Checking the penguin population model estimator for bias

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Abstract

A bootstrap approach is used to check the Robben Island penguin model estimator bias. The results show that there is little evidence for this. The distributions of estimates from the bootstrapping process are well-centred on the penalized maximum likelihood estimates.

Introduction

A key aspect of model-testing is to check whether the estimator used can "recover itself". In other words, if the best estimate from the model is actually reality, and the types of data fitted are generated from that reality (operating model) and then fitted using the original estimator, how do the distributions of results correspond to the underlying values of the operating model.

Method and results

The estimator tested is the penguin population model described in MARAM IWS/DEC10/PA/P6 with the logistic transformation. The maximum penalized likelihood estimates of the adult female moult counts $\hat{N}_y$ and the proportion of immature birds in the moult counts $\hat{J}_y$ are input to the bootstrap routine. Pseudo-datasets are then generated from the best estimates as follows:

$$\begin{aligned} N_{y,i}^{\text{pseudo}} &= \hat{N}_y e^{X_{y,i}} \qquad & X_{y,i} &\sim N\left(0,\sigma_M^2\right) \\ J_{y,i}^{\text{pseudo}} &= \hat{J}_y e^{Y_{y,i}} \qquad & Y_{y,i} &\sim N\left(0,\sigma_J^2\right) \end{aligned} \tag{1}$$

The standard deviations are the same as those used in the estimator: $\sigma_M = 0.2$ and $\sigma_J = 0.1$. Note that the lower estimates from the model fit residuals are not used because they are negatively biased as a result of the relatively few degrees of freedom for the penalized maximum likelihood estimate used.

The estimator then finds the best fit to each pseudo-dataset. The distributions of the estimated μ and η parameters are shown in Figure 1. The spread of estimated values for the time series of adult female moult counts, juvenile proportions in the counts, annual survival and reproductive success are displayed in Figure 2 to Figure 5.

This process is identical to a standard parametric bootstrap procedure, which is implemented here to check for bias in the estimator.

Discussion

Of key interest are the μ parameters which define the relationships in the model between demographic parameters and prey abundance. The upper plots in Figure 1 show that the bootstrap distributions are well centred on the penalized maximum likelihood estimates, indicating minimal bias in the estimator.

Note that this process has considered the effects of observation errors in the moult count data only. A more comprehensive test would need to consider also alternative realisations of the random effects for the underlying operating model.

Reference

MARAM IWS/DEC10/PA/P6. Robinson W, Butterworth DS. 2010. Penguin population models for Robben Island.

Figures

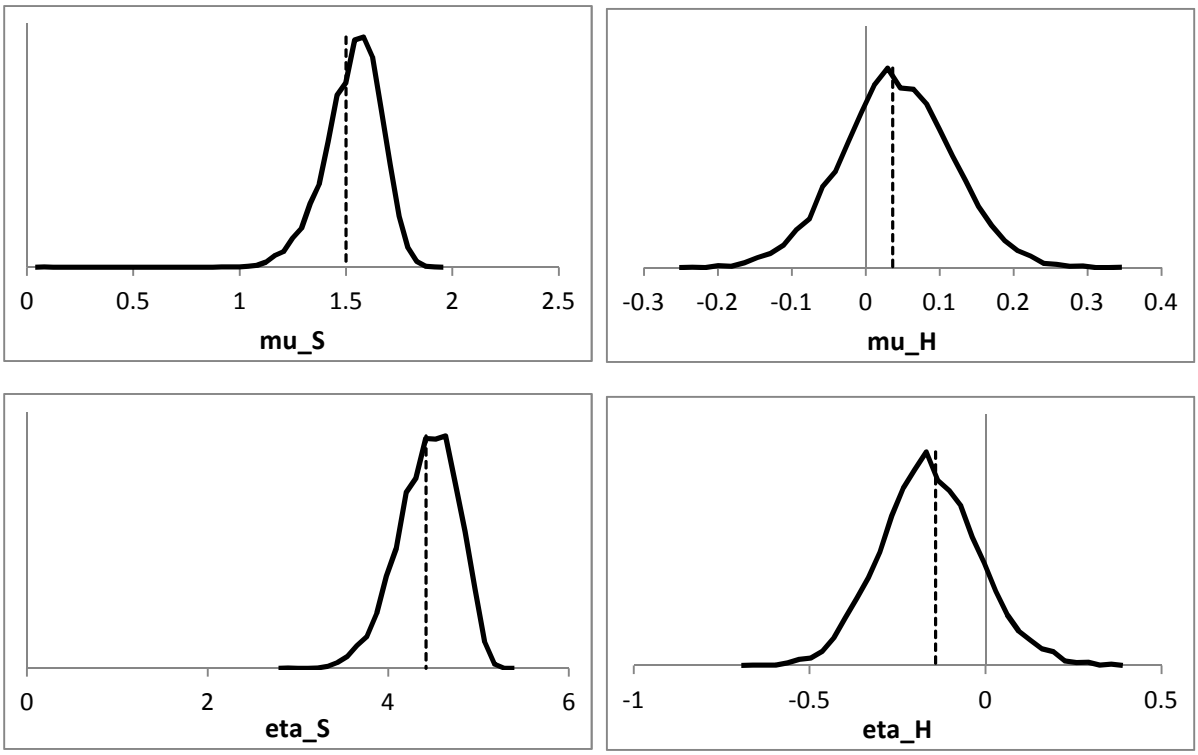

Figure 1: Distributions of the parameter estimates for the relationships with fish abundance obtained from the bootstrapped data. The "true" values are indicated by dashed lines.

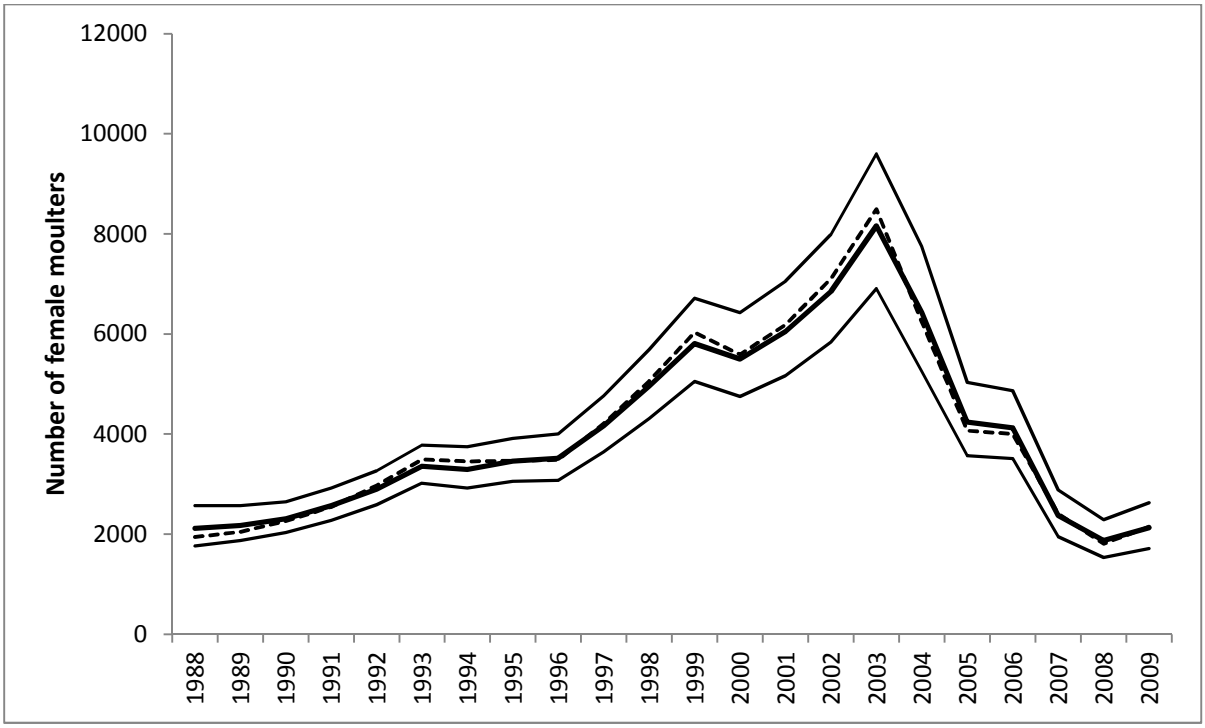

Figure 2: 5th percentile, median and 95th percentile of the estimated adult female moult counts from the bootstrapped data. The dashed line indicates the maximum penalized likelihood estimates.

Figure 3: 5th percentile, median and 95th percentile of the estimated proportion of immature penguins in the moult counts from the bootstrapped data. The dashed line indicates the maximum penalized likelihood estimates.

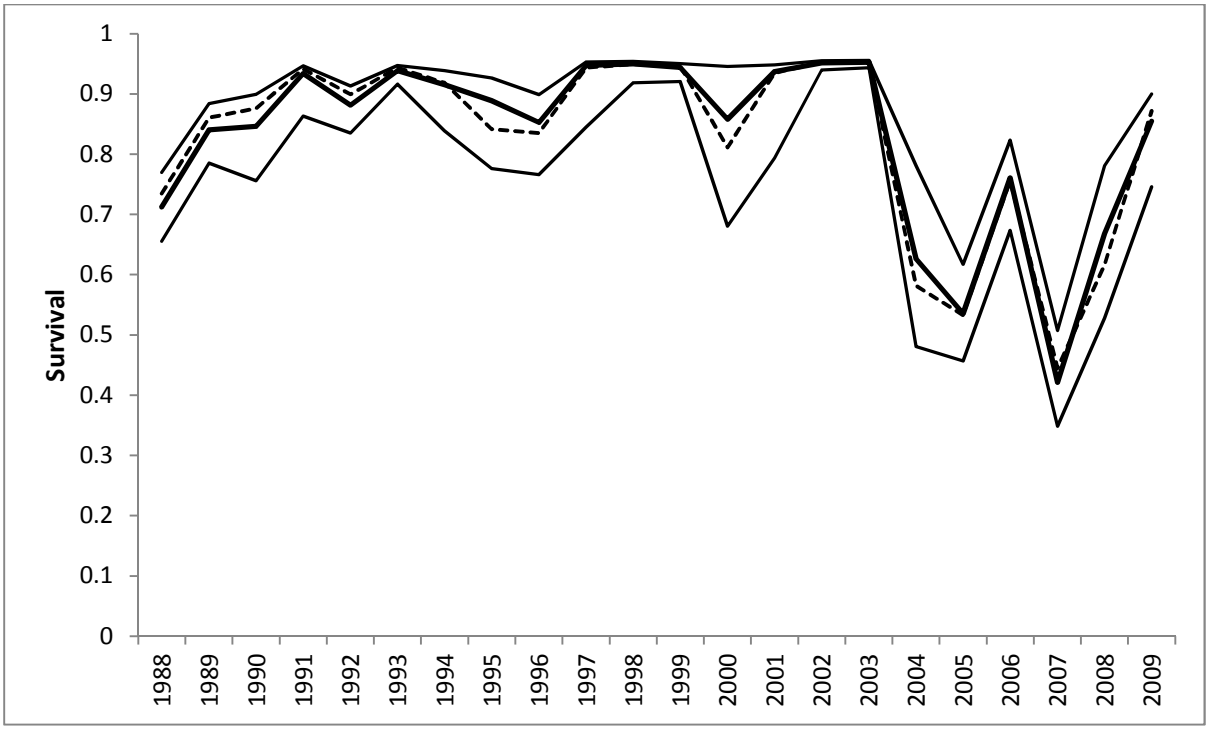

Figure 4: 5th percentile, median and 95th percentile of the estimated annual adult penguin survival rates from the bootstrapped data. The dashed line indicates the maximum penalized likelihood estimates.

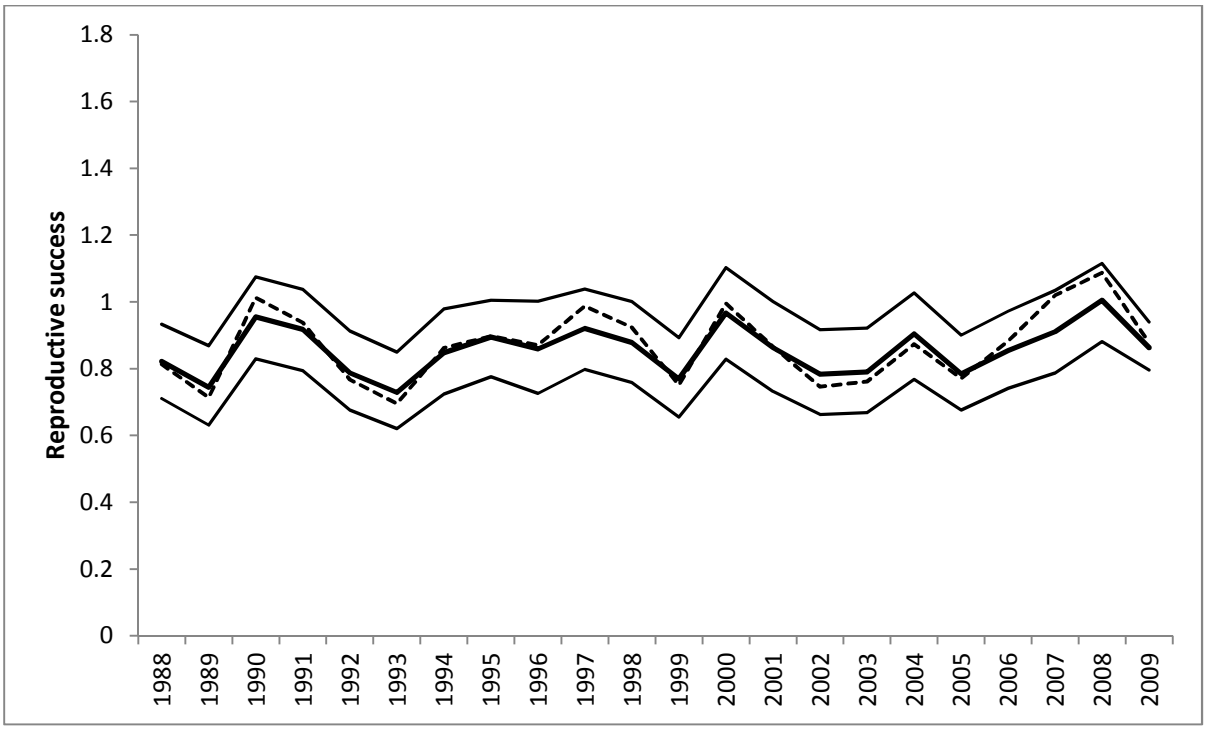

Figure 5: 5th percentile, median and 95th percentile of the estimated annual penguin reproductive success rates from the bootstrapped data. The dashed line indicates the maximum penalized likelihood estimates.