Assessment of the South African Anchovy Resource

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Introduction

The assessment of the South African anchovy resource has been updated from the last assessment (Cunningham and Butterworth 2004) to take account of new data and data adaptations as follows:

i) an update in the time series of November spawner biomass and May recruitment estimates from acoustic surveys, such that the new time series reflects uncapped estimates of biomass based on new target strength calculations throughout,

ii) a new method of using a monthly cut-off length to split recruits from 1-year-olds in the commercial catch (previously recruits and 1-year-olds were assumed to be caught in different months), and

iii) new data for 2004 to 2006 which were not included in the last assessment conducted in 2004.

In addition, this assessment has been modified from previous assessments to include:

iv) a plus group of age 4 (previously anchovy were assumed to spawn at age 4 and then die),

v) accounting for the introduction of the additional season by assuming the juvenile catch was taken in a pulse on 15th June prior to 1999 and on 15th July from 1999 onwards, and

iv) the adult catch now assumed to be taken in a pulse on 1st April (previously assumed to be taken halfway between November and March).

This document details the updated assessment model and gives the assessment results for the base case and robustness tests.

Population Dynamics Model

The population dynamics model used for the South African anchovy resource is detailed in Appendix A. The data used in this assessment are listed in Cunningham *et al*. 2007a. The prior distributions for the estimated parameters were chosen to be relatively uninformative. A range of combinations of adult and juvenile natural mortality rates were examined using this model in order to select realistic values for the base case.

Robustness Tests

The following robustness tests were selected to test the sensitivity of the model, and later the OMP, to assumptions made:

A_0 – base case assessment ($M_j^A = 1.2$ and $M_{ad}^A = 0.9$, see results section)

A_{M1} – alternative natural mortality: $M_j^A = 1.2$ and $M_{ad}^A = 1.2$ (see results section)

A_{M2} – alternative natural mortality: $M_j^A = 1.5$ and $M_{ad}^A = 0.9$ (see results section)

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A_{M3} – alternative natural mortality: $M_j^A = 1.5$ and $M_{ad}^A = 1.2$ (see results section)

A_{10} – 10cm cut-off length for calculating the proportion of 1-year-olds in the November survey

$A_{10.5}$ – 10.5cm cut-off length for calculating the proportion of 1-year-olds in the November survey

A_{11} – 11cm cut-off length for calculating the proportion of 1-year-olds in the November survey

A_{kegg1} – negatively biased egg surveys, i.e., $k_g^A = 0.75$ (testing assumption 7 of Appendix A)

A_{kegg2} – positively biased egg surveys, i.e., $k_g^A = 1.25$ (testing assumption 7 of Appendix A)

A_{lam1} – fix the additional variance (over and above the survey sampling CV) associated with the recruit survey $\left(\lambda_r^A\right)^2 = 0$

A_{lam2} – fix the additional variance (over and above the survey sampling CV) associated with the November survey $\left(\lambda_N^A\right)^2 = 0.02$

A_{HS} – hockey stick stock-recruitment curve with the inflection point estimated (inflection point equal to 20% of K in base case)

A_{BH} – Beverton Holt stock-recruitment curve

A_R – Ricker stock-recruitment curve

For A_{HS}, the prior distribution for the inflection point in the hockey stick curve (see equation A.4) as a proportion of carrying capacity $\frac{b^A}{K^A} \sim U(0,1)$ is introduced.

For A_{BH}, the equation (A.4) is replaced by:

$$N_{y,0}^A = \frac{\alpha^A SSB_{y,N}^A}{\beta^A + SSB_{y,N}^A} e^{\varepsilon_y^A} \qquad y = 1980, \ldots, 2005$$

In order to work with biologically meaningful parameters, the stock-recruit relationship was re-parameterised in terms of carrying capacity, K^A, (pre-exploitation spawning biomass) and the "steepness" of the stock-recruitment relationship, h, which is the proportion of the virgin recruitment that is realised at a spawning biomass level of 20% of virgin spawning biomass:

$\alpha = \frac{4h\overline{N}_0}{5h-1}$ and $\beta = \frac{K^A(1-h)}{5h-1}$, where

$$\overline{N}_0 = \frac{K^A}{\sum_{a=1}^{3} \overline{w}_a^A e^{-M_j^A-(a-1)M_{ad}^A} + \overline{w}_{4+} e^{-M_j^A-3M_{ad}^A} \frac{1}{1-e^{-M_{ad}^A}}}.$$

For A_R, equation (A.4) is replaced by:

$$N_{y,0}^A = \vartheta^A SSB_{y,N}^A e^{-\eta^A SSB_{y,N}^A} e^{\varepsilon_y^A} \qquad y = 1980, \ldots, 2005$$

and equation (A.8) is replaced by:

$$K^A = \frac{1}{\eta^A}\ln\left\{\vartheta^A e^{\frac{1}{2}\left(\sigma_r^A\right)^2}\left[\sum_{a=1}^{3}\overline{w}_a^A e^{-M_{ju}^A-(a-1)M_{ad}^A} + \overline{w}_{4+}^A e^{-M_{ju}^A-3M_{ad}^A}\frac{1}{1-e^{-M_{ad}^A}}\right]\right\}$$

In addition, the prior distributions for the two stock-recruitment parameters in A_R are changed to $\ln\left(\vartheta^A\right) \sim U(0,8)$ and $\ln\left(\frac{\eta^A}{1+\eta^A}\right) \sim U(-10000{,}10000)$ to be relatively uninformative.

Bayesian Estimation

The objective function consisting of the negative log likelihood equation (A.7) added to the negative of the 32 log prior distributions[1] was minimised using AD Model Builder (Otter Research Ltd. 2000) to fit the model to the observed data and estimate the parameters at the posterior mode. The posterior probability distributions were estimated using Markov Chain Monte Carlo (Gelman *et al*. 1995) in AD Model Builder. Two chains of 20 million samples were run for the purposes of testing convergence, with one chain beginning at the posterior mode and the other starting from a random vector. A burn-in of one million was discarded and the remaining chain was thinned by 1 in every 1000 to decrease any autocorrelation. Results presented in this document are based on a random sample of 5 000 from the 19 000-long chain begun at the posterior mode after burn-in and thinning. A smaller sample will be used as input to the OMP testing framework due to run-time constraints.

Convergence of the chains was tested using the BOA (Bayesian Output Analysis) package (Smith 2003) and the diagnostics from the tests of Geweke (1992), Gelman and Rubin (1992), Raftery and Lewis (1992) and Heidelberger and Welch (1983) were good, indicating convergence of the chain. The autocorrelations for each estimable parameter and cross-correlations between the parameters were also low.

Results

Natural Mortality

Table 1 lists the various contributions to the objective function at the posterior mode for the full range of combinations of juvenile and adult natural mortality tested. The following criteria were used to distinguish "reasonable" from "unrealistic" combinations (unrealistic combinations are shaded in Table 1):

- $M_j^A \geq M_{ad}^A$;
- the ratio $k_r^A / k_N^A \in [0.5, 1.0]$, as the November spawner biomass survey is expected to have a greater coverage of the full distribution of the resource than the May recruit survey so that the latter should reflect a smaller relative bias.

[1] Prior distributions were placed on all estimated parameters: recruitment residuals, ε_y^A, $y = 1984,...,2005$, and standard deviation thereof, $\left(\sigma_r^A\right)^2$, the log of multiplicative bias factors for the surveys, $\ln\left(k_N^A\right)$, $\ln\left(k_r^A\right)$, and $\ln\left(k_p^A\right)$, additional recruit survey variance, $\left(\lambda_r^A\right)^2$, the log of maximum recruitment on the Hockey Stick stock recruitment curve $\ln(a^A)$, and initial numbers at age in November 1983, $N_{1983,a}^A$, $a = 0,\ldots,3$.

One further "reality check" was provided by the criterion that the multiplicative bias for the proportion-at-age 1 in the November survey, k_p^A, should not be markedly different from 1.

There was little change in the posterior distribution as M_j^A changed for a given M_{ad}^A, while the posterior distribution indicated an improved fit to the data for increasing M_{ad}^A. This latter feature, however, seems to reflect an artefact of the assessment in that a higher natural mortality results in a higher loss of "memory" of cohorts, making the November survey data easier to fit. Considering k_p^A then, the following combinations were chosen for a base case and robustness tests:

- $M_j^A = 1.2$ and $M_{ad}^A = 0.9$ (base case)
- $M_j^A = 1.2$ and $M_{ad}^A = 1.2$ (robustness test)
- $M_j^A = 1.5$ and $M_{ad}^A = 0.9$ (robustness test)
- $M_j^A = 1.5$ and $M_{ad}^A = 1.2$ (robustness test)

Base Case at Posterior Mode

The model fit to the data at the posterior mode is shown in Figure 1 for acoustic spawner biomass, Figure 2 for DEPM estimates of spawner biomass, Figure 3 for recruitment and Figure 4 for the proportion of 1-year-olds in the November survey. The model predicted November spawner biomass and recruitment at the posterior mode are shown in Figure 5, together with the model estimated hockey-stick stock-recruitment curve. The inflection point and maximum recruitment of the estimated curve are lower than that estimated by the last assessment (Table 5), with recruitments in November 1999 and 2000 being clear outliers.

Robustness Tests

The model parameters, contributions to the objective function and key model outputs at the posterior mode for the robustness tests are given in Tables 2a and 2b. Tables 3a and 3b repeat the results assuming $M_{ad}^A = 1.2$ (corresponding to A_{M1}) in the robustness tests. The alternative stock recruit curves are shown in Figure 6 for $M_{ad}^A = 0.9$ and Figure 7 for $M_{ad}^A = 1.2$. There were two cases which resulted in a substantial overall improvement in the posterior at the mode. In the case of A_{lam2}, the larger additional variance on the November survey results in a significant improvement in the fit to the recruit survey ($\left(\lambda_r^A\right)^2$ was estimated to be much smaller than in A_0) and the proportion-at-age 1 in the November survey at the expense of fitting to the November survey spawner biomass. Given the confidence scientists place in the November survey, and the lack of fit of A_{lam2} to the November survey, this case was not considered more plausible than the chosen base case A_0. In the case of A_{BH}, the improved posterior mode was obtained primarily through an improved fit to the stock-recruitment curve, but the hockey-stick form was retained for the base case for comparability with previous work.

When developing OMP-04, the risk threshold used was 10% of the average adult biomass between November 1984 and November 1999. This value is reported in Tables 2b and 3b. The risk threshold differs for A_{Kegg1} and A_{Kegg2}, the robustness tests to sensitivity to the bias in the egg surveys, and consequently in the November acoustic surveys.

Base Case Posterior Distributions

The posterior means and CVs of the model parameters and some key outputs for A_0 are given in Table 4, with the posterior distributions of key model outputs to be used in the testing of the new OMP shown in Figure 8. Table 5 lists some key model parameters and outputs from all the robustness tests, assuming $M_j^A = 1.2$ and $M_{ad}^A = 0.9$ in line with the base case assumption.

Implications for the OMP

Samples from the posterior distributions of key model parameters and outputs, including those presented in Tables 4 and 5 and Figure 8 will be used to develop the new OMP. For comparative purposes, therefore, Table 6 gives some key model parameters and outputs at the joint posterior mode for A_0, together with those from the last assessment used to develop OMP-04. In relation to measurement of risk to the resource, it should be noted that the standard deviation in recruitment residuals is estimated to be higher than that used to develop OMP-04, while carrying capacity is lower. The average spawner biomass between 1984 and 1999, used to define risk to develop OMP-04 is 7% higher than previously. Figure 9 shows the November spawner biomass over time in relation to carrying capacity and 10% of the average 1984 to 1999 biomass, the risk threshold used to tune OMP-04. It is clear from Figure 9 that the anchovy spawner biomass at the posterior mode has never dropped below 10% of its 1984 to 1999 average over the past 23 years, while it has historically dropped below the average 1984 to 1999 biomass 35% of the time (Figures 5 and 9). To place this in context with the last assessment, the annual November biomass posterior distributions are given for this assessment and the last assessment in Figure 10. Table 6 lists the mean of these distributions and the annual probability of falling below the average 1984 to 1999 biomass. The probability of historically being below 10% of the average 1984 to 1999 biomass was zero in all years for both assessments. Figure 11 shows the harvest rate over time at the posterior mode, calculated as the proportion of observed catch by mass to model predicted spawner biomass.

In order to obtain a clearer understanding of the changes in the perception of the anchovy resource over time, two retrospective-type analyses were run. In these two cases, the A_0 model was re-fit to the data only up to November 1999, A_{1999}, and only up to November 2003, A_{2003}. All other assumptions were the same as A_0. The comparative fits to the November spawner biomass and May recruitment are given in Figure 12, while some key model outputs for use in developing the OMP are given in Table 7. These results indicate that the standard deviation in recruitment residuals has changed as more data have become available. Other changes, however, such as that in the average November 1984 to 1999 biomass and bias on the surveys, are due to other aspects such as the refinement to the acoustic survey biomass series (Cunningham *et al*. 2007b) and a change in natural mortality (although this is also linked to the inclusion of a plus-group).

Summary

This document has detailed the updated assessment of the South African anchovy resource and provided results of the base case hypothesis and robustness tests. The posterior distributions resulting from the base case hypothesis and some key robustness tests will be used as input into the testing framework for the combined management procedure for sardine and anchovy currently under development.

Cunningham and Butterworth (2007) suggested that the modelling of anchovy account for within-year variation in the pattern of recruitment, centred around 1st November, using the observed mean weight of recruits from the May recruit survey. Although the base case model results show a negative correlation between the mean weight of recruits and the ratio of projected (using observed May recruitment) to observed November 1-year-olds, this was not statistically significant;, nor did using a von Bertalanffy growth curve to back-predict birth dates prove successful. Nevertheless, we suggest that the OMP be tested with an alternative anchovy TAC rule which incorporates the mean weight of recruits as a partial predictor of recruitment strength. The hope would be that with more information included in the rule, a greater knowledge of true recruitment would be obtained and hence a better anticipated performance, but it might turn out that such lesser bias is more than countered by increased variance.

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Table 1. The contributions to the objective function at the posterior mode for a range of combinations of juvenile, M_j^A, and adult, M_{ad}^A, natural mortality. The ratio of the multiplicative bias in the recruit survey to that in the November survey, k_r^A / k_N^A, and the multiplicative bias in the proportion-at-age 1 in the November survey, k_p^A, are given for diagnostic purposes. Shaded cells represent unrealistic choices in terms of the criteria applied.

M_j^A	M_{ad}^A	Posterior	$-\ln(L_{Nov})$	$-\ln(L_{Egg})$	$-\ln(L_{Rec})$	$-\ln(L_{Prop})$	-ln(Prior)	k_r^A	k_N^A	k_r^A / k_N^A	k_p^A
0.6	0.6	96.312	12.232	6.069	15.384	34.163	28.464	2.014	1.079	1.866	0.994
0.9	0.6	96.051	11.206	5.887	15.724	34.535	28.698	2.014	1.168	1.725	0.991
1.2	0.6	96.110	11.099	5.888	15.852	34.564	28.708	1.799	1.181	1.523	0.991
1.5	0.6	96.191	11.127	5.896	15.944	34.541	28.683	1.579	1.183	1.336	0.992
1.8	0.6	96.288	11.133	5.901	16.064	34.528	28.663	1.387	1.184	1.172	0.992
2.1	0.6	Model could not fit									
0.6	0.9	69.263	2.113	4.557	9.581	25.171	27.841	1.560	1.200	1.300	0.916
0.9	0.9	69.301	2.170	4.569	9.625	25.130	27.808	1.371	1.203	1.140	0.917
1.2	0.9	69.370	2.195	4.576	9.717	25.103	27.779	1.206	1.206	1.000	0.917
1.5	0.9	69.468	2.188	4.579	9.857	25.090	27.754	1.060	1.209	0.877	0.917
1.8	0.9	69.595	2.151	4.576	10.044	25.090	27.734	0.933	1.213	0.769	0.917
2.1	0.9	69.749	2.086	4.569	10.273	25.102	27.718	0.821	1.216	0.675	0.917
0.6	1.2	53.383	-3.187	3.704	6.027	19.270	27.569	1.227	1.211	1.013	0.847
0.9	1.2	53.409	-3.144	3.715	6.066	19.237	27.534	1.079	1.215	0.888	0.847
1.2	1.2	53.477	-3.132	3.720	6.170	19.216	27.502	0.950	1.220	0.779	0.847
1.5	1.2	53.585	-3.151	3.720	6.335	19.207	27.474	0.836	1.224	0.683	0.847
1.8	1.2	53.731	-3.198	3.714	6.558	19.208	27.449	0.736	1.229	0.599	0.847
2.1	1.2	53.914	-3.272	3.704	6.834	19.220	27.428	0.648	1.233	0.526	0.847
0.6	1.5	44.405	-6.738	3.089	4.101	16.450	27.503	1.046	1.217	0.860	0.789
0.9	1.5	44.421	-6.695	3.100	4.127	16.422	27.467	0.921	1.222	0.754	0.789
1.2	1.5	44.485	-6.681	3.106	4.228	16.401	27.432	0.811	1.227	0.661	0.789
1.5	1.5	44.597	-6.698	3.106	4.400	16.387	27.401	0.715	1.233	0.580	0.789
1.8	1.5	44.754	-6.741	3.100	4.640	16.381	27.373	0.630	1.238	0.509	0.789
2.1	1.5	44.955	-6.808	3.089	4.943	16.382	27.349	0.555	1.243	0.446	0.789
0.6	1.8	39.762	-8.753	2.704	2.798	15.618	27.395	0.934	1.218	0.767	0.729
0.9	1.8	39.772	-8.703	2.717	2.810	15.592	27.356	0.822	1.224	0.672	0.744
1.2	1.8	39.836	-8.685	2.724	2.908	15.570	27.319	0.725	1.229	0.589	0.744
1.5	1.8	39.952	-8.698	2.724	3.088	15.552	27.285	0.639	1.235	0.517	0.744
1.8	1.8	40.119	-8.738	2.719	3.345	15.539	27.253	0.563	1.241	0.453	0.787
2.1	1.8	40.335	-8.803	2.708	3.673	15.531	27.225	0.496	1.247	0.398	0.744
0.6	2.1	37.583	-9.907	2.491	1.895	15.865	27.240	0.860	1.217	0.706	0.709
0.9	2.1	37.594	-9.858	2.504	1.908	15.840	27.201	0.757	1.223	0.619	0.744
1.2	2.1	37.663	-9.845	2.510	2.015	15.819	27.164	0.667	1.229	0.543	0.709
1.5	2.1	37.789	-9.864	2.510	2.212	15.801	27.130	0.588	1.236	0.476	0.709
1.8	2.1	37.969	-9.912	2.504	2.493	15.786	27.097	0.519	1.242	0.418	0.709
2.1	2.1	38.200	-9.986	2.493	2.851	15.775	27.068	0.457	1.249	0.366	0.709

Table 2a. The contributions to the objective function at the posterior mode for the robustness tests, shown together with the key difference in each robustness test from the base case A_0. Other than A_{M1}, A_{M2} and A_{M3}, the robustness tests have the same natural mortality as A_0. Key parameter values estimated at the joint posterior mode are also given. Fixed values are given in ***bold****. Numbers are reported in billions.*

	A_0	A_{M1}	A_{M2}	A_{M3}	A_{10}	$A_{10.5}$	A_{11}	A_{Kegg1}	A_{Kegg2}	A_{lam1}	A_{lam2}	A_{HS}	A_{BH}	A_R
SR curve	Hockey Stick											HS, est b^A	Beverton-Holt	Ricker
Ageing Method	Prosch ALK				10cm cut-off	10.5cm cut-off	11cm cut-off							
k_g^A	= 1							= 0.75	= 1.25					
$(\lambda_r^A)^2$	Estimate									= 0				
$(\lambda_N^A)^2$	= 0										= 0.02			
M_j^A	**1.2**	**1.2**	**1.5**	**1.5**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**
M_{ad}^A	**0.9**	**1.2**	**0.9**	**1.2**	**0.9**	**0.9**	**0.9**	**0.9**	**0.9**	**0.9**	**0.9**	**0.9**	**0.9**	**0.9**
Posterior	69.37	53.48	69.47	53.58	67.60	68.83	74.23	69.37	69.37	74.63	61.72	68.24	66.08	76.25
$-\ln(L_{Nov})$	2.19	-3.13	2.19	-3.15	-4.07	-1.67	0.22	2.20	2.19	22.49	17.83	3.15	5.52	0.85
$-\ln(L_{Egg})$	4.58	3.72	4.58	3.72	3.52	3.83	4.20	4.58	4.58	7.60	8.51	4.90	5.23	4.36
$-\ln(L_{Rec})$	9.72	6.17	9.86	6.33	14.05	12.09	10.65	9.72	9.72	-3.12	-9.72	9.02	7.63	10.48
$-\ln(L_{Prop})$	25.10	19.22	25.09	19.21	24.02	25.17	30.62	25.10	25.10	20.71	18.67	24.76	23.99	25.65
-ln(Prior)	27.78	27.50	27.75	27.47	30.07	29.41	28.53	27.78	27.78	26.96	26.44	26.41	23.71	34.90
$N_{1983,0}^A$	132.1	151.18	177.90	203.48	27.4	45.7	110.2	176.2	105.7	139.4	139.1	132.5	133.5	130.1
$N_{1983,1}^A$	124.0	157.44	123.77	157.07	208.9	187.6	135.0	165.3	99.2	134.8	137.3	124.7	126.0	121.8
$N_{1983,2}^A$	0.0005	0.0005	0.0005	0.0005	0.0002	0.0003	0.0006	0.0006	0.0004	0.0002	0.0003	0.0004	0.0003	0.0006
$N_{1983,3}^A$	0.0004	0.0004	0.0004	0.0004	0.0001	0.0002	0.0005	0.0005	0.0003	0.0002	0.0002	0.0003	0.0002	0.0005
k_N^A	1.206	1.220	1.209	1.224	1.237	1.227	1.212	0.904	1.508	1.167	1.133	1.212	1.191	1.219
k_r^A	1.206	0.950	1.060	0.836	1.315	1.274	1.232	0.904	1.507	1.111	1.084	1.207	1.173	1.228
k_r^A/k_N^A	1.000	0.779	0.877	0.683	1.063	1.039	1.016	1.000	1.000	0.952	0.957	0.996	0.985	1.007
k_p^A	0.970	0.870	0.970	0.870	0.586	0.826	0.999	0.970	0.970	0.989	0.996	0.972	0.974	0.967
$(\sigma_p^A)^2$	0.519	0.311	0.519	0.311	0.473	0.523	0.839	0.519	0.519	0.354	0.297	0.504	0.471	0.545
$(\lambda_N^A)^2$	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.020**	**0.000**	**0.000**	**0.000**
$(\lambda_r^A)^2$	0.111	0.072	0.113	0.074	0.177	0.144	0.123	0.111	0.111	**0.000**	0.000	0.102	0.086	0.121

Table 2b. Further model outputs and stock recruit parameters used in developing the OMP at the joint posterior mode for the robustness tests. Other than A_{M1}, A_{M2} and A_{M3}, the robustness tests have the same natural mortality as A_0. Numbers are reported in billions and biomass in thousands of tons.

	A_0	A_{M1}	A_{M2}	A_{M3}	A_{10}	$A_{10.5}$	A_{11}	A_{Kegg1}	A_{Kegg2}	A_{lam1}	A_{lam2}	A_{HS}	A_{BH}	A_R
$N^A_{2006,1}$	55.4	71.6	55.1	71.3	49.3	51.5	53.7	73.8	44.3	59.7	61.7	56.1	59.3	54.8
$N^A_{2006,2}$	39.8	39.5	39.7	39.4	43.3	41.0	40.3	53.1	31.9	35.9	36.0	39.3	38.9	41.1
$N^A_{2006,3}$	11.5	8.3	11.4	8.3	9.2	10.4	11.1	15.3	9.2	14.3	17.0	11.7	12.4	11.3
$N^A_{2006,4+}$	17.7	7.8	17.6	7.7	16.7	17.1	17.4	23.6	14.1	22.3	24.2	17.8	18.6	17.3
$\overline{B}^A_{Nov}$ [2]	1096.3	1093.7	1094.6	1091.4	1088.2	1088.5	1092.9	1461.8	876.9	1101.0	1110.8	1087.2	1097.4	1091.2
K^A	1838.9	1858.3	1832.9	1850.2	1831.2	1841.0	1846.7	2451.8	1471.1	1932.7	1962.4	2099.9	1107.0	2955.6
a^A / ϑ^A	212.5	269.4	286.1	362.4	194.4	200.6	207.9	283.3	170.0	229.3	236.5	253.3		1.0
b^A / η^A	367.8	371.7	366.6	370.0	366.2	368.2	369.3	490.4	294.2	386.5	392.5	1192.9		0.0
h													0.227	
σ^A_r	0.855	0.845	0.854	0.844	0.949	0.921	0.885	0.855	0.855	0.824	0.805	0.804	0.711	1.182
η^A_{2005}	-0.169	-0.147	-0.171	-0.150	-0.180	-0.174	-0.172	-0.169	-0.169	-0.177	-0.178	-0.381	-0.729	-0.511
s^A_{cor}	0.551	0.552	0.550	0.550	0.478	0.495	0.523	0.551	0.551	0.603	0.621	0.415	0.310	0.718

[2] OMP-04 was developed using Risk defined as "the probability that adult anchovy biomass falls below 10% of the average adult anchovy biomass between November 1984 and November 1999 at least once during the projection period of 20 years".

Table 3a. The contributions to the objective function at the posterior mode for the robustness tests, shown together with the key difference in each robustness test from the base case A_0. The difference from Table 2a is that here the robustness tests have the same natural mortality as A_{M1}. Key parameter values estimated at the joint posterior mode are also given. Fixed values are given in ***bold****. Numbers are reported in billions.*

	A_0	A_{M1}	A_{10}	$A_{10.5}$	A_{11}	A_{Kegg1}	A_{Kegg2}	A_{lam1}	A_{lam2}	A_{HS}	A_{BH}	A_R
SR curve	Hockey Stick									HS, est b^A	Beverton-Holt	Ricker
Ageing Method	Prosch ALK		10cm cut-off	10.5cm cut-off	11cm cut-off							
k_g^A	= 1					= 0.75	= 1.25					
$(\lambda_r^A)^2$	Estimate							= 0				
$(\lambda_N^A)^2$	= 0								= 0.02			
M_j^A	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**
M_{ad}^A	**0.9**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**
Posterior	69.37	53.48	58.40	57.67	60.55	53.48	53.47	57.05	49.93	52.08	49.54	58.73
-ln(L_{Nov})	2.19	-3.13	-7.81	-6.85	-4.89	-3.13	-3.13	10.61	10.26	-2.55	-0.58	-3.98
-ln(L_{Egg})	4.58	3.72	2.90	2.98	3.37	3.72	3.72	5.88	6.72	4.01	4.28	3.59
-ln(L_{Rec})	9.72	6.17	9.78	9.00	7.17	6.17	6.17	-3.70	-9.66	5.63	4.28	6.73
-ln(L_{Prop})	25.10	19.22	24.59	23.65	26.73	19.22	19.22	17.11	15.80	19.09	18.67	19.50
-ln(Prior)	27.78	27.50	28.93	28.89	28.17	27.50	27.50	27.15	26.80	25.90	22.88	32.89
$N_{1983,0}^A$	132.1	151.18	31.7	52.3	123.0	201.6	121.0	157.8	160.0	151.4	151.0	148.8
$N_{1983,1}^A$	124.0	157.44	265.5	245.3	174.1	209.9	126.0	165.3	168.8	157.5	156.9	154.8
$N_{1983,2}^A$	0.0005	0.0005	0.0003	0.0004	0.0007	0.0006	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005
$N_{1983,3}^A$	0.0004	0.0004	0.0002	0.0003	0.0005	0.0005	0.0003	0.0003	0.0003	0.0003	0.0003	0.0004
k_N^A	1.206	1.220	1.244	1.242	1.226	0.915	1.525	1.178	1.144	1.226	1.202	1.230
k_r^A	1.206	0.950	0.999	0.994	0.966	0.712	1.187	0.891	0.873	0.953	0.928	0.961
k_r^A/k_N^A	1.000	0.779	0.803	0.800	0.788	0.779	0.779	0.756	0.763	0.778	0.772	0.782
k_p^A	0.970	0.870	0.486	0.717	0.906	0.870	0.870	0.873	0.875	0.870	0.870	0.869
$(\sigma_p^A)^2$	0.519	0.311	0.497	0.458	0.598	0.311	0.311	0.259	0.231	0.308	0.297	0.319
$(\lambda_N^A)^2$	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.020**	**0.000**	**0.000**	**0.000**
$(\lambda_r^A)^2$	0.111	0.072	0.111	0.102	0.082	0.072	0.072	**0.000**	0.000	0.067	0.056	0.078

Table 3b. Further model outputs and stock recruit parameters used in developing the OMP at the joint posterior mode for the robustness tests. The difference from Table 2b is that here the robustness tests have the same natural mortality as A_{M1}. Numbers are reported in billions and biomass in thousand of tons.

	A_0	A_{M1}	A_{10}	$A_{10.5}$	A_{11}	A_{Kegg1}	A_{Kegg2}	A_{lam1}	A_{lam2}	A_{HS}	A_{BH}	A_R
$N^A_{2006,1}$	55.4	71.6	68.7	68.7	70.7	95.5	57.3	75.7	76.8	72.4	76.1	71.5
$N^A_{2006,2}$	39.8	39.5	42.3	41.2	40.1	52.7	31.6	37.5	35.7	39.2	38.9	40.4
$N^A_{2006,3}$	11.5	8.3	6.9	7.4	8.0	11.1	6.7	9.8	11.7	8.4	8.9	8.2
$N^A_{2006,4+}$	17.7	7.8	7.4	7.4	7.7	10.4	6.2	9.4	10.2	7.8	8.2	7.7
$\overline{B}^A_{Nov}$	1096.3	1093.7	1088.5	1087.4	1091.2	1458.4	874.9	1103.0	1112.3	1083.8	1094.1	1089.3
K^A	1838.9	1858.3	1853.8	1860.4	1867.6	2477.6	1486.6	1948.1	1978.5	2200.0	1257.9	3007.4
a^A / ϑ^A	212.5	269.4	255.8	257.1	264.8	359.2	215.5	285.7	293.2	334.8		1.00
b^A / η^A	367.8	371.7	370.8	372.1	373.5	495.5	297.3	389.6	395.7	1264.3		0.0007
h											0.229	
σ^A_r	0.855	0.845	0.901	0.900	0.871	0.845	0.845	0.831	0.818	0.785	0.685	1.079
η^A_{2005}	-0.169	-0.147	-0.128	-0.133	-0.139	-0.147	-0.147	-0.154	-0.170	-0.421	-0.771	-0.572
s^A_{cor}	0.551	0.552	0.497	0.496	0.522	0.552	0.552	0.593	0.610	0.351	0.223	0.665

Table 4. Means and CVs at the joint posterior mode of model parameters and key model outputs for the base case A_0.

Parameter	Mean	CV	Parameter	Mean	CV
k_N^A	1.20	0.14	ε_{1984}^A	-0.480	-0.90
k_R^A	1.31	0.19	ε_{1985}^A	0.744	0.45
k_p^A	0.93	0.06	ε_{1986}^A	0.018	22.94
$\left(\lambda_r^A\right)^2$	0.376	0.49	ε_{1987}^A	-0.451	-0.90
$N_{1983,0}^A$	167.8	0.47	ε_{1988}^A	-1.620	-0.27
$N_{1983,1}^A$	123.2	0.44	ε_{1989}^A	-0.641	-0.61
$N_{1983,2}^A$	0.005	0.58	ε_{1990}^A	1.088	0.27
$N_{1983,3}^A$	0.005	0.56	ε_{1991}^A	-0.373	-1.16
$N_{2006,1}^A$	54.1	0.38	ε_{1992}^A	-0.933	-0.45
$N_{2006,2}^A$	46.8	0.27	ε_{1993}^A	-1.612	-0.27
$N_{2006,3}^A$	10.4	0.34	ε_{1994}^A	-0.569	-0.65
$N_{2006,4+}^A$	16.7	0.20	ε_{1995}^A	-1.330	-0.34
$B_{2006,N}^A$	2106.3	0.00	ε_{1996}^A	0.196	2.01
$\overline{B}_{Nov}^A$	1152.4	0.14	ε_{1997}^A	-0.140	-2.98
a^A	228.4	0.45	ε_{1998}^A	0.559	0.64
b^A	556.7	0.74	ε_{1999}^A	1.786	0.19
η_{2005}^A	-0.208	-1.95	ε_{2000}^A	1.753	0.21
σ_r^A	1.109	0.19	ε_{2001}^A	0.544	0.82
s_{cor}^A	0.448	0.21	ε_{2002}^A	0.465	0.94
K^A	2783	0.74	ε_{2003}^A	-0.078	-5.63
			ε_{2004}^A	0.551	0.71
			ε_{2005}^A	-0.238	-1.93

Table 5. The MCMC chain length, thinning and burn-in used to get a sample from the posterior distribution for the robustness tests. The posterior means and CVs of key model parameters and outputs are also shown.

	A_0		A_{M1}		A_{M2}		A_{M3}		A_{10}		$A_{10.5}$		A_{11}	
Total chain length	20 000 000		20 000 000		20 000 000		20 000 000		20 000 000		90 000 000		20 000 000	
Thinning	1 000		1 000		1 000		1 000		1 000		3 000		1 000	
Chain excluded (eg for burn-in)	1 000 000		8 000 000		11000 000		11000 000		8 500 100		52 00 000		12 500 000	
Length of chain used for posterior	19 000		12 000		9 000		9 000		11 500		12 500		6 500	
Parameter	Mean	CV	Mean	CV	Mean	CV	Mean	CV	Mean	CV	Mean	CV	Mean	CV
k_N^A	1.20	0.14	1.22	0.15	1.20	0.15	1.20	0.15	1.21	0.14	1.21	0.14	1.20	0.14
k_R^A	1.31	0.19	1.01	0.19	1.16	0.20	1.16	0.20	1.39	0.19	1.38	0.20	1.35	0.18
k_p^A	0.93	0.06	0.86	0.05	0.93	0.06	0.93	0.06	0.59	0.11	0.81	0.09	0.95	0.07
$(\lambda_r^A)^2$	0.376	0.49	0.251	0.55	0.378	0.49	0.378	0.49	0.472	0.48	0.431	0.45	0.401	0.46
$N_{1983,0}^A$	167.8	0.47	180.4	0.41	222.5	0.44	222.5	0.44	66.5	0.86	93.2	0.69	151.0	0.52
$N_{1983,1}^A$	123.2	0.44	167.6	0.42	123.3	0.43	123.3	0.43	212.1	0.29	178.8	0.32	130.6	0.43
$N_{1983,2}^A$	0.005	0.58	0.005	0.57	0.005	0.57	0.005	0.57	0.005	0.60	0.005	0.60	0.005	0.61
$N_{1983,3}^A$	0.005	0.56	0.005	0.59	0.005	0.57	0.005	0.57	0.005	0.59	0.005	0.57	0.005	0.58
$N_{2006,1}^A$	54.1	0.38	72.7	0.31	52.3	0.39	52.3	0.39	49.3	0.42	50.3	0.39	51.8	0.38
$N_{2006,2}^A$	46.8	0.27	44.3	0.24	47.0	0.28	47.0	0.28	49.0	0.26	47.3	0.27	47.7	0.28
$N_{2006,3}^A$	10.4	0.34	7.6	0.30	10.4	0.33	10.4	0.33	8.8	0.38	9.6	0.36	9.9	0.36
$N_{2006,4+}^A$	16.7	0.20	7.5	0.21	16.7	0.20	16.7	0.20	16.7	0.19	16.7	0.20	16.7	0.19
$\overline{B}_{Nov}^A$	1152	0.14	1145	0.14	1146	0.13	1146	0.13	1160	0.13	1149	0.13	1147	0.13
a^A	228.4	0.45	301.8	0.45	314.3	0.48	314.3	0.48	218.7	0.51	231.5	0.64	302.3	1.22
b^A	556.7	0.74	562.7	0.75	600.6	0.86	600.6	0.86	610.8	0.86	650.8	1.04	987.4	1.85
η_{2005}^A	-0.21	1.95	-0.18	1.98	-0.25	1.65	-0.25	1.65	-0.23	1.78	-0.24	1.73	-0.24	1.64
σ_r^A	1.11	0.19	1.06	0.20	1.13	0.20	1.13	0.20	1.21	0.19	1.19	0.19	1.19	0.20
s_{cor}^A	0.45	0.21	0.46	0.20	0.44	0.23	0.44	0.23	0.39	0.26	0.40	0.25	0.41	0.28
K^A	2783	0.74	2814	0.75	3003	0.86	3003	0.86	3054	0.86	3254	1.04	4937	1.85

Table 5 (continued).

	A_0		A_{Kegg1}		A_{Kegg2}		A_{lam1}		A_{lam2}		A_{HS}		A_{BH}		A_R	
Total chain length	20 000 000		20 000 000		20 000 000		20 000 000		20 000 000		20 000 000		Convergence		Convergence	
Thinning	1 000		1 000		1 000		1 000		1 000		1 000		not yet		not yet	
Chain excluded (eg for burn-in)	1 000 000		8000 000		6 000 000		6 000 000		12 000 000		6 000 000		achieved		achieved	
Length of chain used for posterior	19 000		12 000		14 000		14 000		6 000		14 000					
Parameter	Mean	CV	Mean	CV	Mean	CV	Mean	CV	Mean	CV	Mean	CV	Mean	CV	Mean	CV
k_N^A	1.20	0.14	0.90	0.14	1.44	0.13	1.15	0.14	1.12	0.15	1.19	0.14				
k_R^A	1.31	0.19	1.00	0.21	1.53	0.16	1.12	0.14	1.09	0.15	1.29	0.18				
k_p^A	0.93	0.06	0.94	0.06	0.93	0.06	0.97	0.05	0.98	0.04	0.94	0.06				
$(\lambda_r^A)^2$	0.376	0.49	0.371	0.52	0.370	0.48	0.000		0.000[3]		0.331	0.49				
$N_{1983,0}^A$	167.8	0.47	228.4	0.44	136.9	0.49	166.1	0.42	165.3	0.38	164.3	0.47				
$N_{1983,1}^A$	123.2	0.44	161.8	0.43	103.2	0.42	140.8	0.34	145.5	0.33	125.0	0.41				
$N_{1983,2}^A$	0.005	0.58	0.005	0.57	0.005	0.59	0.005	0.56	0.005	0.57	0.005	0.56				
$N_{1983,3}^A$	0.005	0.56	0.005	0.56	0.005	0.58	0.005	0.57	0.005	0.58	0.005	0.60				
$N_{2006,1}^A$	54.1	0.38	71.5	0.39	45.3	0.36	62.0	0.21	63.2	0.21	57.7	0.35				
$N_{2006,2}^A$	46.8	0.27	62.5	0.28	38.5	0.27	37.3	0.24	37.4	0.25	45.4	0.28				
$N_{2006,3}^A$	10.4	0.34	13.8	0.35	8.5	0.34	14.5	0.21	17.4	0.23	10.7	0.33				
$N_{2006,4+}^A$	16.7	0.20	22.3	0.20	14.0	0.18	22.9	0.16	24.9	0.17	17.3	0.20				
$\overline{B}_{Nov}^A$	1152	0.14	1528	0.13	951	0.13	1142	0.13	1152	0.13	1144	0.13				
a^A	228.4	0.45	299.3	0.42	187.3	0.42	244.7	0.30	257.1	0.34	977.5	0.79				
b^A	556.7	0.74	710.1	0.61	454.0	0.67	487.3	0.46	513.7	0.58	6835.0	0.86				
η_{2005}^A	-0.21	1.95	-0.22	1.89	-0.20	1.89	-0.16	1.81	-0.19	1.57	-0.55	0.84				
σ_r^A	1.11	0.19	1.11	0.18	1.11	0.19	0.96	0.18	0.94	0.20	0.99	0.17				
s_{cor}^A	0.45	0.21	0.45	0.20	0.45	0.21	0.59	0.09	0.61	0.09	0.24	0.51				
K^A	2783	0.74	3551	0.61	2270	0.67	2436	0.46	2569	0.58	9810	0.82				

[3] $(\lambda_r^A)^2$ was fixed to zero (the value at the posterior mode) in order for the chain to converge.

Table 6. The mean posterior annual November biomass for this assessment and the previous assessment, together with the annual probability of November biomass being below the average 1984 to 1999 biomass.

Year	Mean November Biomass		Probability of November Biomass being below 1984-1999 average	
	2007 Assessment	2004 Assessment	2007 Assessment	2004 Assessment
1984	1406	1313	0.17	0.32
1985	1131	1134	0.58	0.64
1986	1879	2016	0.00	0.00
1987	1683	1678	0.00	0.00
1988	1231	1241	0.26	0.33
1989	762	719	1.00	1.00
1990	648	646	1.00	1.00
1991	1782	1923	0.00	0.00
1992	1463	1673	0.01	0.00
1993	966	1082	0.98	0.83
1994	642	631	1.00	1.00
1995	486	494	1.00	1.00
1996	519	435	1.00	1.00
1997	990	1038	0.82	0.77
1998	1155	1170	0.53	0.55
1999	1711	1713	0.00	0.01
2000	3946	3759	0.00	0.00
2001	4819	5388	0.00	0.00
2002	3881	3983	0.00	0.00
2003	2909	3131	0.00	0.00
2004	2160		0.00	
2005	2372		0.00	
2006	1844		0.00	

Table 7. A comparison of key parameters and outputs at the joint posterior mode for the updated anchovy base case assessment, A_0, to the previous assessment and to retrospective-type analyses A_{2003} and A_{1999}. Biomass is given in thousands of tons and numbers in billions.

	Previous Assessment (used to develop OMP-04)			A_0	A_{2003}	A_{1999}
Starting numbers at age	$N^A_{2003,1}$	131.752	$N^A_{2006,1}$	55.4		
	$N^A_{2003,2}$	45.570	$N^A_{2006,2}$	39.8		
	$N^A_{2003,3}$	62.684	$N^A_{2006,3}$	11.5		
			$N^A_{2006,4+}$	17.7		
Starting observed spawner biomass	$B^A_{2003,N}$	3669	$B^A_{2006,N}$	2106		
Juvenile natural mortality	M^A_j	0.9 (fixed)	M^A_j	1.2 (fixed)	1.2 (fixed)	1.2 (fixed)
Adult natural mortality	M^A_{ad}	0.9 (fixed)	M^A_{ad}	0.9 (fixed)	0.9 (fixed)	0.9 (fixed)
Biases for November survey	k^A_N	1.384	k^A_N	1.206	1.212	1.192
Bias for recruit survey	k^A_r	0.984	k^A_r	1.206	1.130	1.077
Stock-recruitment parameters	a^A	227.7	a^A	212.5	218.4	156.9
	b^A	461.3	b^A	367.8	402.2	232.9
	K^A	2307	K^A	1839	2011	1165
Last estimated recruitment residual	η^A_{2002}	0.877	η^A_{2005}	-0.169	1.112	1.212
Recruitment residual standard deviation	σ^A_r	0.740	σ^A_r	0.855	0.938	0.616
Recruitment serial correlation	s^A_{cor}	0.565	s^A_{cor}	0.551	0.596	0.169
Average 1984 – 1999 biomass	$\overline{B}^A_{Nov}$	1023	$\overline{B}^A_{Nov}$	1096	1098	1100

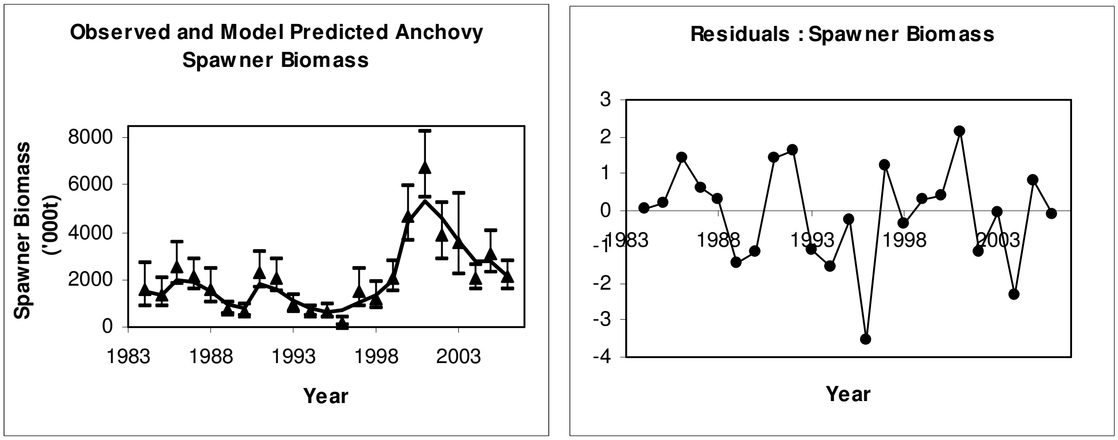

Figure 1. Acoustic survey observed and model predicted November anchovy spawner biomass from 1984 to 2006 for A_0. The observed indices are shown with 95% confidence intervals. The residuals from the fit are given in the right hand plot.

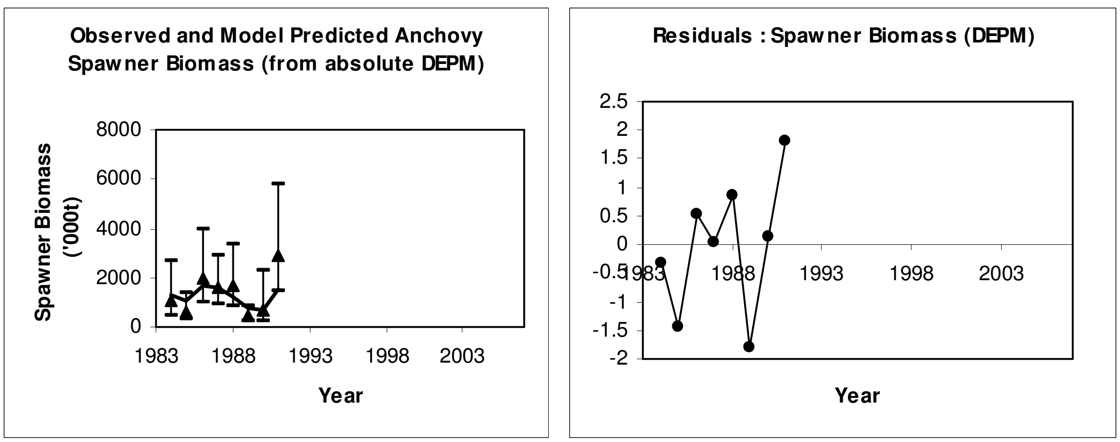

Figure 2. Egg survey observed and model predicted November anchovy spawner biomass from 1984 to 1991 for A_0. The observed indices are shown with 95% confidence intervals. The residuals from the fit are given in the right hand plot.

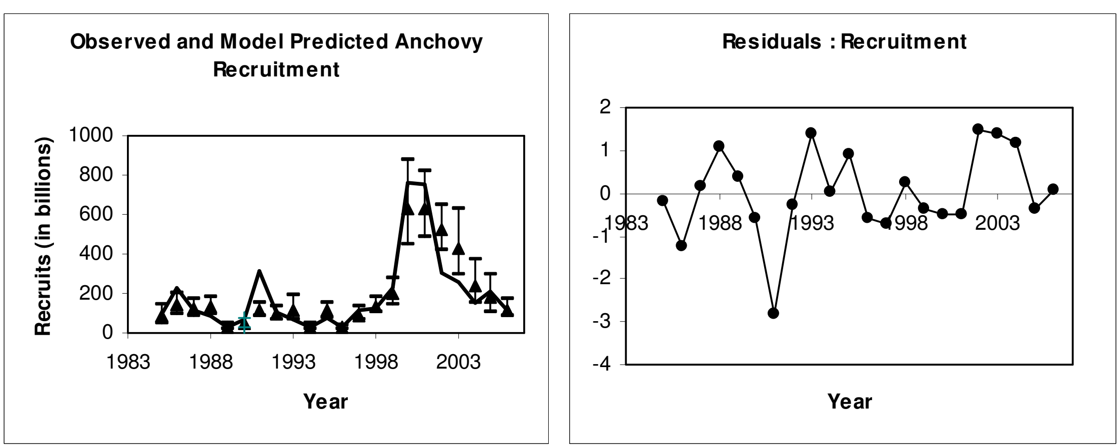

Figure 3. Observed and model predicted anchovy recruitment numbers from May 1985 to May 2006 for A_0. The observed indices are shown with 95% confidence intervals. The residuals from the fit are given in the right hand plot.

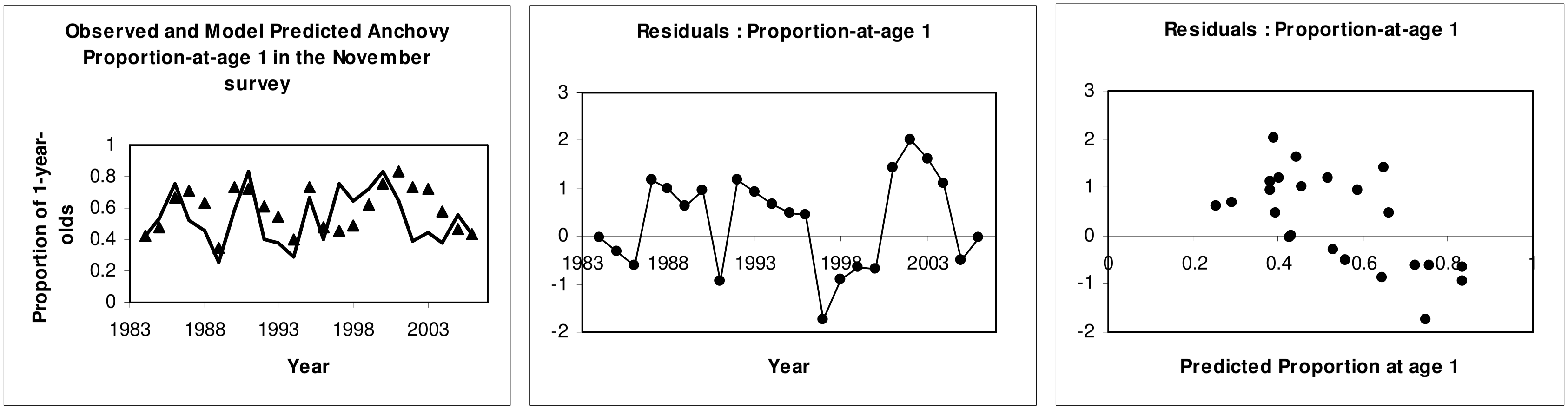

Figure 4. Observed and model predicted proportion of 1-year-olds in the November survey from 1984 to 2006 for A_0. The residuals from the fit are given in the right hand plots, against year and against predicted proportions at age 1.

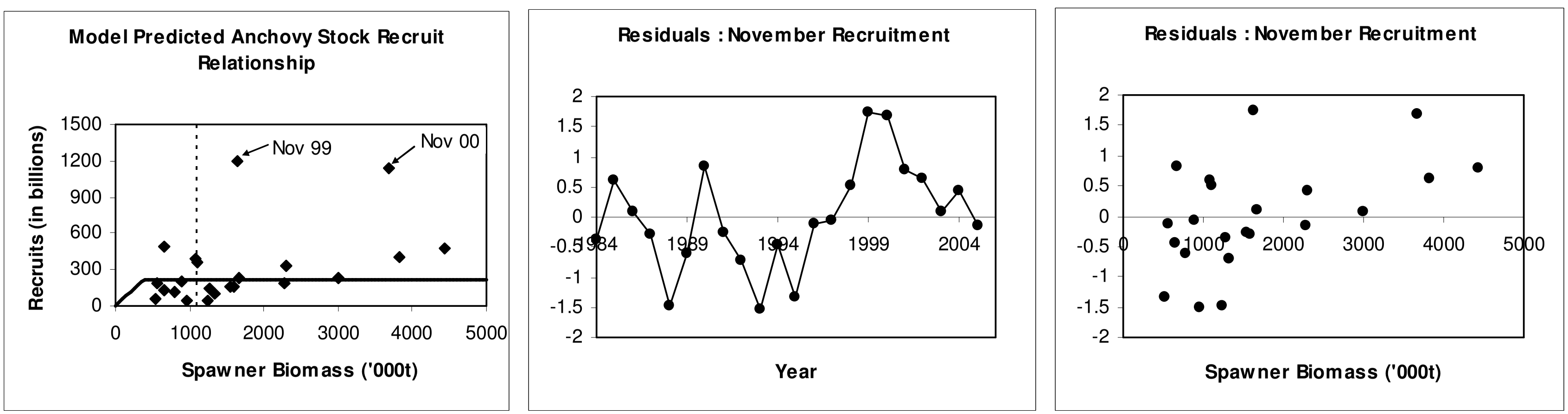

Figure 5. Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to November 2005 for A_0, with the 'hockey-stick' stock-recruit curve. The dashed line indicates the average 1984 to 1999 spawner biomass (used in the definition of risk in OMP-04). The residuals from the fit are given in the right hand plots, against year and against spawner biomass.

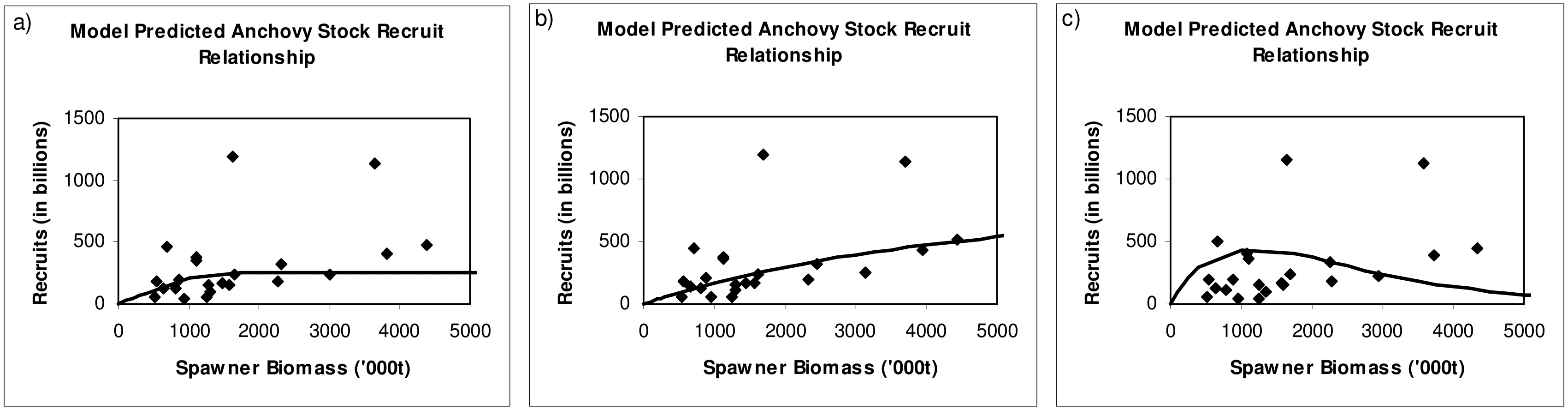

Figure 6. Stock recruit relationships for a) A_{HS}, b) A_{BH} and c) A_R, assuming $M_{ad}^{A} = 0.9$.

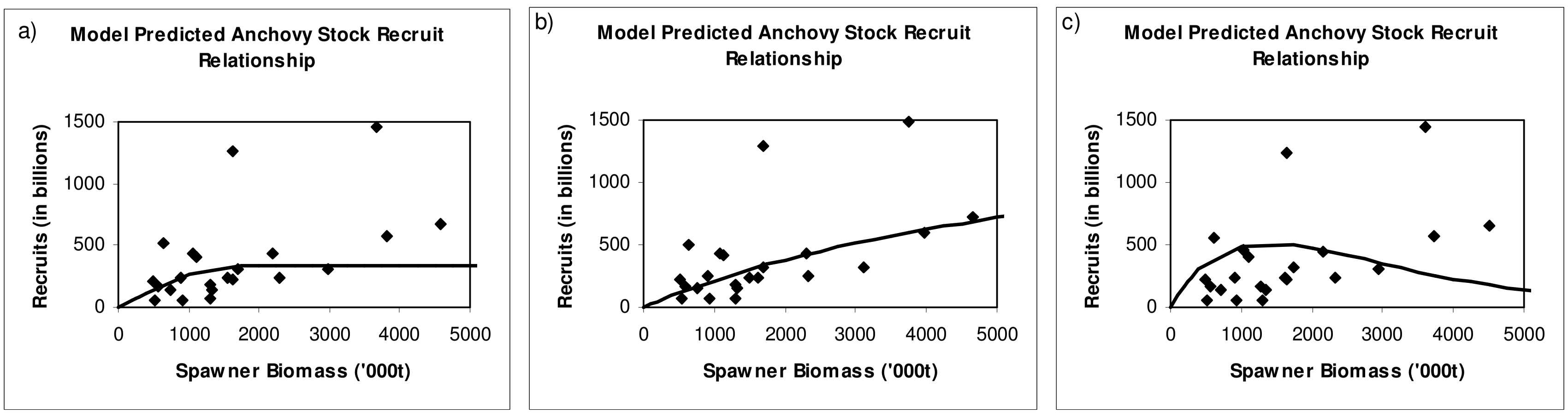

Figure 7. Stock recruit relationships for a) A_{HS}, b) A_{BH} and c) A_R, assuming $M_{ad}^{A} = 1.2$.

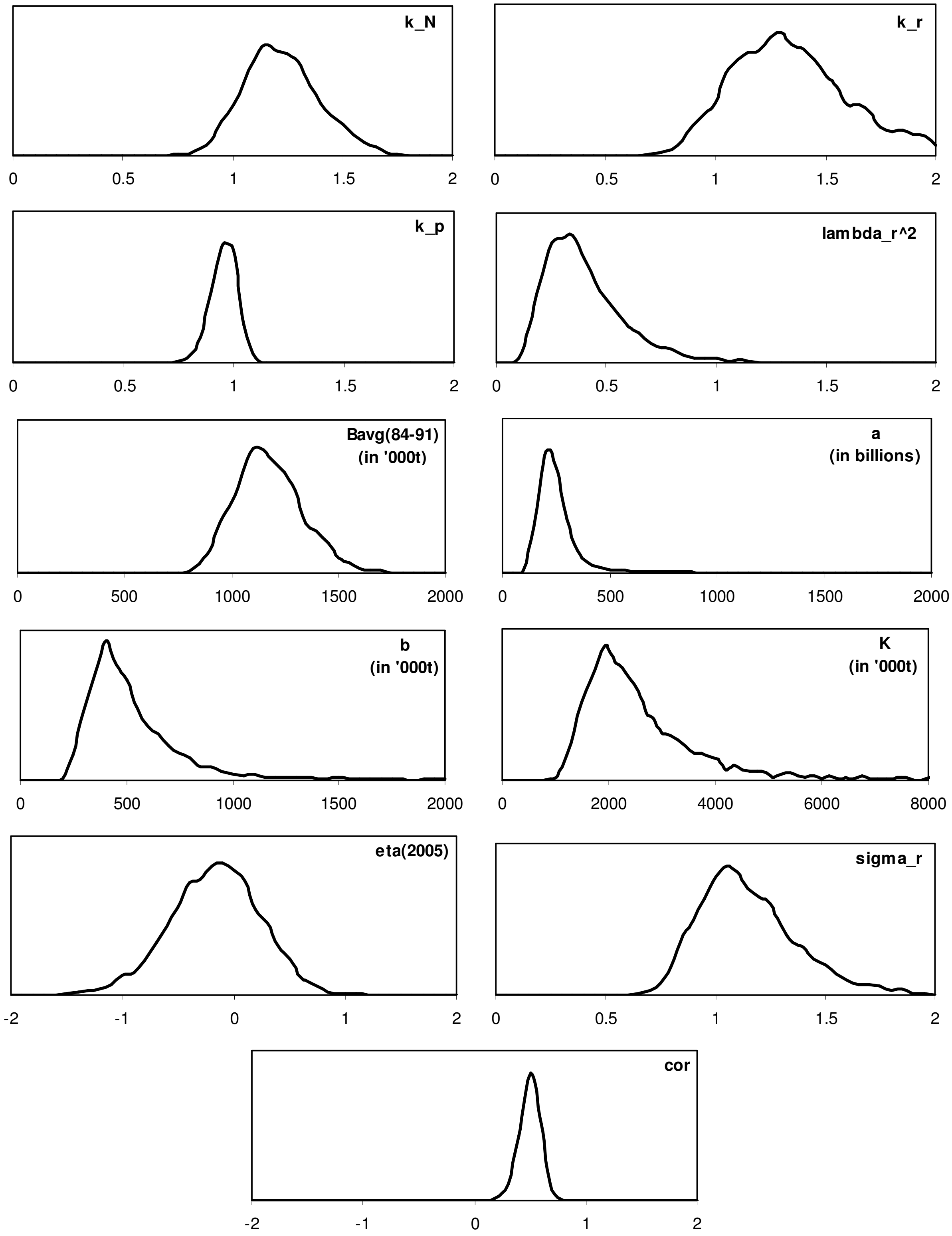

Figure 8. Posterior distributions for key base case model parameters and outputs.

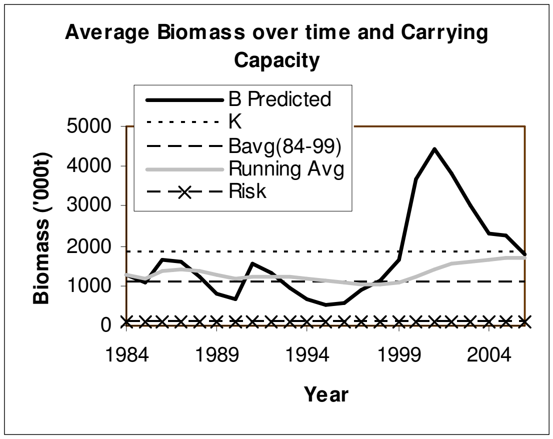

Figure 9. The base case model predicted November anchovy spawner biomass, plotted against carrying capacity, the average November 1984 to 1999 spawner biomass and 10% of this average. This last quantity was used as the risk threshold in developing OMP-04. The running average spawner biomass is also shown.

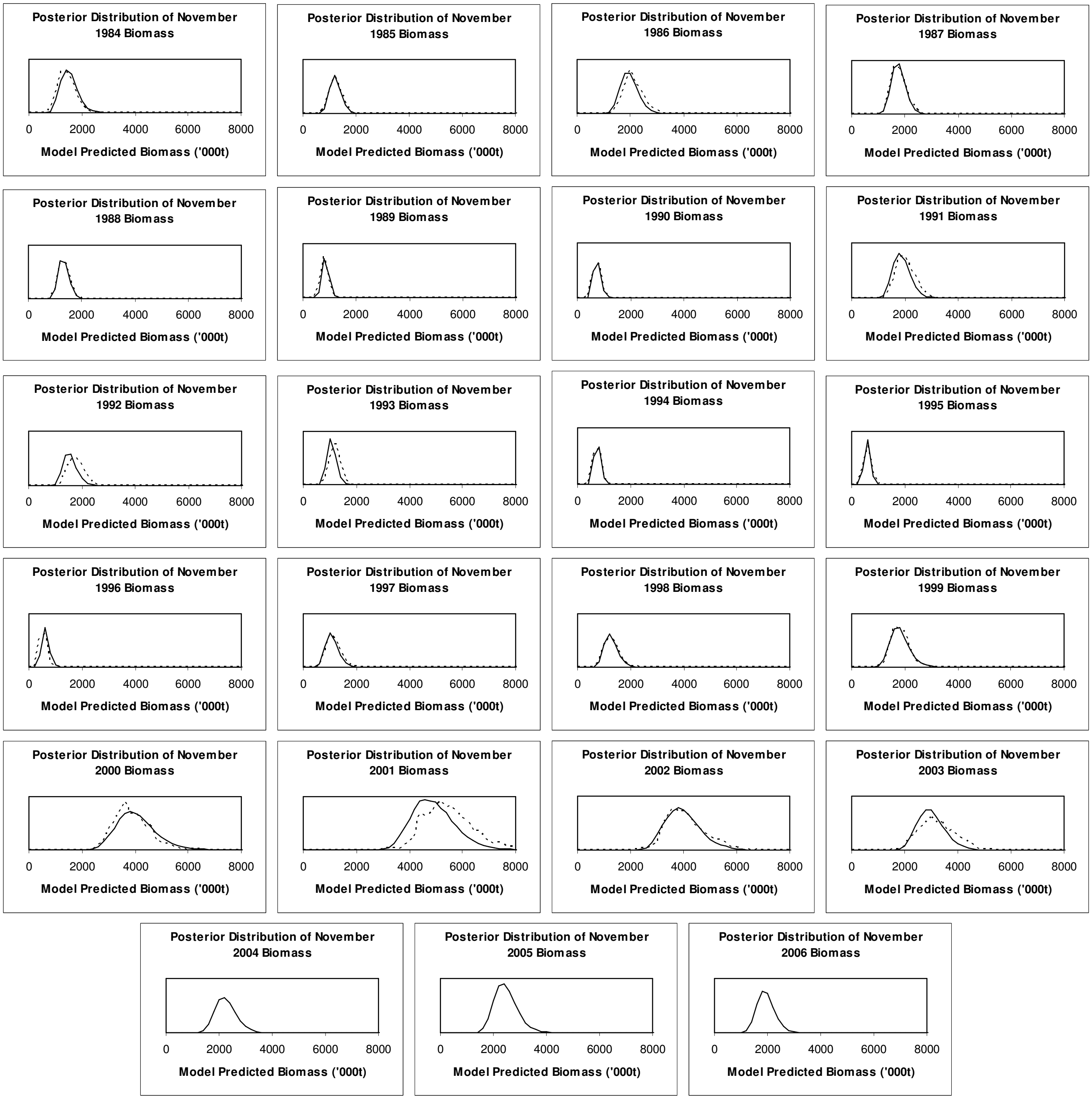

Figure 10. The posterior pdfs of annual November biomass from this assessment (solid line) and the last assessment (dashed line).

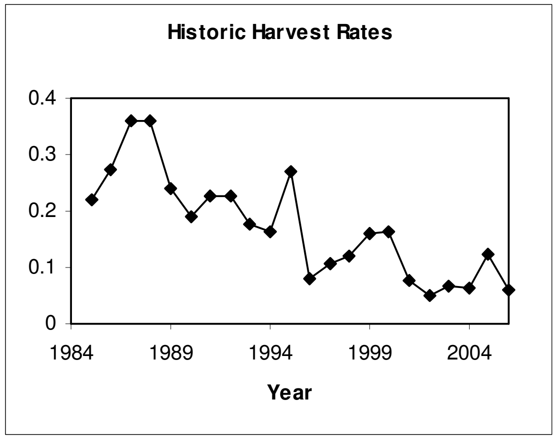

Figure 11. The historic harvest rate (catch by mass to spawner biomass) on anchovy from the base case model.

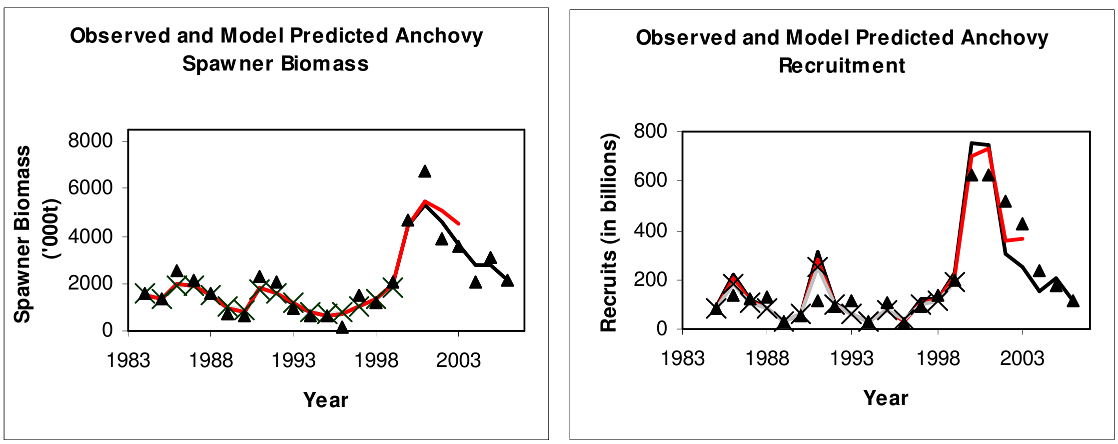

Figure 12. Acoustic survey observed and model predicted a) November anchovy spawner biomass and b) anchovy recruit numbers for the base case A_0 (black line), and retrospectives A_{2003} (red line) and A_{1999} (grey line with black crosses).

APPENDIX A: Bayesian Assessment Model for the South African Anchovy Resource

Model Assumptions

1) All fish have a theoretical birthdate of 1 November.
2) Anchovy spawn for the first time (and are called adult anchovy) when they turn one year old.
3) A plus group of age 4 is used, thus assuming that natural mortality is the same for age 4 and older ages.
4) Two acoustic surveys are held each year: the first takes place in November and surveys the adult stock; the second is in May/June (known as the recruit survey) and surveys juvenile anchovy.
5) The November acoustic survey provides a relative index of abundance of unknown bias.
6) The recruit survey provides a relative index of abundance of unknown bias.
7) The egg survey observations (derived from data collected during the earlier November surveys) provide absolute indices of abundance.
8) The survey designs have been such that they result in survey estimates of abundance whose bias is invariant over time.
9) Pulse fishing occurs five months after 1 November for 1-year-old anchovy; for 0-year-old anchovy this occurs 7½ months after 1 November prior to 1999, and 8½ months after 1 November from 1999 onwards; these two ages (0 and 1) are the only ages targeted by the fishery.
10) Catches are measured without error. (Selectivity of age 0 and age 1 anchovy varies from year to year. This would prove problematic were model predicted catch to be estimated and fitted to observed catch, but here the observed catches-at-age are directly incorporated into the dynamics.)
11) Natural mortality is year-invariant for juvenile and adult fish, and age-invariant for adult fish.

Population Dynamics

Assuming that 1-year-olds are caught in a pulse at 1 April and that 0-year-olds are caught in a pulse at 1 June up to 1998 and 1 July thereafter, the basic dynamic equations for anchovy are as follows.

Numbers-at-age at 1 November

$$N_{y,1}^A = (N_{y-1,0}^A e^{-(7.5)M_j^A/12} - C_{y,0}^A)e^{-(4.5)M_j^A/12} \qquad y = 1984,\ldots,1998$$

$$N_{y,1}^A = (N_{y-1,0}^A e^{-(8.5)M_j^A/12} - C_{y,0}^A)e^{-(3.5)M_j^A/12} \qquad y = 1999,\ldots,2006$$

$$N_{y,2}^A = (N_{y-1,1}^A e^{-5M_{ad}^A/12} - C_{y,1}^A)e^{-7M_{ad}^A/12} \qquad y = 1984,\ldots,2006$$

$$N_{y,3}^A = N_{y-1,2}^A e^{-M_{ad}^A} \qquad y = 1984,\ldots,2006$$

$$N_{y,4+}^A = N_{y-1,3}^A e^{-M_{ad}^A} \qquad y = 1984$$

$$N_{y,4+}^A = N_{y-1,3}^A e^{-M_{ad}^A} + N_{y-1,4+}^A e^{-M_{ad}^A} \qquad y = 1985,\ldots,2006 \qquad \text{(A.1)}$$

where

$N^A_{y,a}$ is the number (in billions) of anchovy of age *a* at the beginning of November in year *y*;

$C^A_{y,a}$ is the number (in billions) of anchovy of age *a* caught from 1 November in year $y-1$ to 31 October in year y;

M^A_j is the natural mortality (in year^{-1}) of juvenile anchovy (i.e. fish of age 0); and

M^A_{ad} is the natural mortality (in year^{-1}) of adult anchovy (i.e. fish of age 1+).

Biomass associated with the November survey

$$\hat{B}^A_{y,N} = \sum_{a=1}^{4+} N^A_{y,a} w^A_{y,a} \qquad y = 1984,\ldots,2006 \qquad \text{(A.2)}$$

where:

$\hat{B}^A_{y,N}$ is the biomass (in thousand tons) of adult anchovy at the beginning of November in year *y*, which are taken to be associated with the November survey; and

$w^A_{y,a}$ is the mean mass (in grams) of anchovy of age *a* sampled during the November survey of year *y*.

Anchovy are assumed to mature at age 1 and thus the spawning stock biomass is:

$$SSB^A_{y,N} = \sum_{a=1}^{4+} N^A_{y,a} w^A_{y,a} \qquad y = 1984,\ldots,2006 \qquad \text{(A.3)}$$

Recruitment

For the base case assessment a Hockey-Stick (or Single-Sloped) stock-recruitment curve is assumed. Recruitment at the beginning of November is assumed to fluctuate lognormally about the stock-recruitment curve. Thus recruitment in November is given by:

$$N^A_{y,0} = \begin{cases} a^A e^{\varepsilon^A_y} & \text{,if SSB}^A_{y,N} \geq b^A \\ \dfrac{a^A}{b^A} SSB^A_{y,N} e^{\varepsilon^A_y} & \text{,if SSB}^A_{y,N} < b^A \end{cases} \qquad y = 1980,\ldots,2005 \qquad \text{(A.4)}$$

where

a^A is the maximum recruitment (in billions);

b^A is the spawner biomass below which the expectation for recruitment is reduced below the maximum; and

ε^A_y is the annual lognormal deviation of anchovy recruitment.

Number of recruits at the time of the recruit survey

The following equation projects $N^A_{y,0}$ to the start of the recruit survey, taking natural and fishing mortality into account, and assuming pulse fishing of juveniles half way between 1 November and the start of the recruit survey.

$$\hat{N}^A_{y,r} = (N^A_{y-1,0} e^{-0.5(6+t^A_y)M^A_j/12} - C^A_{y,0bs}) e^{-0.5(6+t^A_y)M^A_j/12} \qquad y = 1984,\ldots,2006 \qquad \text{(A.5)}$$

where

$\hat{N}^A_{y,r}$ is the number (in billions) of juvenile anchovy at the time of the recruit survey in year *y*;

$C^A_{y,0bs}$ is the number (in billions) of juvenile anchovy caught between 1 November and the day before the start of the recruit survey in year *y*;

t^A_y is the time lapsed (in months) between 1 May and the start of the recruit survey that provided the estimate $N^A_{y,rec}$ in year *y*.

Proportions of 1-year-olds associated with November survey

$$\hat{p}^A_{y,1} = \frac{N^A_{y,1}}{\sum_{a=1}^{4+} N^A_{y,a}} \qquad y = 1984,\ldots,2006 \qquad \text{(A.6)}$$

where

$\hat{p}^A_{y,1}$ is the proportion of 1-year-old anchovy at the beginning of November in year *y*, which is taken to be associated with the November survey.

Fitting the Model to Observed Data (Likelihood)

The observations are assumed to be log-normally distributed, and sampling CVs (squared) of the untransformed survey observations are used to approximate the "sampling" component of the total variance of the corresponding log-distributions. The proportions of 1-year-olds are first logit-transformed before being used in the likelihood[4]. Thus we have:

$$\begin{aligned}
-\ln L = {} & \tfrac{1}{2}\sum_{y=1984}^{2006}\left\{\frac{\left(\ln B^A_{y,N} - \ln(k^A_N \hat{B}^A_{y,N})\right)^2}{(\sigma^A_{y,Nov})^2 + (\lambda^A_N)^2} + \ln\left[2\pi\left((\sigma^A_{y,Nov})^2 + (\lambda^A_N)^2\right)\right]\right\} \\
& + \tfrac{1}{2}\sum_{y=1984}^{1991}\left\{\frac{\left(\ln B^A_{y,egg} - \ln(k^A_g \hat{B}^A_{y,N})\right)^2}{(\sigma^A_{y,egg})^2} + \ln\left[2\pi(\sigma^A_{y,egg})^2\right]\right\} \\
& + \tfrac{1}{2}\sum_{y=1985}^{2006}\left\{\frac{\left(\ln N^A_{y,r} - \ln(k^A_r \hat{N}^A_{y,r})\right)^2}{(\sigma^A_{y,rec})^2 + (\lambda^A_r)^2} + \ln\left[2\pi\left((\sigma^A_{y,rec})^2 + (\lambda^A_r)^2\right)\right]\right\} \\
& + \frac{1}{2}\sum_{y=1984}^{2006}\left\{\frac{\left(\ln\left(p^A_{y,1}/\left(1-p^A_{y,1}\right)\right) - \ln\left(k^A_p \hat{p}^A_{y,1}/\left(1-k^A_p \hat{p}^A_{y,1}\right)\right)\right)^2}{\left(\sigma^A_p\right)^2} + \ln\left[2\pi\left(\sigma^A_p\right)^2\right]\right\}
\end{aligned} \qquad \text{(A.7)}$$

where

$B^A_{y,N}$ is the acoustic survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year *y*, with associated CV $\sigma^A_{y,Nov}$ and constant of proportionality (multiplicative bias) k^A_N;

[4] This transformation proved adequate, resulting in no heteroscedasticity in the residuals of the logit transformation.

$B^A_{y,egg}$ is the egg survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year *y*, with associated CV $\sigma^A_{y,egg}$ and constant of proportionality k^A_g;

$N^A_{y,rec}$ is the acoustic survey estimate (in billions) of anchovy recruitment from the recruit survey in year *y*, with associated CV $\sigma^A_{y,rec}$ and constant of proportionality k^A_r;

$p^A_{y,1}$ is an estimate of the proportion (by number) of 1-year-old anchovy in the November survey of year *y*, derived by one of two methods (*meth*=Prosch uses the Prosch age length keys, and *meth*=10/10.5/11cm uses a cut-off length in the raised length frequencies for the corresponding survey);

k^A_p is a multiplicative bias associated with the proportion of 1-year-olds in the November survey;

$(\lambda^A_{N/r})^2$ is the additional variance (over and above the survey sampling CV $\sigma^A_{y,Nov/rec}$ that reflects survey inter-transect variance) associated with the November/recruit surveys;

σ^A_p is the standard deviation associated with the proportion of 1-year-olds in the November survey, which is estimated in the fitting procedure by:

$$\sigma^A_p = \sqrt{\sum_{y=1984}^{2006}\left[\ln\left(p^A_{y,1}\middle/\left(1-p^A_{y,1}\right)\right)-\ln\left(k^A_p\hat{p}^A_{y,1}\middle/\left(1-k^A_p\hat{p}^A_{y,1}\right)\right)\right]^2 \Big/ \sum_{y=1984}^{2006}1}$$

Fixed Parameters

Four parameters are fixed externally in this assessment (see main text for reasons and for variations for robustness tests):

M^A_j and M^A_{ad} (values given in main text), $\left(\lambda^A_N\right)^2 = 0$, and $k^A_g = 1$, as the egg survey estimates of abundance are assumed to be absolute.

In the base case assessment, it is assumed that $b^A = 0.2K^A$, where carrying capacity, K^A , taken to be the biomass value where replacement line and the stock recruit function intersect, is defined as:

$$K^A = a^A e^{\frac{1}{2}\left(\sigma^A_r\right)^2}\left[\sum_{a=1}^{3}\overline{w}^A_a e^{-M^A_j-(a-1)M^A_{ad}} + \overline{w}_{4+}e^{-M^A_j-3M^A_{ad}}\frac{1}{1-e^{-M^A_{ad}}}\right] \qquad \text{(A.8)}$$

(calculated assuming maximum recruitment in the absence of fishing) where

$\overline{w}^A_a$ is the average of $w^A_{y,a}$ defined above.

The $e^{\frac{1}{2}\left(\sigma^A_r\right)^2}$ factor (see below for definition) in the above equation corrects for log-normal distribution bias.

Estimable Parameters and Prior Distributions

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve:

$$\varepsilon^A_y \sim N\left(0,\left(\sigma^A_r\right)^2\right) \;, \qquad y = 1984,\ldots,2005$$

The remaining estimable parameters are defined as having the following near non-informative prior distributions:

$\ln(k_N^A) \sim U(-100,0.7)$ (upper bound corresponding to $k_N^A = 2$)

$\ln(k_r^A) \sim U(-100,0.7)$ (upper bound corresponding to $k_r^A = 2$)

$\ln(k_p^A) \sim U(-100,0.7)$ (upper bound corresponding to $k_p^A = 2$)

$(\lambda_r^A)^2 \sim U(0,100)$

$(\sigma_r^A)^2 \sim U(0,10)$

$\ln(a^A) \sim U(0,8)$ (given the lack of *a priori* information on the scale of a^A, a log-scale was used)

$N_{1983,a}^A \sim U(0,500)$, $a = 0,\ldots,3$

Further Outputs

Recruitment serial correlation:

$$s_{cor}^A = \frac{\sum_{y=1984}^{2004} \varepsilon_y \varepsilon_{y+1}}{\sqrt{\left(\sum_{y=1984}^{2004} \varepsilon_y^2\right)\left(\sum_{y=1984}^{2004} \varepsilon_{y+1}^2\right)}} \qquad \text{(A.9)}$$

and the standardised recruitment residual value for 2005:

$$\eta_{2005}^A = \frac{\varepsilon_{2005}^A}{\sigma_r^A}. \qquad \text{(A.10)}$$

are also required as input into the OMP.