

Age-reading error matrices for *Merluccius paradoxus* and *M. capensis*

Rebecca A Rademeyer

October 2009

Summary

The revised methodology for hake assessments currently under development requires inputs on the relative biases amongst different readers of hake ages from otoliths, and the extent of variability of these readings. This document details the methodology applied to determine these ageing error matrices.

Introduction

The current stock assessment models used for the South African *M. paradoxus* and *M. capensis* resources are age structured models, making use of ageing data, either in the form of catch-at-age data or more recently directly as age-length keys. Until now the assumption has been made that age classes are determined without error, when in fact some level of misclassification is often to be expected. Age-reading error occurs when estimates of age based on reading hard structures such as otoliths differ from the true age of the animal concerned. There are two sources of uncertainty in the relationship between the ages obtained from reading otoliths and the true age of the animal: bias and imprecision. Ageing bias occurs when there is a systematic difference between the true age of an animal and the age assigned to it, whereas ageing imprecision occurs when age-reading errors occur at random (Punt *et al.*, 2008).

Errors in ageing can be taken into account by supplying an ageing-error matrix (Fournier and Archibald, 1982; Richards *et al.*, 1992; Punt *et al.*, 2008), which defines the probability of assigning a particular age to a fish with a given true age. The method described in Punt *et al.* (2008) is used here to construct such matrices for the two hake species for use in upcoming assessments.

Data and Method

Punt *et al.* (2008) model the probability of reader i (of I readers) assigning an animal of true age a an age of a' , $P^i(a'|a)$, by assuming that both the ageing bias and the age-reading error standard deviation depend on the reader and the true age of the animal and that age-reading error is normally distributed about the expected age (i.e., the expected age given any bias in age reading):

$$P^i(a'|a, \varphi) \propto \exp \left[\frac{-(a' - b_a^i(\varphi))^2}{2(\sigma_a^i(\varphi))^2} \right] \quad (1)$$

where

- b_a^i is the expected age when reader i determines the age of an animal of true age a ,
- σ_a^i is the standard deviation for reader i of the age-reading error for animals of true age a , and
- φ is the vector of parameters that determines the age-reading error matrix.

The ageing bias is modelled by:

$$b_a = \begin{cases} b_L + (b_H - b_L) \frac{1 - e^{-\lambda(a-L)}}{1 - e^{-\lambda(H-L)}} & \text{if } \lambda \neq 0 \\ b_L + (b_H - b_L) \frac{a-L}{H-L} & \text{if } \lambda = 0 \end{cases} \quad (2)$$

where

- b_L is the expected age of animal of prespecified minimum age L ,
 b_H is the expected age of animal of prespecified maximum age H , and
 λ determines the extent of nonlinearity between the true age and the expected age (note that $\lambda = 0$ reflects the special case of linear dependence).

The age-reading error standard deviation is modelled by:

$$\sigma_a = \begin{cases} \sigma_L + (\sigma_H - \sigma_L) \frac{1 - e^{-\alpha(a-L)}}{1 - e^{-\alpha(H-L)}} & \text{if } \alpha \neq 0 \\ \sigma_L + (\sigma_H - \sigma_L) \frac{a-L}{H-L} & \text{if } \alpha = 0 \end{cases} \quad (3)$$

where

- σ_L is the age-reading error standard deviation for a prespecified minimum age L ,
 σ_H is the age-reading error standard deviation for a prespecified maximum age H , and
 α determines the extent of nonlinearity between age and the age-reading error standard deviation (note that $\alpha = 0$ reflects the special case of linear dependence).

The values for the parameters that determine the age-reading error matrix for each reader are estimated by maximizing the following likelihood function:

$$L(A|\beta, \varphi) = \prod_{j=1}^J \sum_{a=L}^H \beta_a \prod_{i=1}^I P^i(a_{i,j}|a, \varphi) \quad (4)$$

where

- $a_{i,j}$ is the age assigned by reader i to the j th otolith,
 A is the entire data set of otolith readings, and
 β_a are nuisance parameters that can be interpreted as the relative frequency of animals of (true) age a in the sample (rather than in the population from which the sample was taken).

In general, not all otoliths are read by all readers. Therefore, the likelihood function is more generally the product of eqn (4) over sets of otoliths that were all read by the same group of readers and a separate set of β 's is estimated for each such set of otoliths.

For this hake case the ageing error matrices were computed for each species separately. The data were aggregated over sex and over all sources of data (survey, commercial offshore and commercial longline). For each species, the data were divided into three groups of three readers:

- Alexia Daniels (AD), Luke Bester (LB) and 'Unknown Reader' (UR),
- Kevin Gradie (KG), John Prinsloo (JP) and Andy Payne/Dave Japp (AP/DJ) (these two readers have been aggregated as they read otoliths only when KG and JP did not agree, i.e. relatively very few data available), and
- Phoeby Mullins (PM), Teressa Akkers (TA) and Kashif Booley (KB).

Table 1 give details on the data available for each group.

Results and Discussion

In each group of three readers, one reader at least was assumed to be unbiased, as the age-reading errors would be confounded otherwise (Punt *et al.*, 2008). For each group of three readers, Akaike's information criterion (AIC) was used to select among alternative models (including which reader should be assumed to be unbiased).

The final models for age-reading error are summarised in Table 2, while Figs 1 and 2 show plots of the age-reading error matrices for each reader and species. Fig. 3 plots the estimated ageing bias for each reader within each set of three readers. The fact that in some instances there is a bias for the true age zero is connected to the use of a minus group.

References

- Fournier D and Archibald CP. 1982. A general theory for analyzing catch at age data. *Can. J. Fish. Aquat. Sci.* 39: 1195-1207.
- Punt AE, Smith DC, KrusicGolub K and Robertson S. 2008. Quantifying age-reading error for use in fisheries stock assessments, with application to species in Australia's southern and eastern scalefish and shark fishery. *Can. J. Fish. Aquat. Sci.* 65: 1991-2005.
- Richards LJ, Schnute JT, Kronlund AR and Beamish RJ. 1992. Statistical models for the analysis of ageing error. *Can. J. Fish. Aquat. Sci.* 49: 1801-1815.

Table 1: Number of aged hake by species for each reader.

Source	Year	<i>M. capensis</i>			<i>M. paradoxus</i>		
		UR	AD	LB	UR	AD	LB
WC summer	1999	314	351	358	324	299	299
WC summer	2006				263	465	465
WC summer	2007		369	369		554	554
WC summer	2008		451	451		409	409
WC winter	2004		808	808			
SC spring	2006					243	243
SC autumn	1999		265	264		139	139
SC autumn	2005					192	192
SC autumn	2007		626	626		358	358
SC autumn	2008		638	638		214	214

Source	Year	<i>M. capensis</i>			<i>M. paradoxus</i>		
		KG	JP	AP/DJ	KG	JP	AP/DJ
WC summer	1992	389	389	33	310	310	44
WC summer	1993	351	351	62	311	311	49
WC summer	1994	282	282	6	290	290	4
WC summer	1995	0	368		0	303	0
SC autumn	1992	329	329	91	40	40	5
SC autumn	1993	407	407	40	95	95	23
SC autumn	1994	390	391	83	72	69	27
Comm Offshore	1992	260	260	28	521	521	46
Comm Offshore	1993	115	115	17	645	645	75
Comm Offshore	1994	126	126	5	330	330	38
Comm Longline	1994				314	314	9

Source	Year	<i>M. capensis</i>			<i>M. paradoxus</i>		
		PM	TA	KB	PM	TA	KB
WC autumn	1999	408	406	400	140	140	140

Table 2: Selected model for age-reading error for each reader and species.

	<i>M. paradoxus</i>		<i>M. capensis</i>	
	bias	precision	bias	precision
AD	Eqn 2	Eqn 3	Eqn 2	Eqn 3
LB	Eqn 2	Eqn 3	Eqn 2	Eqn 3
UR	Unbiased	Eqn 3	Unbiased	Eqn 3
KG	Eqn 2	Eqn 3	Eqn 2	Eqn 3
JP	Eqn 2	Eqn 3	Eqn 2	Eqn 3
AP/DJ	Unbiased	Eqn 3	Unbiased	Eqn 3
PM	Linear	Eqn 3	Eqn 2	Eqn 3
TA	Unbiased	Eqn 3*	Unbiased	Eqn 3
KB	As PM	As PM	As PM	Eqn 3*

* Eqn 3 pertains to the coefficient of variation rather than the standard deviation.

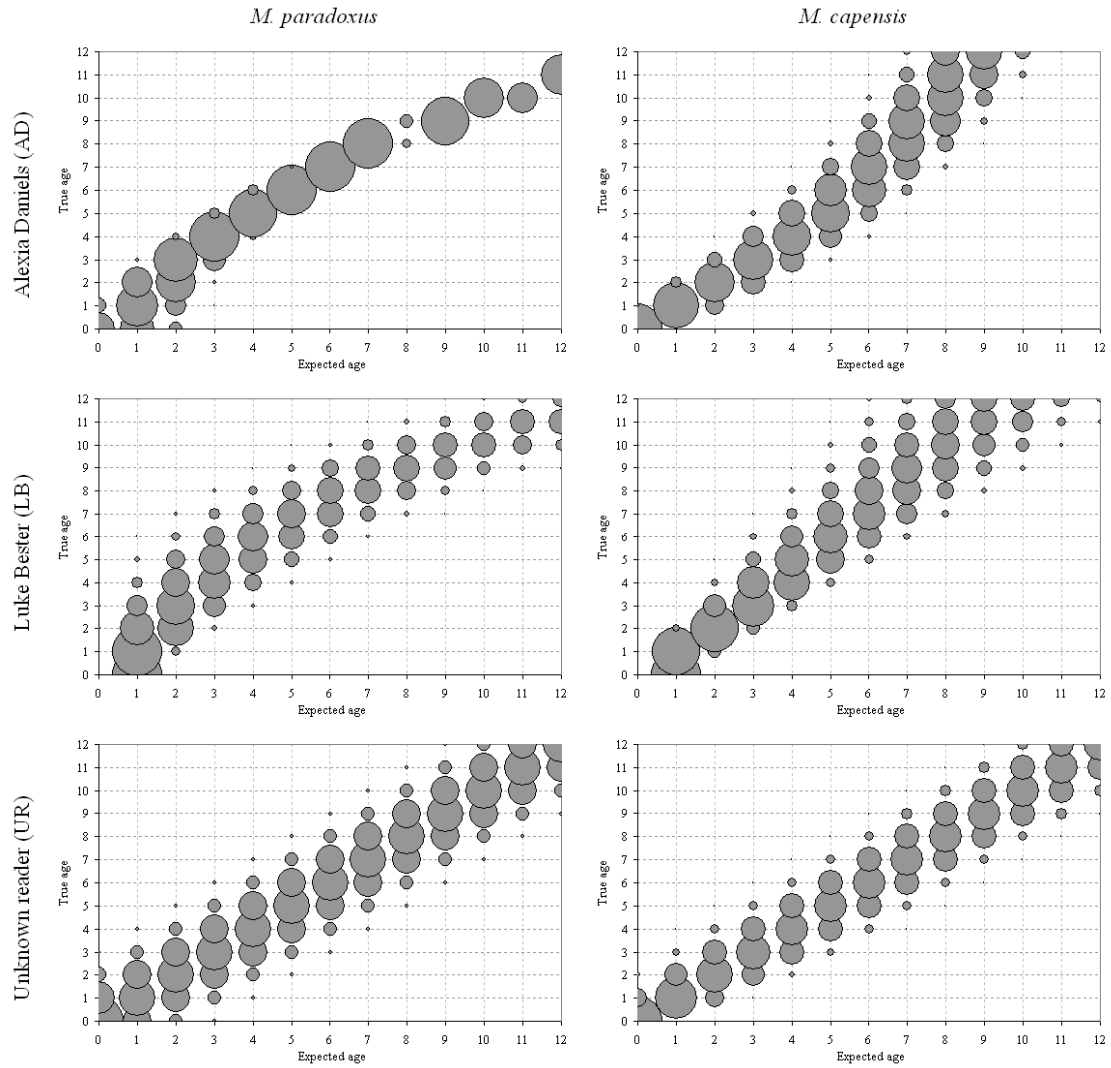


Fig. 1: Plots of the ageing-error matrices ('true' vs. expected age – the area of the bubble represents the proportion expected at each age) for *M. paradoxus* and *M. capensis*, for the two current readers (AD and LB) and the 'unknown reader'.

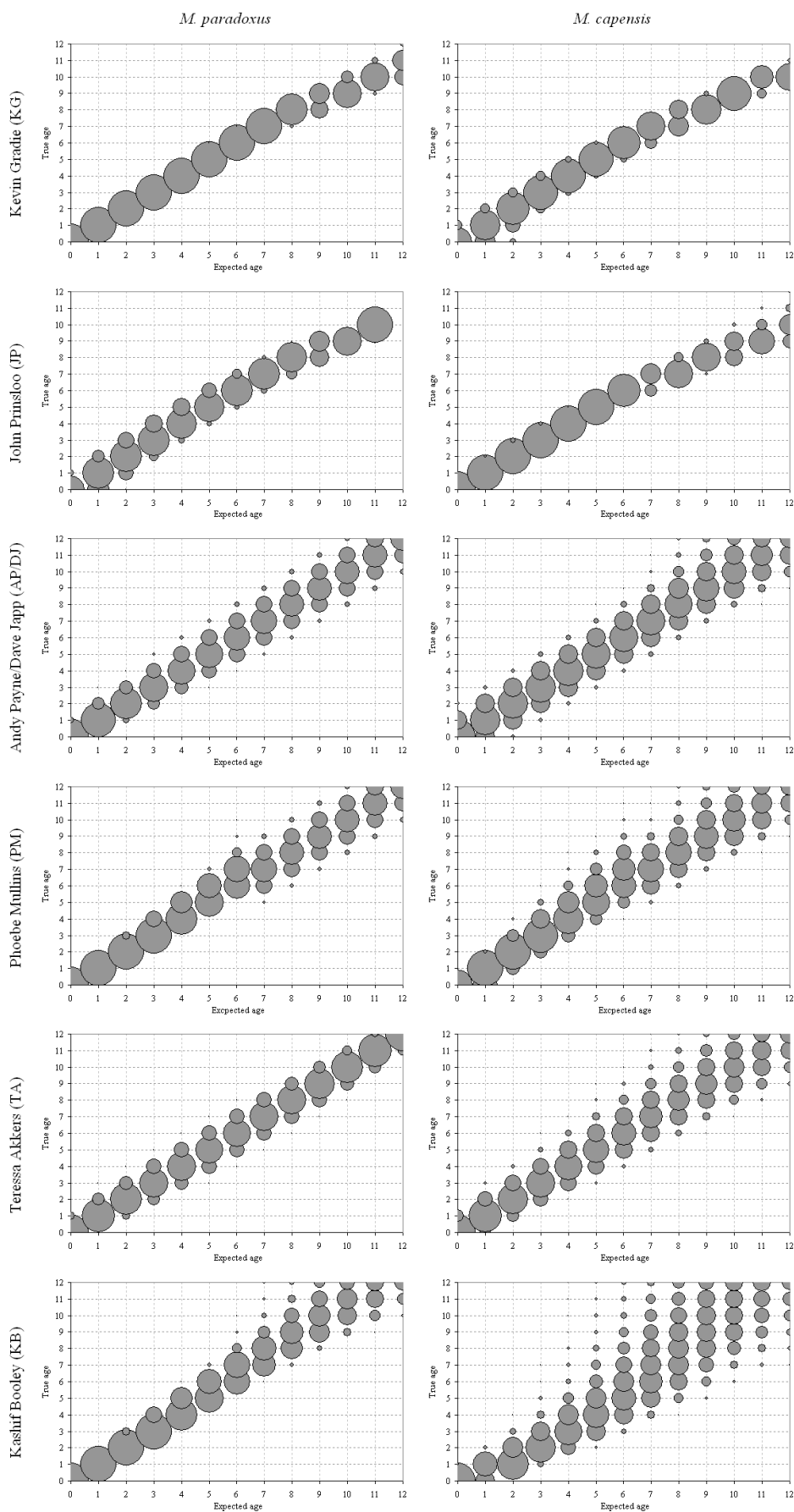


Fig. 2: Plots of the ageing-error matrices ('true' vs. expected age – the area of the bubble represents the proportion expected at each age) for *M. paradoxus* and *M. capensis*.

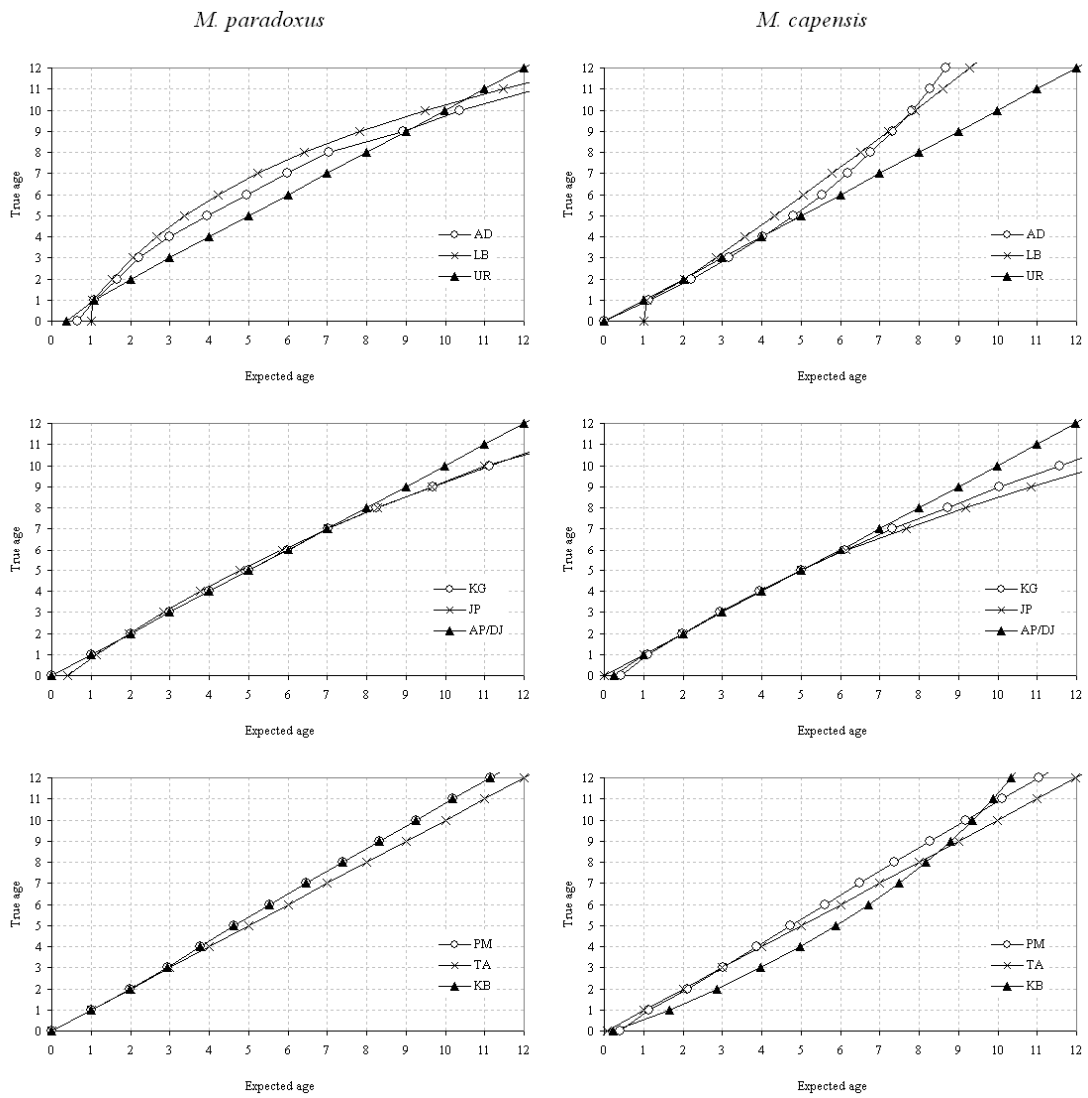


Fig. 3: Plots of the true vs. mean expected age across readers for *M. paradoxus* and *M. capensis*.