Updated Sardine Assessment

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The previous full assessment of the SA sardine resource, used to develop OMP-08, was tuned to data up to and including November 2006 (Cunningham and Butterworth 2007, with further undocumented updates). Since then, 3 further years of below average recruitment have been evident from the May recruitment surveys, together with the subsequent low November survey biomass estimates. This document presents an update of the sardine assessment (posterior modes only), now taking data up to August 2009 into account. This is to obtain a better understanding of the current status of the population and assist in 2010 directed sardine fishery planning.

Methods

The new data used in this assessment, other than those used in the previous assessment and documented in Cunningham *et al*. 2007, are detailed in Appendix A.

The base case model is identical to the model implemented to produce the Bayesian posterior distributions used to develop OMP-08 (Cunningham and Butterworth 2007, with further undocumented updates). This is the case for which the model was fitted to survey estimates of November biomass and May recruitment only. In the absence of ageing data, selectivity was not estimated in this model. Instead it was set at 0.43 for age 1 and at 1 for the remaining ages, corresponding to that estimated by the 'full model' which included catch-at-length data. The additional variance parameters, λ_N^S and λ_R^S are set to zero and the proportions-at-age for the initial year are fixed ($Nprop_0^S = 0.31$, $Nprop_1^S = 0.23$, $Nprop_2^S = 0.45$, $Nprop_3^S = Nprop_{4+}^S = 0.01$).

Due to time constraints, only model runs to provide posterior modes have been carried out. The base case model used the historic average November survey weights-at-age in 2007, 2008 and 2009, and sardine bycatch was calculated assuming average April-August 2009 anchovy catch in September and October 2009 (see Appendix A). The sensitivity tests carried out included:

1) "Weight": November survey weights-at-age for 2007 and 2008 estimated from currently available ALKs for these years are used, with the constraint that the weight-at-age 5+ cannot be less than that at age 4.

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2) "Bycatch": Sardine bycatch was calculated assuming double the average April-August 2009 anchovy catch in September and October 2009.
3) Alternative fixed values for natural mortality, maintaining the assumption that juvenile natural mortality $M_0^S = M_a^S + 0.2$, $a = 1, \ldots, 5+$.

Results and Discussion

The model fits to the November 1+ biomass and May recruitment are shown in Figures 1 and 2, respectively. The updated model fits the data well and predicts slightly lower November 1+ biomass and May recruitment in 2005 and 2006 compared to the previous assessment. The model predictions are very similar to those obtained when the sardine assessment was updated in November 2008 (Figures 1 and 2; de Moor and Butterworth 2008). The residuals from the model fits to these data are shown in Figures 3 and 4. The inclusion of three further years of data has resulted in the estimated maximum recruitment of the hockey stick stock recruitment curve being decreased substantially from around 78 billion to 48 billion recruits (Table 1, Figure 5). The residuals about this stock recruitment curve are shown in Figure 6. The standard deviation of the (non-peak years) recruitment residuals is estimated to have increased from around 0.4 to 0.6 (Table 1). This is to be expected as the November 2003 to 2007 recruitments have been much lower than would have been anticipated.

There is little difference between the base case assessment and the sensitivity test assuming a higher sardine bycatch in September and October 2009 (Table 1). The November 2009 1+ biomass is predicted to be 52 000t smaller under the sensitivity test assuming alternative weights-at-age in November 2007 and 2008.

Making the assumption that natural mortality, *M*, has remained unchanged between 1984 and 2009, Table 2a shows that the model achieves a better fit to the data (plus priors) for higher values of *M*. However, the ability of these models to achieve a better fit for higher values of *M* has been noted before; it arises primarily because with a higher *M*, there is less "memory" in the population and the November survey biomass tends towards representing only a single year-class so that there is greater flexibility to fit the data more closely. It has been considered that the ratio of estimated multiplicative bias in the recruit survey to that in the November survey should realistically fall in the range [0.5; 1.0]. This ratio falls outside this range for high values of *M* in Table 2b. There is some indication that the data provide support for a lower *M* pre-2000 than post-2000 (Table 2a,b). However, firm conclusions as to which values of natural mortality are most appropriate for sardine should await the full revised assessment. This rough update has been based on a model which excludes ageing data, and thus its use to distinguish between alternative values of *M* is problematic.

The annual losses to predation as estimated by the base case assessment are given in Table 3 (see Appendix B for the formulae used).

Summary

In summary, the results from this updated assessment are broadly similar to what were obtained from the previous assessment using data up to November 2006, and also the update in November 2008. There is nothing sufficiently "unusual" to suggest other than the continued application of OMP-08, except perhaps the increase in the recruitment residual variance.

The base case assessment predicts a November 2009 1+ biomass of 513 000t (taking bias into account), with a standard error of 163 000t. If this biomass was observed in November 2009, the 2010 directed sardine TAC would be 90 000t (the minimum TAC constraint). It should be noted, however, that this assessment does predict a higher November 1+ biomass than those estimated by the surveys (though within the confidence interval of those estimates) from November 2005 onwards. However, a lower 1+ biomass estimate will not lead to the directed sardine TAC being reduced below the minimum of 90 000t unless the survey estimate drops below the exceptional circumstances threshold of 300 000t. Due to the slightly better recruitment in May 2009, compared to immediately preceding years, the model predicts a biomass increase of 18% from that observed in 2008 (Table 1). An 18% increase to the November 2008 survey estimate of 384 000t would be 452 000t.

Acknowledgements

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References

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Table 1. *Key model parameter values and model outputs estimated at the joint posterior mode (see Glossary for definitions). Fixed values are given in bold. Numbers are reported in billions and biomass in thousands of tonnes.*

Parameter	Previous Assessment (Data up to Nov 2006)		Updated Assessment (Data up to Oct 2008)	Updated Assessment (Data up to Aug 2009)		
	Base Case	Input to OMP08	Base Case	Base Case	"Weight"	"Bycatch"
M_0^S	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**
M_a^S, $a=1,\ldots,4$	**0.8**	**0.8**	**0.8**	**0.8**	**0.8**	**0.8**
M_{5+}^S	**0.8**	**0.8**	**0.8**	**0.8**	**0.8**	**0.8**
-ln(posterior)	9.28		15.81	15.84	17.77	15.84
k_N^S	0.75	0.75	0.75	0.76	0.75	0.76
k_r^S	0.34	0.34	0.37	0.38	0.38	0.38
k_r^S / k_N^S	0.46	0.46	0.50	0.50	0.51	0.50
$(\lambda_N^S)^2$	0	**0**	**0**	**0**	**0**	**0**
$(\lambda_r^S)^2$	0	**0**	**0**	**0**	**0**	**0**
$\overline{B}_{Nov}^S$	555.7	552.8	571.3	550.7	557.5	550.7
$\hat{B}_{2008,N}^S$	N/A	N/A	611	573	529	573
$\hat{B}_{2009,N}^S$	N/A	N/A	N/A	674	622	673
$\hat{B}_{2009,N}^S / \hat{B}_{2008,N}^S$	N/A	N/A	N/A	1.18	1.18	1.17
K_{normal}^S	2874.5	2859.0	1878.4	1941.0	1987.0	1941.0
K_{peak}^S	4500.9	4500.2	4389.2	4307.2	4377.2	4307.2
a^S	77.7	77.9	46.3	48.1	48.2	48.1
b^S	788.6	786.0	507.4	549.8	557.4	549.8
c^S	72.6	72.7	67.5	65.4	65.8	65.4
σ_r^S (peak years)	0.42 (1.27)	**0.4** (1.27)	0.60 (1.30)	0.59 (1.32)	0.63 (1.33)	0.59 (1.32)
η_{2005}^S or η_{2007}^S	-1.032	-1.043	-1.359	0.160	0.291	0.160
s_{cor}^S	0.319	0.329	0.333	0.341	0.344	0.341

Table 2a. *The negative joint posterior mode for alternative values of juvenile and adult natural mortality from 1984 to 1999 (rows) and 2000 to 2009 (columns). The base case is* $M_0^S = 1.0$ *and* $M_a^S = 0.8$, $a = 1,\ldots,5+$ *(shown in bold). In all cases* $M_0^S = M_a^S + 0.2$.

		M_a^S from 2000 to 2009				
		0.6	0.7	0.8	0.9	1.0
M_a^S from 1984 to 1999	0.6	20.455		12.577		12.275
	0.7		17.578	14.059		10.394
	0.8			**15.836**		10.448
	0.9			17.424	14.758	11.561
	1.0			19.439		13.530

Table 2b. *The ratio of estimated multiplicative bias in the recruit survey to that in the November survey for alternative values of juvenile and adult natural mortality from 1984 to 1999 (rows) and 2000 to 2009 (columns). The base case is* $M_0^S = 1.0$ *and* $M_a^S = 0.8$, $a = 1,\ldots,5+$ *(shown in bold). In all cases* $M_0^S = M_a^S + 0.2$. *The grey cells correspond to those ratios outside the range of [0.5,1.0] that has been considered realistic.*

		M_a^S from 2000 to 2009				
		0.6	0.7	0.8	0.9	1.0
M_a^S from 1984 to 1999	0.6	0.77		0.58		0.46
	0.7		0.61	0.54		0.43
	0.8			**0.50**		0.40
	0.9			0.47	0.41	0.37
	1.0			0.44		0.35

Table 3. *The annual estimated loss to predation (in '000t) compared to the annual catch (in '000t).*

Year	Loss to *M*	Catch	Catch: Loss to *M*
1984	97.068	29.500	0.30
1985	63.240	29.600	0.47
1986	81.530	35.400	0.43
1987	83.380	33.528	0.40
1988	163.076	36.330	0.22
1989	125.625	34.741	0.28
1990	161.195	57.420	0.36
1991	236.044	52.961	0.22
1992	205.432	55.067	0.27
1993	251.841	51.085	0.20
1994	344.826	94.854	0.28
1995	375.543	121.151	0.32
1996	657.848	107.852	0.16
1997	639.408	119.374	0.19
1998	1079.710	133.331	0.12
1999	1076.094	131.919	0.12
2000	983.334	135.197	0.14
2001	898.296	191.530	0.21
2002	1522.702	260.883	0.17
2003	1902.140	289.994	0.15
2004	2706.592	373.826	0.14
2005	1513.662	246.712	0.16
2006	715.416	217.281	0.30
2007	398.111	139.500	0.35
2008	254.699	90.917	0.36
2009	266.539	90.000	0.34

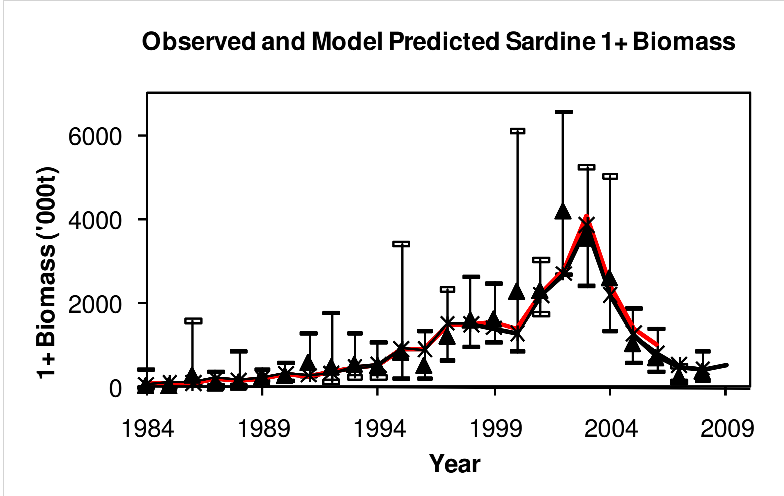

Figure 1. *Observed and model predicted sardine 1+ biomass from the previous assessment (red line) and the base case updated assessment (black line). The model predicted sardine 1+ biomass from the updated assessment in November 2008 is also given for comparison (thin black line with crosses).*

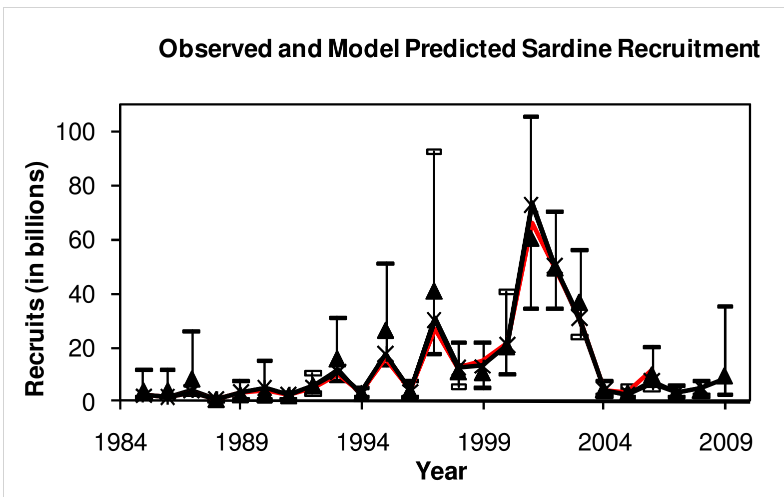

Figure 2. *Observed and model predicted sardine May recruitment from the previous assessment (red line) and the base case updated assessment (black line). The model predicted sardine recruitment from the updated assessment in November 2008 is also given for comparison (thin black line with crosses).*

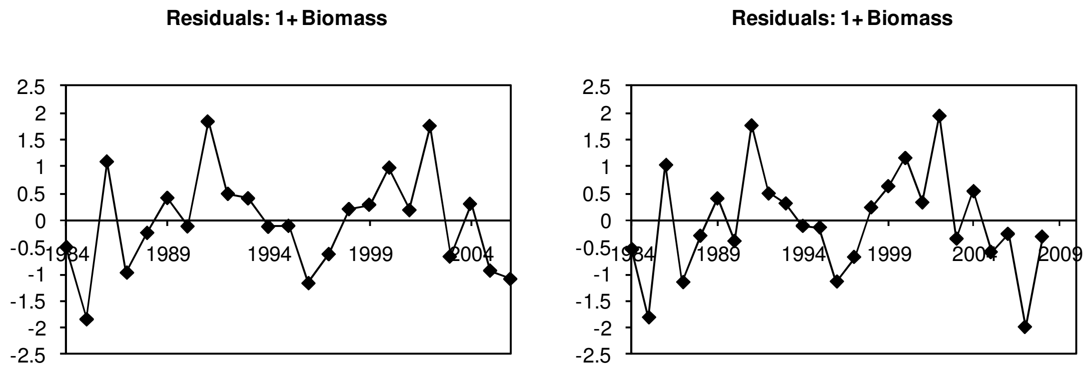

Figure 3. *Standardised residuals of the model fit to the November 1+ biomass data from the previous assessment (left panel) and the base case updated assessment (right panel).*

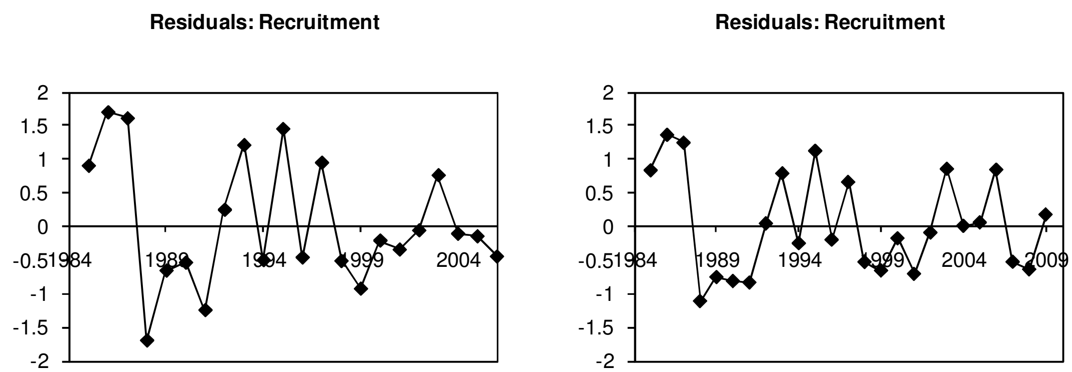

Figure 4. *Standardised residuals of the model fit to the May recruitment data from the previous assessment (left panel) and the base case updated assessment (right panel).*

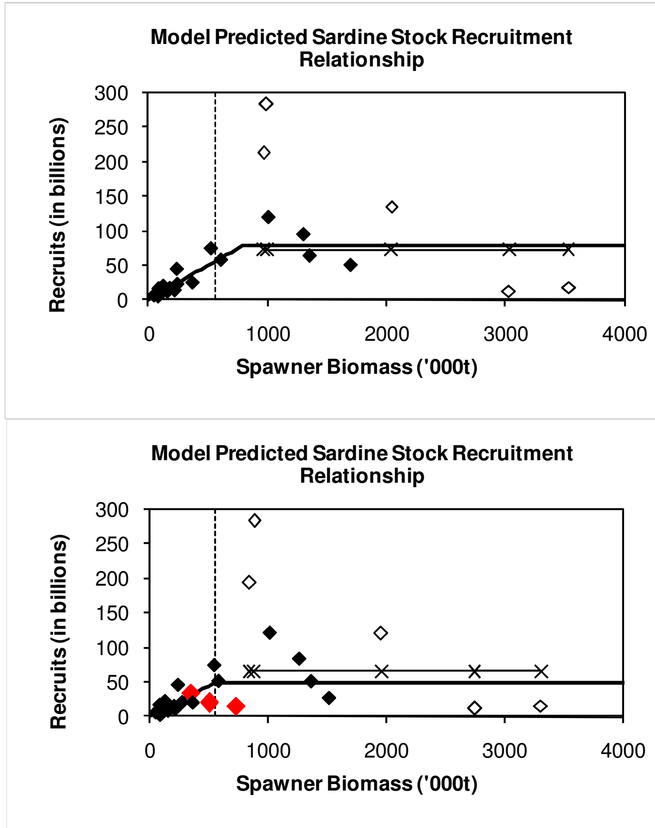

Figure 5. *Model predicted sardine recruitment (in November) plotted against spawner biomass from November 1984 to November 2005 (previous assessment, upper panel) and to November 2008 (updated assessment, lower panel), with the 'hockey-stick' stock-recruit curve and the constant relationship between 2000 and 2004 also shown. The open diamonds denote the 2000 to 2004 November spawner biomass and recruitment. The red diamonds in the lower panel indicate the recruitment in November 2006, 2007 and 2008. The dashed line indicates the average 1991 to 1994 1+ biomass (used in the definition of risk in OMP-08).*

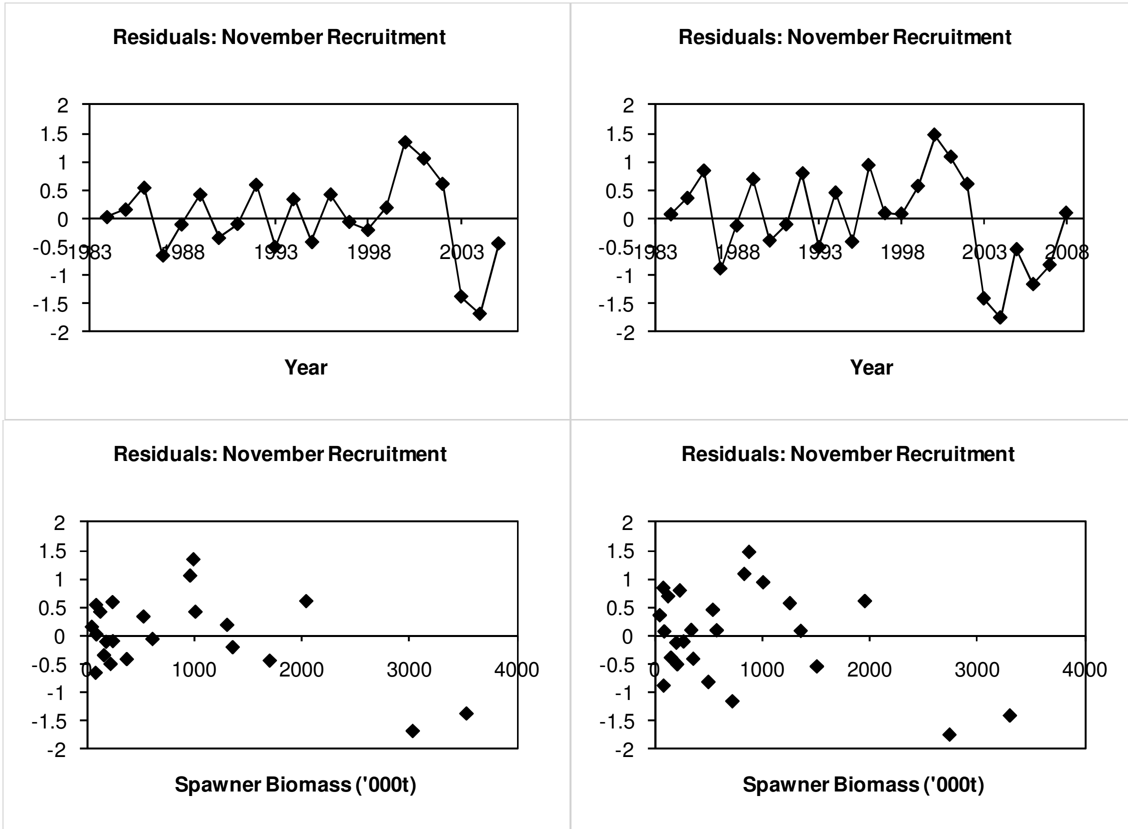

Figure 6. *Standardised November recruitment residuals from the previous assessment (left panels) and the base case updated assessment (right panels), plotted against time (upper panels) and against spawner biomass (lower panels).*

Glossary of Model Parameters

M_a^S the rate of natural mortality (in year^{-1}) of sardine of age a.

$k_{N/r}^S$ the constant of proportionality (multiplicative bias) associated with the November / recruit survey.

$(\lambda_{N/r}^S)^2$ the additional variance (over and above the survey sampling CV $\sigma_{y,Nov/rec}^S$ that reflects survey inter-transect variance) associated with the November/recruit surveys.

$\overline{B}_{Nov}^S$ the average 1+ sardine biomass between November 1991 and November 1994; OMP-04 was developed using Risk defined as "the probability that 1+ sardine biomass falls below the average 1+ sardine biomass between November 1991 and November 1994 at least once during the projection period of 20 years".

$\hat{B}_{y,N}^S$ the biomass (in thousand tonnes) of adult sardine at the beginning of November in year y, associated with the November survey.

K_{normal}^S the carrying capacity during "normal" years.

K_{peak}^S the carrying capacity during "peak" abundance (2000-2004).

a^S the maximum recruitment (in billions) (i.e. median of the distribution in question).

b^S the spawner biomass above which expected recruitment is constant in the hockey stick model.

c^S the constant recruitment (distribution median) during the "peak" years of 2000 to 2004.

σ_r^S the standard deviation in the annual lognormal deviation of sardine recruitment.

η_y^S the standardised recruitment residual value for year y.

s_{cor}^S the recruitment serial correlation.

$Nprop_a^S$ the proportion numbers-at-age a in the initial year of the model (November 1983).

Appendix A: Updated Data

Acoustic Survey Data

The new data included in this assessment are listed in Table A.

Table A. The hydroacoustic survey data from 2007 to 2009.

November Acoustic Survey			May Recruitment Survey			
Year	1+ Biomass ('000t)	CV	Year	Commencement Date	Recruitment (billions)	CV
2007	256.73	0.345	2007	18th May	2.937	0.342
2008	384.08	0.422	2008	21st May	3.852	0.325
			2009	15th May	9.207	0.679

The ALKs for November 2007 and 2008 consisted of 225 and 161 samples. There was a lack of larger sardine available for ageing – only 5 were aged age 4+ in 2007 and none were aged age 4+ in 2008. The November survey weights-at-age calculated using these ALKs may therefore be unreliable. As a base case assumption, therefore, the historic average weights-at-age are used and the currently available weights-at-age for 2007 and 2008 are used as a sensitivity test, with the constraint that the weight-at-age 5+ cannot be less than that at age 4.

Table B. November survey weights-at-age (in grams)

	Age 1	Age 2	Age 3	Age 4	Age 5+
Average	32.38	58.56	83.61	92.70	108.82
2007	54.67	86.08	96.60	110.79	(82.66)[1]
2008	34.11	59.69	68.41	105.53	(82.66)[2]

Commercial Catch Data

In the last sardine assessment (Cunningham and Butterworth 2007), 0-year old and 1+-year-old quarterly catch tonnage was calculated using the "full" sardine assessment that was fit to commercial catch proportions-at-length data, and input into the "short" assessment which was not fit to commercial catch data.

As the "full" sardine assessment has not been updated, the quarterly catch tonnage split between 0-year-olds and 1+-year-olds for the additional years now added (Nov 2006- Oct 2009) has been calculated as follows:

[1] 110.79g is used instead.
[2] 105.53g is used instead.

The data available for these calculations include the number of fish in length class *l* in month *m*, $N^c_{l,m}$, and the observed tonnage in month *m*, $ObsT_m$ from November 2006 to August 2009[3]. The August 2009 RLF was applied to September and October 2009. The assumption was made that the directed sardine TAC will be filled by the end of October 2009 and that the proportion of sardine bycatch with anchovy in August was maintained in September and October 2009. As two alternative assumptions were made w.r.t. the anchovy catch in September and October (see de Moor and Butterworth 2009), this results in two alternatives for sardine bycatch tonnage in September and October 2009.

Expected mass (in kilograms) by length class (in centimetres) and month is calculated as: $EM_{l,m} = 0.0096 \times l_{mid}^{3.075} \times N^c_{l,m}$

where l_{mid} is the mid-point of the length class considered and $N^c_{l,m}$ is the number of fish in length class l in month m.

Adjusted mass by length class and month is calculated as: $AM_{l,m} = \dfrac{EM_{l,m}}{\sum_l EM_{l,m}} \times ObsT_m$

Average monthly adjusted mass per individual fish by length class and month is calculated as:

$$\overline{AM}_{l,m} = \frac{AM_{l,m}}{N^c_{l,m}} = \frac{\dfrac{EM_{l,m}}{\sum_l EM_{l,m}} \times ObsT_m}{N^c_{l,m}}$$

Making the assumption that all sardine <15.5cm are juveniles and those >=15.5cm are 1+ adults, the average juvenile and adult mass per individual fish by month is calculated as:

$$FM_m^{juv} = \frac{\sum_{l<15.5} \overline{AM}_{l,m} \times N^c_{l,m}}{\sum_{l<15.5} N^c_{l,m}} \text{ and } FM_m^{ad} = \frac{\sum_{l\geq 15.5} \overline{AM}_{l,m} \times N^c_{l,m}}{\sum_{l\geq 15.5} N^c_{l,m}}$$

The juvenile catch tonnage by month is therefore $FM_m^{juv} \times \sum_{l<15.5} N^c_{l,m}$, while the 1+ adult catch tonnage by month is $FM_m^{ad} \times \sum_{l\geq 15.5} N^c_{l,m}$.

A check is performed on the calculations such that:

$$FM_m^{juv} \times \sum_{l<15.5} N^c_{l,m} + FM_m^{ad} \times \sum_{l\geq 15.5} N^c_{l,m} = ObsT_m .$$

The recruit catch between 1 May and the day before the survey was estimated as follows:

The data available for these calculations include the number of fish in length class *l* for this period and the associated observed tonnage. The adjusted mass by length class, average adjusted mass per individual fish by length class, and average juvenile mass per individual fish is calculated as above, but using only

[3] August 2009 bycatch tonnage was raised by 200t, the amount estimated to be outstanding when the data were collated (van der Westhuizen pers. comm.).

the length classes up to and including 10cm in 2007 and 12cm in 2008 and 2009[4]. Dividing this average mass into the observed tonnage gives the estimated recruit catch in billions prior to the survey.

Table C. New sardine catch data used in the updated assessment. The values in brackets in the last row are the sardine catch under the alternative assumption of double the average April-August anchovy catch in September and October 2009 (which affects the amount of bycatch assumed to be caught).

	Quarterly recruit catch ('000t)	Quarterly 1+ catch ('000t)	Recruit catch from 1 May to the day before the survey (billions)
Nov06-Jan07	2.208	31.196	
Feb07-Apr07	3.236	47.032	
May07-Jul07	4.435	49.202	0.053422
Aug07-Oct07	1.104	22.907	
Nov07-Jan08	0.836	7.178	
Feb08-Apr08	1.286	34.743	
May08-Jul08	3.483	24.728	0.109123
Aug08-Oct08	1.743	10.770	
Nov08-Jan09	3.602	10.941	
Feb09-Apr09	1.926	52.006	
May09-Jul09	2.906	28.444	0.032551
Aug09-Oct09	5.923 (6.787)	4.650 (5.328)	

[4] The cut-off lengths were determined by a modal progression analysis, as for former years (Janet Coetzee pers. comm.)

Appendix B: Calculation of Loss to Predation

The assessment model assumes catch is taken in a pulse mid-way between the four quarters of the year. The loss in numbers of fish of age *a* to predation in year *y* is given by:

$$
\begin{aligned}
P_{y,a} = {} & N^S_{y-1,a-1}\left(1 - e^{-M^S_{a-1}/8}\right) + \left(N^S_{y-1,a-1} e^{-M^S_{a-1}/8} - \hat{C}^S_{y,1,a-1}\right)\left(1 - e^{-M^S_{a-1}/4}\right) \\
& + \left(\left(N^S_{y-1,a-1} e^{-M^S_{a-1}/8} - \hat{C}^S_{y,1,a-1}\right) e^{-M^S_{a-1}/4} - \hat{C}^S_{y,2,a-1}\right)\left(1 - e^{-M^S_{a-1}/4}\right) \\
& + \left(\left(\left(N^S_{y-1,a-1} e^{-M^S_{a-1}/8} - \hat{C}^S_{y,1,a-1}\right) e^{-M^S_{a-1}/4} - \hat{C}^S_{y,2,a-1}\right) e^{-M^S_{a-1}/4} - C^S_{y,3,a-1}\right)\left(1 - e^{-M^S_{a-1}/4}\right) \\
& + \left(\left(\left(\left(N^S_{y-1,a-1} e^{-M^S_{a-1}/8} - \hat{C}^S_{y,1,a-1}\right) e^{-M^S_{a-1}/4} - \hat{C}^S_{y,2,a-1}\right) e^{-M^S_{a-1}/4} - C^S_{y,3,a-1}\right) e^{-M^S_{a-1}/4} - C^S_{y,4,a-1}\right)\left(1 - e^{-M^S_{a-1}/8}\right)
\end{aligned}
$$

which simplifies to:

$$
P_{y,a} = N^S_{y-1,a-1}\left(1 - e^{-M^S_{a-1}}\right) - \hat{C}^S_{y,1,a-1}\left(1 - e^{-7M^S_{a-1}/8}\right) - \hat{C}^S_{y,2,a-1}\left(1 - e^{-5M^S_{a-1}/8}\right) - \hat{C}^S_{y,3,a-1}\left(1 - e^{-3M^S_{a-1}/8}\right) - \hat{C}^S_{y,4,a-1}\left(1 - e^{-M^S_{a-1}/8}\right)
$$

$$y = 1984, \ldots, 2006, \; a = 1, \ldots, 4$$

$$
\begin{aligned}
P_{y,5} = {} & N^S_{y-1,4}\left(1 - e^{-M^S_4}\right) - \hat{C}^S_{y,1,4}\left(1 - e^{-7M^S_4/8}\right) - \hat{C}^S_{y,2,4}\left(1 - e^{-5M^S_4/8}\right) - \hat{C}^S_{y,3,4}\left(1 - e^{-3M^S_4/8}\right) - \hat{C}^S_{y,4,4}\left(1 - e^{-M^S_4/8}\right) \\
& + N^S_{y-1,5}\left(1 - e^{-M^S_5}\right) - \hat{C}^S_{y,1,5}\left(1 - e^{-7M^S_5/8}\right) - \hat{C}^S_{y,2,5}\left(1 - e^{-5M^S_5/8}\right) - \hat{C}^S_{y,3,5}\left(1 - e^{-3M^S_5/8}\right) - \hat{C}^S_{y,4,5}\left(1 - e^{-M^S_5/8}\right)
\end{aligned}
$$

The loss in biomass of fish of age *a* to predation in year *y* is therefore given by:

$$
\begin{aligned}
P_{y,a} = {} & N^S_{y-1,a-1}\left(1 - e^{-M^S_{a-1}}\right) w_{y-1,a-1} - \hat{C}^S_{y,1,a-1}\left(1 - e^{-7M^S_{a-1}/8}\right)\left(\frac{7}{8} w_{y-1,a-1} + \frac{1}{8} w_{y,a}\right) \\
& - \hat{C}^S_{y,2,a-1}\left(1 - e^{-5M^S_{a-1}/8}\right)\left(\frac{5}{8} w_{y-1,a-1} + \frac{3}{8} w_{y,a}\right) - \hat{C}^S_{y,3,a-1}\left(1 - e^{-3M^S_{a-1}/8}\right)\left(\frac{3}{8} w_{y-1,a-1} + \frac{5}{8} w_{y,a}\right) \\
& - \hat{C}^S_{y,4,a-1}\left(1 - e^{-M^S_{a-1}/8}\right)\left(\frac{1}{8} w_{y-1,a-1} + \frac{7}{8} w_{y,a}\right)
\end{aligned}
$$

$$y = 1984, \ldots, 2006, \; a = 1, \ldots, 4$$

$$
\begin{aligned}
P_{y,5} = {} & N^S_{y-1,4}\left(1 - e^{-M^S_4}\right) w_{y-1,4} - \hat{C}^S_{y,1,4}\left(1 - e^{-7M^S_4/8}\right)\left(\frac{7}{8} w_{y-1,4} + \frac{1}{8} w_{y,5}\right) \\
& - \hat{C}^S_{y,2,4}\left(1 - e^{-5M^S_4/8}\right)\left(\frac{5}{8} w_{y-1,4} + \frac{3}{8} w_{y,5}\right) - \hat{C}^S_{y,3,4}\left(1 - e^{-3M^S_4/8}\right)\left(\frac{3}{8} w_{y-1,4} + \frac{5}{8} w_{y,5}\right) \\
& - \hat{C}^S_{y,4,4}\left(1 - e^{-M^S_4/8}\right)\left(\frac{1}{8} w_{y-1,4} + \frac{7}{8} w_{y,5}\right) + N^S_{y-1,5}\left(1 - e^{-M^S_5}\right) w_{y-1,5} - \hat{C}^S_{y,1,5}\left(1 - e^{-7M^S_5/8}\right)\left(\frac{7}{8} w_{y-1,5} + \frac{1}{8} w_{y,5}\right) \\
& - \hat{C}^S_{y,2,5}\left(1 - e^{-5M^S_5/8}\right)\left(\frac{5}{8} w_{y-1,5} + \frac{3}{8} w_{y,5}\right) - \hat{C}^S_{y,3,5}\left(1 - e^{-3M^S_5/8}\right)\left(\frac{3}{8} w_{y-1,5} + \frac{5}{8} w_{y,5}\right) \\
& - \hat{C}^S_{y,4,5}\left(1 - e^{-M^S_5/8}\right)\left(\frac{1}{8} w_{y-1,5} + \frac{7}{8} w_{y,5}\right)
\end{aligned}
$$

The assumption is made that $w_{1983,a} = w_{1984,a}$, $a = 1, \ldots, 5+$.

The total loss in sardine biomass to predation in year *y* is then given by:

$$P_y = \sum_{a=1}^{5+} P_{y,a} \; .$$