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On Drawing Inferences Concerning Trends in Selectivity with Age from Tag-Recapture Information

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Abstract

It is shown that the implications of dome shaped selectivity for tag recovery proportions as a function of age depend on whether the drop in selectivity at large age arises from a gear selection effect or is surrogating emigration. A simple extraction of summary statistics from tag-recapture data is suggested to throw further light on the mechanisms actually in play for various stocks.

Selectivity Arising from Gear Selection Effects

For simplicity, and also because this captures the essence of the issue, we consider only ages ($a \geq A$) above which selectivity (S_a) declines exponentially ($S_{a+1} = S_a e^{-\alpha}$), as assumed (for example) in Butterworth and Rademeyer (2008). Note that this includes the case where selectivity is flat ($\alpha = 0$).

Let $n_a(t)$ be the number of tagged animals remaining in the population and available for capture at a time t after an initial number n_A^0 were first tagged when of age A. Then:

$$\frac{dn_A}{dt} = -\left(M + e^{-\alpha t}F\right)n_A \tag{1}$$

$$\frac{dc_A}{dt} = e^{-\alpha t}Fn_A \tag{2}$$

where

$S(t) = e^{-\alpha t}$ (S at age A is taken to be 1),

F = fishing mortality rate at age A,

M = natural mortality rate (taken to be age independent),

$c_A(t)$ = rate at which tags are recaptured at time t.

First consider the case $\alpha = 0$:

$$n_A(t) = n_A^0 e^{-(F+M)t} \tag{3}$$

$$c_A^a = \int_a^{a+1} Fn(t)dt = n_A^0 e^{-(F+M)(a-A)} \frac{F}{F+M}\left[1 - e^{-(F+M)}\right] \tag{4}$$

$$c_A^{all} = \int_{a=A}^{\infty} Fn(t)dt = n_A^0 \frac{F}{F+M} \tag{5}$$

$$R_A = c_A^{all} / n_A^o = F/(F+M) \tag{6}$$

where

c_A^a is the number of tags recaptured from fish of age *a* (i.e. *a*-*A* years after they were tagged),

c_A^{all} is the total number of tags recaptured at any time after tagging, and

R_A is the proportion of fish tagged at age *A* that are recaptured (i.e. notation as in Miller *et al.* (2008)).

If instead we consider n_{A+1}^0 fish tagged at age *A*+1, equations (5) and (6) become:

$$c_A^{all} = n_{A+1}^0 \frac{F}{F+M} \tag{7}$$

$$R_{A+1} = F/(F+M) \tag{8}$$

i.e. the proportion of tags returned R_A is independent of *A*, corresponding to the data plotted in Fig. 2 of Miller *et al*. (2008).

Now consider the $\alpha > 0$ case:

$$n_A(t) = n_A^0 e^{-\left(Mt+F(1-e^{-\alpha t})/\alpha\right)} \tag{9}$$

$$c_A^{all} = \int_0^\infty F e^{-\alpha t} n_A^0 e^{-\left(Mt+F(1-e^{-\alpha t})/\alpha\right)} dt \tag{10}$$

$$R_A = F \int_0^\infty e^{-\left(Mt+\alpha t+F(1-e^{-\alpha t})/\alpha\right)} dt \tag{11}$$

and for fish tagged at age *A*+1:

$$R_{A+1} = F e^{-\alpha} \int_0^\infty e^{-\left(Mt+\alpha t+Fe^{-\alpha}(1-e^{-\alpha t})/\alpha\right)} dt \tag{12}$$

To a good approximation (true in the limit of $F << M + \alpha$)

$$R_{A+1} \approx e^{-\alpha} R_A \tag{13}$$

This exponential decline with *A* reflects what Miller *et al*. (2008, Fig. 1) show for yellowtail flounder for the selectivity form estimated by Butterworth and Rademeyer (2008) (for which $e^{-\alpha} = 0.44$). This decline is **not** shown by the tag return data (Fig. 2 of Miller *et al*. (2008)). Hence these tag data are not consistent with a declining selectivity at age **if** this selectivity pattern arises purely from a gear effect, with all tagged fish of a particular age in the population equally likely to be recaptured.

Selectivity Surrogating Emigration

Selectivity at age (S_a) in the ASPM model of Butterworth and Rademeyer (2008) is the combination of availability and gear selection effects. Consider the situation where fish of age A and above move out of the area where they are potentially recaptured by the fishery at an annual proportional rate E, and fishing mortality F is independent of age a. Then if $N_A(t)$ is the number of fish remaining in the part of the population available for capture by the fishery at a time t after N_A^0 of them reached age A:

$$\frac{dN_A}{dt} = -(M + F + E)N_A \quad (14)$$

$$N_A(t) = N_A^0 e^{-(M+F+E)t} \quad (15)$$

from which it follows that the annual catches of age A and age A+1 fish are respectively:

$$C_A = N_A^0 \frac{F}{M + F + E}\left[1 - e^{-(M+F+E)}\right] \quad (16)$$

$$C_{A+1} = N_A^0 e^{-(M+F+E)} \frac{F}{M + F + E}\left[1 - e^{-(M+F+E)}\right] \quad (17)$$

so that

$$C_{A+1}/C_A = e^{-(M+F+E)} \quad (18)$$

Compare this with the situation of no emigration, but a selectivity S operative at age A+1:

$$\frac{dN_A}{dt} = -(M + F)N_A \qquad \text{for } 0 \le t \le 1 \quad (19)$$

$$\frac{dN_A}{dt} = -(M + SF)N_A \qquad \text{for } 1 \le t \le 2 \quad (20)$$

for which

$$C_A = N_A^0 \frac{F}{M+F}\left[1-e^{-(M+F)}\right] \qquad (21)$$

$$C_{A+1} = N_A^0 e^{-(M+F)} \frac{SF}{M+SF}\left[1-e^{-(M+SF)}\right] \qquad (22)$$

so that

$$C_{A+1}/C_A = e^{-(M+F)} S \frac{M+F}{M+SF} \frac{1-e^{-(M+SF)}}{1-e^{-(M+F)}}$$

$$\approx e^{-(M+F)} S \frac{M+SF}{M+F}$$

$$\approx e^{-(M+F)} S \qquad (23)$$

where the approximations first take the series expansion of the exponentials to 2^{nd} order terms only, and secondly assume either S close to 1 or $SF<<M$.

Comparing equations (18) and (23), it is evident that an apparent decrease in selectivity can be a surrogate for emigration:

$$S \approx e^{-E} \qquad (24)$$

In these circumstances, replication of the analyses of equations (3-6) will yield:

$$R_A = F/(F+M+E) \qquad (25)$$

independent of A, i.e. if dome-shaped selectivity in the assessment is a reflection of emigration, even though S_a as estimated decreases with a, R_A will remain constant as shown by the yellowtail flounder data in Fig. 2 of Miller *et al*. (2008). (Note that in the context of tagged fish, E could include effects of continuous tag shedding and tag-induced additional mortality, as well as emigration itself.)

Thus the tag-recapture data for yellowtail flounder as summarised in Fig. 2 of Miller *et al*. (2008) are not inconsistent with dome shaped selectivity, **if** such a shape arises from an emigration effect. Note that such "emigration" could arise from two possible mechanisms: first emigration to outside the area fished, and secondly net avoidance being more readily accomplished by larger (older) and hence likely stronger swimming fish. The second mechanism mimics emigration because of correlation effects that come into play if tagged fish are captured and recaptured by the same method; this is because the two samples will not be independent samples of all age *a* fish, since amongst such fish, those more adept at net avoidance will be more likely to be absent from the first (tagging) sample as well as the second (recapture) sample.

Inferences from Mean Time to Recovery

The mean time from tagging to recovery is given by:

$$T_A = \int_0^\infty t c_A(t)dt \Big/ \int_0^\infty c_A(t)dt \qquad (26)$$

Under the model of equation (14) [*F* independent of age and emigration]:

$$c_A(t) = n_A^0 F e^{-(M+F+E)t} \qquad (27)$$

which yields:

$$T_A = 1/(M + F + E) \qquad (28)$$

If then *E*=0, one gets two equations:

$$T_A = 1/(M + F) \qquad (29)$$

$$R_A = F/(M + F) \qquad (6)$$

which given T_A and R_A from the data can be solved for the unknowns *M* and *F*.

In simple area-aggregated terms, this is the basis that allows Miller *et al*. (2008) to obtain their Table 1 estimates of *M* and *F*. Given R_A of about 0.08 (their Fig. 2), equation 6 precludes *F* from being too high, and then what must be an effective lowish mean time to recapture forces an (unrealistically) high *M* estimate through equation (29).

However, if the possibility of emigration to an area outside the three considered by Miller *et al.* (2008) is considered:

$$T_A = 1/(M + F + E) \qquad (30)$$

$$R_A = F/(M + F + E) \qquad (31)$$

Here the introduction of E allows a realistic estimate of M (e.g. 0.2) to become compatible with tagging results for R_A and T_A.

This simple analysis thus suggests that the combination of the tagging results for yellowtail flounder and a realistic value of M necessitates emigration $E>0$ and hence could provide confirmatory evidence for dome shaped selectivity.

The presentation of simple area-aggregated R_A and T_A statistics for various choices for A (or length) for which such data are available, together with associated values of F and E from equations (30) and (31) given a fixed choice for M (0.2 perhaps) would therefore seem to have the potential to provide independent evidence for the possibility of dome shaped selectivity for such resources.

References

Butterworth DS and Rademeyer RA. 2008. Application of an age-structured production model to the Georges Bank yellowtail flounder. 2008 GARM Assessment Methodology Meeting Working Paper 2.5.

Miller T, Hart D, Cadrin S, Jacobson L, Legault C and Rago P. 2008. Analyses of tagging data for evidence of decreased fishing mortality for large yellowtail flounder. 2008 GARM Assessment Methodology Meeting Working Paper.