a-1} M_{a'}} + f_m w_m \frac{e^{-\sum_{a'=0}^{m-1} M_{a'}}}{1-e^{-M_m}} \right] \tag{8}$$

The total catch by mass in year y is given by:

$$C_y = \sum_{a=0}^{m} w_{a+\frac{1}{2}} N_{y,a} \frac{S_a F_y}{Z_{y,a}} (1-e^{-Z_{y,a}}) \tag{9}$$

where $w_{a+\frac{1}{2}}$ denotes the mid-year mass of a lobster at age a.

The model estimate of mid-year exploitable biomass is given by:

$$\hat{B}_y = \sum_{a=0}^{m} w_{a+\frac{1}{2}} S_a N_{y,a} e^{-(Z_{y,a})/2} \tag{10}$$

where

$\hat{B}_y$ is the model estimate of exploitable biomass for year y, and

S_a is the fishing selectivity-at-age for age a.

Models that do not allow for the possibility of fluctuations about the stock-recruitment relationship (i.e. those which set $\varsigma_y = 0$ in equation 4) assume that the resource is at the deterministic equilibrium that corresponds to an absence of harvesting at the start of the initial year ($B_{1973}^{sp} = K^{sp}$). For models that allow for that possibility, this assumption together with that of the associated equilibrium age-structure is made for 1973, with the biomass and age-structure thereafter potentially impacted by such fluctuations.

2. The likelihood function

The model is fitted to CPUE and catch-at-age data to estimate model parameters. Contributions by each of these to the negative log-likelihood (-ln$L$) are as follows:

2.1 Relative abundance data (CPUE):

The likelihood is calculated assuming that the observed abundance index is log-normally distributed about its expected value:

$$CPUE_y = qB_y e^{\varepsilon_y} \text{ or } \varepsilon_y = \ln(CPUE_y) - \ln(qB_y) \tag{11}$$

where

$CPUE_y$ is the CPUE abundance index for year y,

B_y is the model estimate of mid-year exploitable biomass for year y given by equation 10,

q is the constant of proportionality (catchability coefficient), and

ε_y from $N(0,\sigma^2)$.

The contribution of the abundance data to the negative of the log-likelihood function (after removal of constants) is given by:

$$-\ln L = \sum_y \left[(\varepsilon_y)^2 / 2\sigma^2 + \ln \sigma \right] \qquad (12)$$

where

σ is the residual standard deviation estimated in the fitting procedure by its maximum likelihood value:

$$\hat{\sigma} = \sqrt{1/n \sum_y \left(\ln CPUE_y - \ln \hat{q}\hat{B}_y \right)^2} \qquad (13)$$

where

n is the number of data points in the CPUE series, and

q is the catchability coefficient, estimated by its maximum likelihood value:

$$\ln \hat{q} = 1/n \sum_y \left(\ln CPUE_y - \ln \hat{B}_y \right) \qquad (14)$$

2.2 "Effort saturation"

When the possibility of "effort saturation" is taken into account, the CPUE abundance relationship of equation 11 is modified as follows:

$$CPUE_y = q_y B_y e^{\varepsilon_y} \text{ or } \varepsilon_y = \ln(CPUE_y) - \ln(q_y B_y) \qquad (15)$$

where

$$q_y = q' / \left[1 + \left(\frac{E_y - E'}{E^* - E'} \right)^{n^*} \right] \qquad \text{if } E_y > E' \qquad (16)$$

$$q_y = q' \qquad \text{if } E_y \le E'$$

where

$CPUE_y$ is the GLM standardised CPUE data given in Table 1,

E_y is the estimated effort given by $\dfrac{C_y}{CPUE_y}$,

$$q' = e^{\left(\sum_{y, E_y < E'} \left(\ln(CPUE_y) - \ln B_y \right) + \sum_{y, E_y \ge E'} \left(\ln(CPUE_y \left[1 + \left(\frac{E_y - E'}{E^* - E'} \right)^{n^*} \right]) - \ln B_y \right) \right) / n}$$

E^* quantifies the extent of "effort saturation",

E' is the threshold effort above which "effort saturation" sets in, and

n^* allows for flexibility in the "effort saturation" relationship.

For this scenario, equation 13 is modified by replacing q with the q_y as defined above.

2.3 Catches-at-age

The contribution of the catch-at-age data to the negative of the log-likelihood function when assuming a log-normal error distribution and when making an adjustment to effectively weight in proportion to sample size is given by:

$$-\ln L = \sum_y \sum_a \left[\ln(\sigma_{age} / \sqrt{p_{y,a}}) + p_{y,a} \left(\ln p_{y,a} - \ln \hat{p}_{y,a} \right)^2 / 2 (\sigma_{age})^2 \right] \qquad (17)$$

where

$p_{y,a} = C_{y,a} / \sum_{a'} C_{y,a'}$ is the observed proportion of fish caught in year y that are of age a,

$\hat{p}_{y,a} = \hat{C}_{y,a} / \sum_{a'} \hat{C}_{y,a'}$ is the model predicted proportion of fish caught in year y that are of age a, where:

$$\hat{C}_{y,a} = N_{y,a} \frac{S_{y,a} F_y}{Z_{y,a}} (1 - e^{-Z_{y,a}}) \tag{18}$$

and σ_{age} is the standard deviation associated with the catch-at-age data, estimated in the fitting procedure by:

$$\hat{\sigma}_{age} = \sqrt{\left[\sum_y \sum_a p_{y,a} (\ln p_{y,a} - \ln \hat{p}_{y,a})^2 / \sum_y \sum_a 1 \right]} \tag{19}$$

Note that allowance is made for a “minus” group (lobsters age 8 and younger) in the catch-at-age contribution to the likelihood function, as well as for a “plus” group (lobsters aged 20 and over).

2.4 Stock-recruitment function residuals:

The assumption that these residuals are log-normally distributed and could be serially correlated defines a corresponding joint prior distribution. This can be equivalently regarded as a penalty function added to the log-likelihood, which for fixed ρ is given by:

$$-\ln L = \sum_{y=y1}^{y2} \left[\frac{\varsigma_y - \rho \varsigma_{y-1}}{\sqrt{1-\rho^2}} \right]^2 / 2\sigma_R^2 \tag{20}$$

where

$\varsigma_y = \rho \tau_{y-1} + \sqrt{1-\rho^2}\,\varepsilon_y$ is the recruitment residual for year y (see equation 4), which is estimated for years *y1* to *y2* if $\rho = 0$, or *y1*+1 to *y2* if $\rho > 0$,

$\varepsilon_y \sim N(0, \sigma_R^2)$,

σ_R is the standard deviation of the log-residuals, which is input, and

ρ is their serial correlation coefficient, which is input.

Note that for the Reference Case assessment, ρ is set equal to zero, i.e. the recruitment residuals are assumed uncorrelated, and σ_R is set equal to 0.4. Because of the absence of informative age data for a wider period, recruitment residuals are estimated for years 1974 to 1997 only for the 2007 assessment.

3 Model parameters

Natural mortality: Natural mortality, M_a, is assumed to be the same (M) for all age classes.

Commercial selectivity-at-age: The following time-invariant logistic curve is assumed for the commercial selectivity:

$$S_a = \frac{1}{1 + e^{(-\ln(19)(a - a_{50})/(a_{95} - a_{50}))}} \tag{21}$$

where

a_{50} years is the age-at-50% selectivity which is estimated, and
a_{95} years is the age-at-95% selectivity which is estimated.

Age-at-maturity: The proportion of lobsters of age *a* that are mature is approximated by $f_a = 1$ for *a* > 9 years (i.e. $f_a = 0$ for a = 0, …,9).

Minimum age: Age 8 it taken to be a minus group.

Maximum age: *m* = 20, and is taken as a plus-group.

Mass-at-age: The mass *w* of a lobster at age *a* is given by:

$$w = \alpha\left[l_\infty\left(1 - e^{-\kappa(a-t_0)}\right)\right]^\beta \tag{22}$$

where the values assumed for the growth parameters are shown in Table 3.

Stock-recruitment relationship: The shape parameter, γ, is fixed to 1, corresponding to a Beverton-Holt form.

4. The Bayesian approach

The Bayesian method entails updating prior distributions for model parameters according to the respective likelihoods of the associated population model fits to the CPUE, catch-at-age and tag-recapture data, to provide posterior distribution for these parameters and other model quantities.

In the case of an age-structured production model, the Bayesian computations require integration over the following priors:

- The 1993 harvest proportion ($P = C_{1993}/B_{1993}$),
- The "steepness" of the stock-recruit relationship (*h*), and
- Natural mortality (M_a), assumed independent of age.
- In addition, we integrate over the two parameters defining the shape of the selectivity-at-age curve (a_{50} and a_{95}).

Furthermore, priors for the parameters characterising the postulated "effort saturation" effects (E^*, E' and n^*) of equation 16 are also required. In applications considered thus far, E' and n^* have been taken as fixed. An effective prior based on the effort saturation experiment leads to the following term:

$$-\ln L = 4 \ln \sigma_E + 2 \tag{23}$$

where σ_E is estimated from the data such that:

$$\sigma_E = \sqrt{SS(E^*)/4} \tag{24}$$

where σ_E is the standard deviation of the residuals.

The *SS*(E^*) term is developed as follows (Butterworth 2000): Considering the "full effort" exerted in Dec-Jan of the 1998/99 experiment as the standard, the extent of effort reduction (λ) and the associated relative change in CPUE (GLM-standardised to adjust for normal monthly trends), $f^{obs}(\lambda)$, were as follows for the four area-period combinations considered in the experiment:

Area-period	λ	$f^{obs}(\lambda)$
East – Feb/Mar	0.93	1.25
East – Apr/May	1.24	1.30
Agulhas – Feb/Mar	1.15	1.04
Agulhas – Apr/May	0.60	0.71

The effort "reduction" factors, λ, above are taken from Groeneveld *et al.* (1999), (specifically Table 2c) for effective effort. The $f^{obs}(\lambda)$ values follow from Tables 1 and 2 of an update of a section of that paper (WG/07/99/SCL16a), by dividing CPUE ratios (in relation to the Dec-Jan values taken as the standard) from the 1998/99 experiment by average values over the preceding 1991/92 to 1997/98 seasons.

To relate this "observed" information to a model for the extent of effort saturation, the formulation of Geromont (2000a), equation 16, is used:

$$\hat{f}(\lambda) = \frac{1 + [(E_{98/99} - E')/(E^* - E')]^{n^*}}{1 + [(\lambda E_{98/99} - E')/(E^* - E')]^{n^*}} \qquad (25)$$

Taking the effort for 1998/99, given by $C_{98/99}/\mathrm{CPUE}_{98/99}$, (see Geromont 2000a, equation 16 and Table 1) to be reflective of the full effort Dec-Jan period of the experiment, sets $E_{98/99}$ above to equal 5255. Geromont (pers. commn) advised values of E'=2500 and n^* = 1 to be typical of those obtained in her fits of the ASPM model with effort saturation. This leaves only the key E^* parameter unspecified, and this is estimated by minimizing the sums of squared differences between the observed $f(\lambda)$ values and those predicted by equation 25 above:

$$SS(E^*) = \sum_{i=1}^{4} \left[f^{obs}(\lambda_i) - \hat{f}(\lambda_i, E^*) \right]^2 \qquad (26)$$

The catchability coefficient (q) and the standard deviations associated with the CPUE and catch-at-age data (σ and σ_{age}) are estimated in the fitting procedure by their maximum likelihood values, rather than integrating over these three parameters as well. This is adequately accurate given reasonable large sample sizes (Walters and Ludwig 1994, Geromont and Butterworth 1995).

Modes of posteriors, obtained by finding the maximum of the product of the likelihood and the priors, are then estimated rather than performing a full Bayesian integration, due to the time intensiveness of the latter.

4.1 Priors

The following prior distributions for *P, h, M,* a_{50}, a_{95} are assumed, as previously agreed to by the Working Group (see also Butterworth 1997 and Groeneveld *et al*. 1997).

P: U[0,1]

h: N(0.95,SD) with SD=0.2, where the normal distribution is truncated at $h = 1$.

M: "tent shaped" function (P1,P2,P3,P4) = (0.05,0.1,0.2,0.3)

a_{50}: U[6,13] yr

a_{95}: U[9,17] yr subject to $a_{95} \geq a_{50}$