# Further work towards the assessment of the South African round herring (Etrumeus whiteheadi) resource using data from 1987 to 2010 

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## Introduction

Round herring (Etrumeus whiteheadi), commonly referred to as Red Eye, is exploited by the South African pelagic fishery. The fishery has been managed with a Precautionary Upper Catch Limit of 100000 t for at least the past eight years. An assessment of this resource is required in order to inform on any future decisions to the management of the round herring.

The assessment of round herring is complicated by the fact that the time series of hydroacoustic survey estimates of abundance have been obtained from surveys which were designed first for anchovy and secondly for sardine. However, Coetzee and Merkle (2009) recently proposed that the time series of survey estimates of abundance are comparable, paving the way for an assessment of this resource to be undertaken. de Moor and Butterworth (2010) presented some initial results from fitting a model to November survey estimates of total biomass, May survey estimates of recruitment numbers and commercial length frequency data.

This document presents further advancements in the assessment of the round herring resource and some results for discussion.

## Population Dynamics Model

The population dynamics model used for the South African round herring resource is detailed in Appendix A. The data used in this assessment are listed in Appendix B.

Consideration of the initial assessment model results (de Moor and Butterworth 2010) together with the available round herring ageing and length frequency data (Durholtz et al. 2010) has resulted in a number of structural changes from the initial assessment model:

- The population is modelled as consisting of three annual (summer, winter and late spring) sub-cohorts. These are modelled to have birthdates of $1^{\text {st }}$ March, $1^{\text {st }}$ June and $1^{\text {st }}$ September each year (equation A.1).
- The total biomass estimated by the May hydroacoustic survey is used in preference to the survey estimate of numbers of recruits only (equation A.17; Table B.1). A May survey selectivity at age is introduced to enable a lower selection of the older age groups to be modelled.

[^0]- A parameter measuring the proportion of the round herring biomass for the fully selected age (age 1 ) as measured by the May survey in comparison to that measured by the November survey is introduced in order to estimate the bias associated with the expected lower coverage achieved by the May survey. All other forms of bias are included in the multiplicative factors $k_{N}^{R H}$ and $k_{r}^{R H}$, and in respect of variance $\lambda_{N}^{R H}$ and $\lambda_{r}^{R H}$.
- The length frequency data available from the November and May surveys are now used to assist estimate model parameters by their inclusion in the likelihood (equation A.17).

As before, the base case assumption of $M_{j}^{R H}=M_{a d}^{R H}=1.3$ is based on unpublished data of Y. Geja and D. Durholtz.

## Current Model Results

The model is able to fit the survey estimated November and May biomasses reasonably well (Figures 1 and 2). The multiplicative bias associated with these surveys is considered to be constant over time (Appendix A). This model estimates the survey coverage of round herring biomass in the May survey to be $22 \%$ of that obtained during the November survey (Table 1).

The model fits to the commercial proportions-at-length are provided in Figure 3, for the minus group, and Figure 4 for the remaining length classes. Figure 5 indicates a pattern of underestimation of the proportion-atlengths $16-17 \mathrm{~cm}$ and an overestimation of the higher and lower (excluding minus group) length classes.

The model fits to the November survey proportions-at-length are provided in Figure 6, for the minus group, and Figure 7 for the remaining length classes. The standardised residuals are plotted in Figure 8. The fit to these data appears better than that for the commercial catch data.

The model fits to the May survey proportions-at-length are provided in Figure 9, for the minus group, and Figure 10 for the remaining length classes. The standardised residuals are plotted in Figure 11.

The selectivities-at-age estimated by the model for the November and May surveys and commercial catch are plotted in Figure 12. This shows that the model allows for a small decrease in selectivity with increasing age in the November survey indicating the majority of the resource is covered in the survey. The sharp decrease in selectivity with increasing age in the May survey indicates that the majority of adults are not sampled in the May survey.

The model estimated length at age distributions are plotted in Figures 13 to 15. The estimated CV about the mean length at age, $\vartheta_{a}$, is about $15 \%$ for age 0 and age $5+$ and $11 \%$ for ages 1 to 4 (Table 1 ). $15 \%$ is on the upper boundary of that assumed a priori (Appendix A). The reason for this relatively high CV on the plus
group will be due large fish. The reason for the relatively high CV on the 0 year olds likely arises because of a combination of their fast growth during their first year and the multiple sub-cohorts.

This initial run of the model estimates the proportion of recruitment occurring in the March sub-cohort to be $15 \%$ and the proportion occurring in the September sub-cohort to be $85 \%$ with no recruitment occurring in June. This is in contrast to the a priori information of June representing peak recruitment with the March and September sub-cohorts resulting from (lower) early and late spawning.

## Discussion

This document has presented further work towards the assessment of South African round herring. The current model results are presented and comments and discussion on these results are welcomed.

Further work will need to test the following:

- Incorporating normal prior distributions on $k_{N}^{R H}$ and $k_{r}^{R H}$, instead of fixed values.
- Although initial model runs suggested a zero selectivity in the commercial catch for ages $3+$, this assumption may need to be retested.
- The current model results estimate the recruitment to occur in two pulses only; $15 \%$ in March and $85 \%$ in September. The CVs on the parameters $p_{1}$ and $p_{2}$ may be large. Some investigation into the effects of alternative values for $p_{1}$ and $p_{2}$, and their inter-annual variation will be conducted. In addition, smaller CVs on the age-to-length matrices for age 0 may result in $p_{2}>0$.
- In some years the model does not predict a peak of similar magnitude to that observed in the length frequency data. This results in an overestimation of proportion-at-length for lower and higher length classes and an underestimation of proportion-at-length around the observed peak. The possibility of introducing variability in time in selectivity at age to account for this will be considered.
- The model assumes that age 0 fish are at most 11 cm long in November and 13 cm long in March (the commercial catch). No such assumption is made in May. Instead the age 0 fish are proportioned by length according to the age-to-length matrix (equation A.15). Such a method did not work well for age 0 fish in the commercial catch in initial model runs without November and May survey length frequencies. Given the three sets of data, it may now be possible to estimate proportions-at-length for age 0 fish using the age-to-length matrix in November and March.

In addition, tests will also be carried out to check the robustness of the model results to assumptions of fixed natural mortality. The robustness of model results will also be tested using data from 1998 to 2010 only, with the recruit survey data extended to cover the area west of Port Alfred.

## Acknowledgements

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## References

Coetzee, J., and Merkle, D. 2009. The extent to which recent redeye acoustic survey abundance indices may be biased by greater survey effort. Unpublished MCM Document MCM/2009/Redeye Fishery Task Team/03. 5pp.
de Moor, C.L., and Butterworth, D.S. 2010. Initial assessment of the South African round herring (Etrumeus whiteheadi) resource using data from 1988 to 2010. MCM Document MCM/2010/SWG-PEL/40. 16pp.

Durholtz, D., de Moor, C.L., and Geja, Y. 2010. Examination of redeye roundherring size and age data towards developing a growth curve for the redeye stock assessment. MCM Document MCM/2010/SWG-PEL/49. 11pp.

Pope, J.G. 1972. An investigation of the accuracy of virtual population analysis using cohort analysis. Res. Bull. Int. Commn NW Atl. Fisheries 9:65-74.

Table 1. Key model parameters at the joint posterior mode and contributions of different likelihoods to the posterior mode.

| Contributions to the joint posterior mode |  |
| :--- | ---: |
| Joint posterior mode | -11.35 |
| L(November biomass) | 4.89 |
| L(May biomass) | 14.09 |
| L(Commercial 13-cm) | -11.46 |
| L(Commercial length frequency) | -1.02 |
| L(November 11-cm) | -17.29 |
| L(November length frequency) | -14.48 |
| L(May 13-cm) Model parameters | -11.56 |
| L(May length frequency) | 25.49 |
|  | 22.6 |
| $N_{1987,1}^{R H}$ | 31.0 |
| $N_{1987,2}^{R H}$ | 0 |
| $N_{1987,3}^{R H}$ | 0 |
| $N_{1987,4}^{R H}$ | 0 |
| $N_{1987,5+}^{R H}$ | 0.15 |
| $p_{1}$ | 0 |
| $p_{2}$ | 0.85 |
| $1-p_{1}-p_{2}$ | 0.22 |
| $k^{\text {coverage }}$ | 0.15 |
| $\vartheta_{0}^{2}$ | 0.11 |
| $\vartheta_{a}^{2}, a=1,2,3,4$ | 0.15 |
| $\vartheta_{5+}^{2}$ |  |



Figure 1. Acoustic survey observed and model estimated November round herring biomass from 1987 to 2009. The observed indices are shown with Hessian-based $95 \%$ confidence intervals. The standardised residuals from the fit are given in the right hand plot.


Figure 2. Acoustic survey observed and model estimated round herring biomass from May/June 1987 to 2010. The observed indices are shown with Hessian-based $95 \%$ confidence intervals. The standardised residuals from the fit are given in the right hand plot.


Figure 3. Observed (symbols) and model estimated (line) round herring proportion-at-length 13- in the commercial catch from 1988 (i.e. June 1987 to May 1988) to 2010 (i.e. June 2009 to May 2010).


Figure 4. Observed and initial model estimated round herring proportion-at-length in the commercial catch from 1988 (i.e. June 1987 to May 1988 ) to 2010 (i.e. June 2009 to May 2010). Note that these proportions do not sum to 1 as they exclude the proportions-at-length 13-, plotted in Figure 3.


Figure 5. Bubble plot of the standardised residuals from the model fit to observed proportion-at-length in the round herring commercial catch from 1988 (i.e. June 1987 to May 1988) to 2010 (i.e. June 2009 to May 2010). Note that 13 cm represents the 13 - group and 21 cm represents the $21+$ group.


Figure 6. Observed (symbols) and initial model estimated (line) round herring proportion-at-length 11- in the November survey from 1987 to 2009.


Figure 7. Observed and model estimated round herring proportion-at-length in the November survey from 1987 to 2009. Note that these proportions do not sum to 1 as they exclude the proportions-at-length 11-, plotted in Figure 6.


Figure 8. Bubble plot of the standardised residuals from the model fit to observed proportion-at-length in the round herring November survey from 1987 to 2009 . Note that 11 cm represents the 11 - group and 19.5 cm represents the $19.5+$ group.


Figure 9. Observed (symbols) and initial model estimated (line) round herring proportion-at-length 13 - in the May survey from 1988 to 2010.

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|  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

Figure 10. Observed and model estimated round herring proportion-at-length in the May survey from 1988 to 2010. Note that these proportions do not sum to 1 as they exclude the proportions-at-length 13-, plotted in Figure 9.


Figure 11. Bubble plot of the standardised residuals from the model fit to observed proportion-at-length in the round herring May survey from 1988 to 2010 . Note that 13 cm represents the 13 - group and 18 cm represents the $18+$ group.


Figure 12. Fixed and model estimated selectivity-at-age in the commercial catch, the November survey and the May survey.

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Figure 13. The model estimate age to length matrix, $A_{a, l}^{c o m}$, representing the proportion of round herring catch-at-age $a$ that fall in the length group $l$.


Figure 14. The model estimate age to length matrix, $A_{a, l}^{N o v}$, representing the proportion of round herring of age $a$ that fall in the length group $l$ in mid-November.


Figure 15. The model estimate age to length matrix, $A_{a, l}^{\text {May }}$, representing the proportion of round herring of age $a$ that fall in the length group $l$ in mid-May.

## APPENDIX A: Bayesian assessment model for the South African round herring ("red eye") resource

## Model Assumptions

1) Fish are modelled to be in one of three sub-cohorts, corresponding to summer recruitment ( 1 March), winter recruitment (1 June) and spring recruitment (1 September).
2) A plus group of age 5 is used. No males older than 4 have been observed, though females up to 8 years of age have been observed (Y. Geja and D. Durholtz pers. comm.).
3) Two acoustic surveys are held each year: the first takes place in November and surveys all fish except the 0 year olds of the September sub-cohort; the second is in May/June and targets 0 year old round herring from the September and June sub-cohorts ${ }^{1}$, but also surveys older fish with lower selectivity.
4) The November acoustic survey provides a relative index of abundance of unknown bias.
5) The May survey provides a relative index of abundance of unknown bias.
6) The survey designs have been such that they result in survey estimates of abundance whose bias is invariant over time (Coetzee and Merkle 2009).
7) Pulse fishing occurs on $1^{\text {st }}$ March for all ages (higher round herring catches have historically been recorded between January and May, with a peak in March).
8) Catches are measured without error.
9) Age 0 fish are at most 11 cm long in November and 13 cm long in March (the commercial catch).
10) Selectivity is assumed to be year, but not age, invariant.
11) Natural mortality is year-invariant for juvenile and adult fish, and age-invariant for adult fish.

## Population Dynamics

The basic dynamic equations for round herring, based on Pope's approximation (Pope, 1972), are as follows, where $y_{1}=1987$ and $y_{n}=2010$. The numbers-at-age of the sub-cohorts represent the numbers of round herring at the time of the sub-cohorts 'birthdate', i.e. 1 March, 1 June or 1 September. The plus group is modelled to age on 1 June each year.

## Numbers-at-age

$$
\begin{align*}
& \hat{N} s u b_{y, a, 1}^{R H}=\hat{N} s u b_{y-1, a-1,1}^{R H} e^{-M_{a-1}}-\hat{C} s u b_{y, a-1,1}^{R H} \quad y_{1}+1 \leq y \leq y_{n}, 1 \leq a \leq 4 \\
& \hat{N} s u b_{y, a, 2}^{R H}=\left(\hat{N} s u b_{y-1, a-1,2}^{R H} e^{-9 M_{a-1} / 12}-\hat{C} s u b_{y, a-1,2}^{R H}\right) e^{-3 M_{a-1} 12} \quad y_{1}+1 \leq y \leq y_{n}, 1 \leq a \leq 4 \\
& \hat{N} s u b_{y, a, 3}^{R H}=\left(\hat{N} s u b_{y-1, a-1,3}^{R H} e^{-6 M_{a-1} / 12}-\hat{C} s u b_{y, a-1,3}^{R H}\right) e^{-6 M_{a-1} / 12} \quad y_{1}+1 \leq y \leq y_{n}, 1 \leq a \leq 4 \\
& \hat{N}_{y, 5+}^{R H}=\left(\hat{N} s u b_{y-1,4,1}^{R H} e^{-M_{4}}-\hat{C} s u b_{y, 4,1}^{R H}\right) e^{-3 M_{4} / 12}+\left(\hat{N} s u b_{y-1,4,2}^{R H} e^{-9 M_{4} / 12}-\hat{C} s u b_{y, 4,2}^{R H}\right) e^{-3 M_{4} / 12}+ \\
& \left(\hat{N} s u b_{y-1,4,3}^{R H} e^{-6 M_{4} / 12}-\hat{C} s u b_{y, 4,3}^{R H}\right) e^{-3 M_{4} / 12}+\left(\hat{N}_{y-1,5+}^{R H} e^{-9 M_{5+} / 12}-\hat{C}_{y, 5+}^{R H}\right) e^{-3 M_{5+} / 12} \\
& y_{1}+1 \leq y \leq y_{n} \tag{A.1}
\end{align*}
$$

[^1]where
$\hat{N} s u b_{y, a, c}^{R H} \quad$ is the number (in billions) of the March $(c=1)$, June ( $c=2$ ) or September ( $c=3$ ) spawning sub-cohort round herring of age $a$ at 1 March ( $c=1$ ), 1 June ( $c=2$ ) or 1 September $(c=3)$ in calendar year $y$;
$\hat{N}_{y, 5+}^{R H} \quad$ is the number (in billions) of round herring of age $5+$ at 1 June in year $y$;
$\hat{C} s u b_{y, a, c}^{R H} \quad$ is the number (in billions) of the March ( $c=1$ ), June ( $c=2$ ) or September ( $c=3$ ) spawning sub-cohort round herring of age $a$ caught from 1 June in year $y-1$ to 31 May in calendar year $y$;
$\hat{C}_{y, 5+}^{R H} \quad$ is the number (in billions) of round herring of age $5+$ caught from 1 June in year $y-1$ to 31 May in year $y$; and
$M_{a} \quad$ is the natural mortality (in year ${ }^{-1}$ ) of round herring of age $a$.

## Biomass associated with the November survey

$$
\begin{align*}
& \hat{N} s u b_{N o v, y, a, 1}^{R H}=\left(\hat{N} s u b_{y, a, 1}^{R H} e^{-6 M_{a} / 12}-\hat{C} s u b_{N o v, y, a, 1}^{R H}\right) e^{-2.5 M_{a} / 12} \quad y_{1} \leq y \leq y_{n}-1,0 \leq a \leq 4 \\
& \hat{N} s u b_{N o v, y, a, 2}^{R H}=\left(\hat{N} s u b_{y, a, 2}^{R H} e^{-3 M_{a} / 12}-\hat{C} s u b_{N o v, y, a, 2}^{R H}\right) e^{-2.5 M_{a} / 12} \quad y_{1} \leq y \leq y_{n}-1,0 \leq a \leq 4 \\
& \hat{N} s u b_{N o v, y, a, 3}^{R H}=\left(\hat{N}_{s u b}^{y, a, 3} R-\hat{C} s u b_{N o v, y, a, 3}^{R H}\right) e^{-2.5 M_{a} / 12} \quad y_{1} \leq y \leq y_{n}-1,0 \leq a \leq 4 \\
& \hat{N}_{N o v, y, 5+}^{R H}=\left(\hat{N}_{y, 5+}^{R H} e^{-3 M_{5+} / 12}-\hat{C}_{N o v, y, 5+}^{R H}\right) e^{-2.5 M_{5+} / 12} \quad y_{1} \leq y \leq y_{n}-1 \\
& \hat{B}_{N o v, y}^{R H}=\sum_{c=1}^{2}\left(S_{0}^{N o v} \times \hat{N} s u b_{N o v, y, 0, c}^{R H} w_{0, c}^{N o v}\right)+\sum_{a=1}^{4} \sum_{c=1}^{3}\left(S_{a}^{N o v} \times \hat{N} s u b_{N o v, y, a, c}^{R H} w_{a, c}^{N o v}\right)+S_{5+}^{N o v} \times \hat{N}_{N o v, y, 5+}^{R H} w_{5+}^{N o v} \\
& y_{1} \leq y \leq y_{n}-1 \tag{A.2}
\end{align*}
$$

where
$\hat{N} s u b_{\text {Nov, }, \text { a, }, c}^{R H}$ is the number (in billions) of the March ( $c=1$ ), June ( $c=2$ ) or September ( $c=3$ ) spawning sub-cohort round herring of age $a$ at mid-November in calendar year $y$;
$\hat{N}_{N o v, y, 5+}^{R H} \quad$ is the number (in billions) of round herring of age 5+ at mid-November in year $y$;
$\hat{C} s u b_{N o v, y, a, c}^{R H} \quad$ is the number (in billions) of the March ( $c=1$ ), June ( $c=2$ ) or September ( $c=3$ ) spawning sub-cohort round herring of age $a$ caught from 1 June to mid-November in calendar year $y$;
$\hat{C}_{\text {Nov, }, \text {, } 5+}^{R H} \quad$ is the number (in billions) of round herring of age $5+$ caught from 1 June to mid-November in year $y$;
$\hat{B}_{N o v, y}^{R H} \quad$ is the biomass (in thousand tons) of $1+$ round herring at mid-November in year $y$, which is taken to be associated with the November survey in year $y$;
$w_{a, c}^{\text {Nov }} \quad$ is the mean mass (in grams) of March $(c=1)$, June $(c=2)$ or September $(c=3)$ spawning sub-cohort round herring of age $a$ during the November survey (see Table B.2d); and $w_{5+}^{\text {Nov }} \quad$ is the mean mass (in grams) of round herring of age $5+$ during the November survey (see Table B.2d).

Biomass associated with the May survey

$$
\begin{array}{ll}
\hat{N} s u b_{M a y, y, a, 1}^{R H}=\left(\hat{N} s u b_{y-1, a, 1}^{R H} e^{-M_{a}}-\hat{C} s u b_{y, a, 1}^{R H}\right) e^{-2.5 M_{a} / 12} & y_{1}+1 \leq y \leq y_{n}, 0 \leq a \leq 4 \\
\hat{N} s u b_{M a y, y, a, 2}^{R H}=\left(\hat{N} s u b_{y-1, a, 2}^{R H} e^{-9 M_{a} / 12}-\hat{C} s u b_{y, a, 2}^{R H}\right) e^{-2.5 M_{a} / 12} & y_{1}+1 \leq y \leq y_{n}, 0 \leq a \leq 4 \\
\hat{N} s u b_{M a y, y, a, 3}^{R H}=\left(\hat{N} s u b_{y-1, a, 3}^{R H} e^{-6 M_{a} / 12}-\hat{C} s u b_{y, a, 3}^{R H}\right) e^{-2.5 M_{a} / 12} & y_{1}+1 \leq y \leq y_{n}, 1 \leq a \leq 4 \\
\hat{N}_{M a y, y, 5+}^{R H}=\left(\hat{N}_{y-1,5+}^{R H} e^{-9 M_{5+} / 12}-\hat{C}_{y, 5+}^{R H}\right) e^{-2.5 M_{5+} / 12} & y_{1}+1 \leq y \leq y_{n} \\
\hat{B}_{\text {May }, y}^{R H}=\sum_{a=0}^{4} \sum_{c=1}^{3}\left(S_{a}^{M a y} \times \hat{N} s u b_{M a y, y, a, c}^{R H} w_{a, c}^{M a y}\right)+S_{5+}^{\text {May }} \times \hat{N}_{\text {May }, y, 5+}^{R H} w_{5+}^{M a y} & \\
& y_{1}+1 \leq y \leq y_{n} \tag{A.3}
\end{array}
$$

where
$\hat{N} s u b_{\text {May, }, \text {,a,c }}^{R H} \quad$ is the number (in billions) of the March $(c=1)$, June $(c=2)$ or September $(c=3)$ spawning sub-cohort round herring of age $a$ at mid-May in calendar year $y$;
$\hat{N}_{\text {May, }, 5+5+}^{R H} \quad$ is the number (in billions) of round herring of age 5+ at mid-May in year $y$;
$\hat{B}_{\text {May,y }}^{R H} \quad$ is the biomass (in thousand tons) of $1+$ round herring at mid-May in year $y$, which are taken to be associated with the May survey;
$w_{a, c}^{\text {May }} \quad$ is the mean mass (in grams) of the March $(c=1)$, June $(c=2)$ or September $(c=3)$ spawning sub-cohort round herring of age $a$ during the May survey (see Table B.2b); and
$w_{5+}^{\text {May }} \quad$ is the mean mass (in grams) of round herring of age 5+ during the May survey (see Table B.2b).

## Catch at age

The annual catch at age by number is given by:

$$
\begin{array}{ll}
\hat{C} s u b_{y, a, 1}^{R H}=\hat{N} s u b_{y-1, a, 1}^{R H} e^{-M_{a}} S_{a} F_{y} & y_{1}+1 \leq y \leq y_{n}, 0 \leq a \leq 4 \\
\hat{C} s u b_{y, a, 2}^{R H}=\hat{N} s u b_{y-1, a, 2}^{R H} e^{-9 M_{a} / 12} S_{a} F_{y} & y_{1}+1 \leq y \leq y_{n}, 0 \leq a \leq 4 \\
\hat{C} s u b_{y, a, 3}^{R H}=\hat{N} s u b_{y-1, a, 3}^{R H} e^{-6 M_{a} / 12} S_{a} F_{y} & y_{1}+1 \leq y \leq y_{n}, 0 \leq a \leq 4 \\
\hat{C}_{y, 5+}^{R H}=\hat{N}_{y-1,5+}^{R H} e^{-9 M_{5+} / 12} S_{5+} F_{y} & y_{1}+1 \leq y \leq y_{n}
\end{array}
$$

where
$S_{a} \quad$ is the commercial selectivity at age $a$, which is assumed to be year-independent; and
$F_{y} \quad$ is the fished proportion in year $y$ for a fully selected age class.

In the equations above the difference in the year subscript between the catch-at-age and initial numbers-at-age is because these numbers-at-age pertain to March/June/September of the previous year, while the catch is assumed to be taken in a pulse on $1^{\text {st }}$ March.

The catch at age by number from 1 June to mid-November, for use in calculating the round herring biomass surveyed, is calculated as follows:

$$
\begin{array}{ll}
\hat{C}_{s u b_{N o v, y, a, 1}^{R H}=\hat{N} s u b_{N o v, y, a, 1}^{R H} e^{-6 M_{a} / 12} S_{a} F_{y}^{N o v}} y_{1} \leq y \leq y_{n}-1,0 \leq a \leq 4 \\
\hat{C} s u b_{N o v, y, a, 2}^{R H}=\hat{N} s u b_{N o v, y, a, 2}^{R H} e^{-3 M_{a} / 12} S_{a} F_{y}^{N o v} & y_{1} \leq y \leq y_{n}-1,0 \leq a \leq 4 \\
\hat{C}_{s u b_{N o v, y, a, 3}^{R H}=\hat{N} s u b_{N o v, y, a, 3}^{R H} S_{a} F_{y}^{N o v}} & y_{1} \leq y \leq y_{n}-1,1 \leq a \leq 4 \\
\hat{C}_{N o v, y, 5+}^{R H}=\hat{N}_{y, 5+}^{R H} e^{-3 M_{5+} / 12} S_{5+} F_{y}^{N o v} & y_{1} \leq y \leq y_{n}-1
\end{array}
$$

The fished proportion for the full year (from 1 June of year $y-1$ to 31 May of year $y$ ) is estimated by:

$$
\begin{align*}
& F_{y}= C_{y}^{\text {ObsTon }} /\left[\sum_{a=0}^{4}\left(w_{a, 1}^{\text {Mar }} \hat{N s u b} b_{y-1, a, 1}^{R H} e^{-M_{a}} S_{a}+w_{a, 2}^{M a r} \hat{N} s u b_{y-1, a, 2}^{R H} e^{-9 M_{a} / 12} S_{a}+w_{a, 3}^{M a r} \hat{N s u b_{y-1, a, 3}^{R H}} e^{-6 M_{a} / 12} S_{a}\right)\right. \\
&\left.+w_{5+}^{\text {Mar }} \hat{N}_{y-1,5+}^{R H} e^{-9 M_{5+} / 12} S_{5+}\right] \\
& y_{1}+1 \leq y \leq y_{n} \tag{A.6}
\end{align*}
$$

And for the catch prior to the November survey (1 June $y$ to mid-November $y$ ) is estimated by:

$$
\begin{align*}
& F_{y}^{\text {Nov }}=C_{y}^{\text {ObsTon }, \text { Nov }} /\left\lfloor w_{0,1}^{S e p} \hat{N} s u b_{y, 0,1}^{R H} e^{-6 M_{0} / 12} S_{0}+w_{0,2}^{S e p} \hat{N} s u b_{y, 0,2}^{R H} e^{-3 M_{0} / 12} S_{0}\right. \\
& \left.\quad+\sum_{a=1}^{4}\left(w_{a, 1}^{S e p} \hat{N} s u b_{y, a, 1}^{R H} e^{-6 M_{a} / 12} S_{a}+w_{a, 2}^{S e p} \hat{N} s u b_{y, a, 2}^{R H} e^{-3 M_{a} / 12} S_{a}+w_{a, 3}^{S e p} \hat{N} s u b_{y, a, 3}^{R H} S_{a}\right)+w_{5+}^{S e p} \hat{N}_{y, 5+}^{R H} e^{-3 M_{5+} / 12} S_{5+}\right] \\
& y_{1} \leq y \leq y_{n}-1 \tag{A.7}
\end{align*}
$$

where
$C_{y}^{\text {ObsTon }} \quad$ is the observed catch tonnage of year $y$ (June $y-1$ to May $y$, see Table B.3).
$C_{y}^{\text {ObsTon,Nov }}$
is the observed catch tonnage prior to the November survey of year $y$ (June $y$ to mid-
November $y$, see Table B.3).
$w_{a, c}^{\text {Mar }} \quad$ is the mean mass (in grams) of the March $(c=1)$, June ( $c=2$ ) or September $(c=3)$ spawning sub-cohort round herring of age $a$ in the commercial catch (see Table B.2a).
$w_{5+}^{\text {Mar }} \quad$ is the mean mass (in grams) of round herring of age $5+$ in the commercial catch (see Table B.2a).
$w_{a, c}^{S e p} \quad$ is the mean mass (in grams) of the March $(c=1)$, June ( $c=2$ ) or September $(c=3)$ spawning sub-cohort round herring of age $a$ in the commercial catch taken prior to the November survey (see Table B.2c).
$w_{5+}^{\text {Sep }} \quad$ is the mean mass (in grams) of round herring of age $5+$ in the commercial catch taken prior to the November survey (see Table B.2c).

## Catch at length

Given the predicted proportion-at-age in the quarterly commercial catch

$$
\begin{array}{ll}
\hat{p} s u b_{y, a, c}^{c o m}=\frac{\hat{C} s u b_{y, a, c}^{R H}}{\sum_{a=0}^{4} \sum_{c=1}^{3} \hat{C}_{s u b_{y, a, c}^{R H}+\hat{C}_{y, 5+}^{R H}}} & y_{1}+1 \leq y \leq y_{n}, 0 \leq a \leq 4, c=1, \ldots, 3 \\
\hat{p}_{y, 5+}^{c o m}=\frac{\hat{C}_{y, 5+}^{R H}}{\sum_{a=0}^{4} \sum_{c=1}^{3} \hat{C} s u b_{y, a, c}^{R H}+\hat{C}_{y, 5+}^{R H}} & y_{1}+1 \leq y \leq y_{n}
\end{array}
$$

and the assumption that all age 0 fish are at most $13 \mathrm{~cm}_{\mathrm{c}}$, the predicted proportion-at-length is then estimated as follows:
$\hat{p}_{y, 13-}^{c o m}=\sum_{c=1}^{3} \hat{p} s u b_{y, 0, c}^{c o m}+\sum_{l=3.5}^{13}\left(\sum_{a=1}^{4} \sum_{c=1}^{3} \hat{p} s u b_{y, a, c}^{c o m} A_{a, l, c}^{c o m}+\hat{p}_{y, 5+}^{c o m} A_{5+, l}^{c o m}\right) \quad y_{1}+1 \leq y \leq y_{n}$
$\hat{p}_{y, l}^{\text {com }}=\sum_{a=1}^{4} \sum_{c=1}^{3} \hat{p} s u b_{y, a, c}^{\text {com }}{ }_{a, l, c}^{\text {com }}+\hat{p}_{y, 5+}^{c o m} A_{5+l}^{\text {com }}$
$y_{1}+1 \leq y \leq y_{n}, l=13.5, \ldots, 20.5 \mathrm{~cm}$
$\hat{p}_{y, 21+}^{c o m}=\sum_{l=21}^{23}\left(\sum_{a=1}^{4} \sum_{c=1}^{3} \hat{p} s u b_{y, a, c}^{\text {com }} A_{a, l, c}^{\text {com }}+\hat{p}_{y, 5+}^{\text {com }} A_{5+, l}^{\text {com }}\right)$

$$
\begin{equation*}
y_{1}+1 \leq y \leq y_{n} \tag{A.9}
\end{equation*}
$$

where the length groups are in $0.5 \mathrm{~cm} \mathrm{~L}_{\mathrm{c}}$ and $A_{a, l, c}^{c o m} \quad$ is the proportion of the March $(c=1)$, June $(c=2)$ or September $(c=3)$ spawning round herring catch-at-age $a$ that fall in the length group $l\left(\right.$ thus $\sum_{l=l \min }^{l \max } A_{a l, c}^{c o m}=1$ ).
$A_{5+l}^{c o m} \quad$ is the proportion of round herring catch-at-age $5+$ that fall in the length group $l$ (thus

$$
\left.\sum_{l=l \min }^{l \max } A_{5+l}^{c o m}=1\right) .
$$

A plus group of 21 cm was chosen to ensure that all observations were non-zero. The matrix $A^{c o m}$ is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$
\begin{array}{ll}
L_{a, 1}^{c o m} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(a+1-t_{0}\right)}\right), \vartheta_{a}^{2}\right) & 1 \leq a \leq 4 \\
L_{a, 2}^{c o m} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(a+9 / 12-t_{0}\right)}\right), \vartheta_{a}^{2}\right) & 1 \leq a \leq 4 \\
L_{a, 3}^{c o m} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(a+6 / 12-t_{0}\right)}\right), \vartheta_{a}^{2}\right) & 1 \leq a \leq 4 \\
\left.L_{5+}^{c o m} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(6-t_{0}\right)}\right), \vartheta_{a}^{2}\right)\right)_{2} &
\end{array}
$$

where

| $L_{\infty}$ | denotes the maximum length of the individual; |
| :---: | :--- |
| $\kappa$ | denotes the annual growth rate; |
| $t_{0}$ | denotes the age at which the growth rate is zero; and |

$\vartheta_{a}^{2} \quad$ denotes the variance about the mean length for age $a$.

## Proportion at length in November

Given the predicted proportion-at-age in mid-November of:

$$
\begin{align*}
& y_{1} \leq y \leq y_{n}-1, a=0, c=1,2 \\
& \hat{p} s u b_{y, a, c}^{N o v}=\frac{S_{a}^{N o v} \times \hat{N} s u b_{N o v, y, a, c}^{R H}}{S_{0}^{N o v} \times\left(\hat{N} s u b_{N o v, y, 0,1}^{R H}+\hat{N} s u b_{N o v, y, 0,2}^{R H}\right)+\sum_{a=1}^{4} \sum_{c=1}^{3}\left(S_{a}^{N o v} \times \hat{N} s u b_{N o v, y, a, c}^{R H}\right)+S_{5+}^{N o v} \times \hat{N}_{N o v, y, 5+}^{R H}} \\
& y_{1} \leq y \leq y_{n}-1,1 \leq a \leq 4, c=1, \ldots, 3 \\
& \hat{p}_{y, 5+}^{N o v}=\frac{S_{5+}^{N o v} \times \hat{N}_{N o v, y, 5+}^{R H}}{S_{0}^{N o v} \times\left(\hat{N} s u b_{N o v, y, 0,1}^{R H}+\hat{N} s u b_{N o v, y, 0,2}^{R H}\right)+\sum_{a=1}^{4} \sum_{c=1}^{3}\left(S_{a}^{N o v} \times \hat{N} s u b_{N o v, y, a, c}^{R H}\right)+S_{5+}^{N o v} \times \hat{N}_{N o v, y, 5+}^{R H}} \\
& y_{1} \leq y \leq y_{n}-1 \tag{A.11}
\end{align*}
$$

and the assumption that all age 0 fish are at most $11 \mathrm{~cm} L_{c}$, the predicted proportion-at-length is then estimated as follows:
$\hat{p}_{y, 13-}^{N o v}=\sum_{c=1}^{2} \hat{p} s u b_{y, 0, c}^{N o v}+\sum_{l=1.5}^{11}\left(\sum_{a=1}^{4}\left(\sum_{c=1}^{3} \hat{p} s u b_{y, a, c}^{N o v} A_{a, l, c}^{N o v}\right)+\hat{p}_{y, 5+5}^{N o v} A_{5+l}^{N o v}\right)$

$$
y_{1} \leq y \leq y_{n}-1
$$

$\hat{p}_{y, l}^{N o v}=\sum_{a=1}^{4} \sum_{c=1}^{3} \hat{p} s u b_{y, a, c}^{N o v} A_{a, l, c}^{N o v}+\hat{p}_{y, 5+}^{N o v} A_{5+l}^{N o v}$
$y_{1} \leq y \leq y_{n}-1, l=11.5, \ldots, 19 \mathrm{~cm}$
$\hat{p}_{y, 19.5+}^{N o v}=\sum_{l=19.5}^{25}\left(\sum_{a=1}^{4} \sum_{c=1}^{3} \hat{p} s u b_{y, a, c}^{\text {Nov }}{ }_{a, l, c}^{\text {Nov }}+\hat{p}_{y, 5+}^{N o v} A_{5+l}^{\text {Nov }}\right) \quad y_{1} \leq y \leq y_{n}-1$
where the length groups are in $0.5 \mathrm{~cm}_{\mathrm{c}}$ and
$A_{a, l, c}^{\text {Nov }} \quad$ is the proportion of the March $(c=1)$, June $(c=2)$ or September $(c=3)$ spawning round herring catch-at-age $a$ that fall in the length group $l$ (thus $\sum_{l=1.5}^{25} A_{a, l, c}^{\text {Nov }}=1$ ) in mid-November.
$A_{5+l}^{\text {Nov }} \quad$ is the proportion of round herring catch-at-age $5+$ that fall in the length group $l$ (thus

$$
\left.\sum_{l=1.5}^{25} A_{5+l}^{N o v}=1\right) \text { in mid-November. }
$$

A plus group of 19.5 cm was chosen to ensure that all observations were non-zero. The matrix $A^{\text {Nov }}$ is calculated under the assumption that length-at-age is normally distributed about the same von Bertalanffy growth curve mentioned above:

[^2]$L_{a, 1}^{N o v} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(a+8.5 / 12-t_{0}\right)}\right), \vartheta_{a}^{2}\right)$
$1 \leq a \leq 4$
$L_{a, 2}^{N o v} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(a+5.5 / 12-t_{0}\right)}\right), \vartheta_{a}^{2}\right)$
$1 \leq a \leq 4$
$L_{a, 3}^{N o v} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(a+2.5 / 12-t_{0}\right)}\right), \vartheta_{a}^{2}\right)$
$1 \leq a \leq 4$
$L_{5+}^{N o v} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(6-t_{0}\right)}\right), \vartheta_{a}^{2}\right)_{3}$

## Proportion at length in May

Given the predicted proportion-at-age in mid-May of:

$$
\begin{aligned}
& \hat{p} s u b_{y, a, c}^{M a y}=\frac{S_{a}^{M a y} \times \hat{N} s u b_{M a y, y, a, c}^{R H}}{\sum_{a=0}^{4} \sum_{c=1}^{3}\left(S_{a}^{M a y} \times \hat{N} s u b_{M a y, y, a, c}^{R H}\right)+S_{5+}^{M a y} \times \hat{N}_{M a y, y, 5+}^{R H}} \\
& y_{1}+1 \leq y \leq y_{n}, 0 \leq a \leq 4, c=1, \ldots, 3
\end{aligned}
$$

$\hat{p}_{y, 5+}^{\text {May }}=\frac{\hat{N}_{M a y, y, 5+}^{R H}}{\sum_{a=0}^{4} \sum_{c=1}^{3}\left(S_{a}^{M a y} \times \hat{N} s u b_{M a y, y, a, c}^{R H}\right)+S_{5+}^{M a y} \times \hat{N}_{M a y, y, 5+}^{R H}}$

$$
\begin{equation*}
y_{1}+1 \leq y \leq y_{n} \tag{A.14}
\end{equation*}
$$

the predicted proportion-at-length is then estimated as follows:
$\hat{p}_{y, 13-}^{M a y}=\sum_{l=2}^{13}\left(\sum_{a=0}^{4}\left(\sum_{c=1}^{3} \hat{p} s u b_{y, a, c}^{M a y} A_{a l, c}^{M a y}\right)+\hat{p}_{y, 5+}^{M a y} A_{5+, l}^{M a y}\right)$
$y_{1}+1 \leq y \leq y_{n}$
$\hat{p}_{y, l}^{\text {May }}=\sum_{a=1}^{4} \sum_{c=1}^{3} \hat{p} s u b_{y, a, c}^{M a y} A_{a l, c}^{M a y}+\hat{p}_{y, 5+}^{M a y} A_{5+l}^{M a y}$
$y_{1}+1 \leq y \leq y_{n}, l=13, \ldots, 18 \mathrm{~cm}$
$\hat{p}_{y, 18+}^{M a y}=\sum_{l=18}^{23}\left(\sum_{a=1}^{4} \sum_{c=1}^{3} \hat{p} s u b_{y, a, c}^{M a y} A_{a, l, c}^{M a y}+\hat{p}_{y, 5+}^{M a y} A_{5+, l}^{M a y}\right)$
$y_{1}+1 \leq y \leq y_{n}$
where the length groups are in $0.5 \mathrm{~cm}_{\mathrm{c}}$ and $A_{a l, c}^{M a y} \quad$ is the proportion of the March $(c=1)$, June $(c=2)$ or September $(c=3)$ spawning round herring catch-at-age $a$ that fall in the length group $l$ (thus $\sum_{l=2}^{23} A_{a, l, c}^{\text {May }}=1$ ) in mid-May.
$A_{5+, l}^{M a y} \quad$ is the proportion of round herring catch-at-age 5+ that fall in the length group $l$ (thus

$$
\left.\sum_{l=1.5}^{25} A_{5+l}^{N o v}=1\right) \text { in mid-May. }
$$

A plus group of 18 cm was chosen to ensure that all observations were non-zero. The matrix $A^{\text {May }}$ is calculated under the assumption that length-at-age is normally distributed about the same von Bertalanffy growth curve mentioned above:
$L_{a, 1}^{\text {May }} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(a+14.5 / 12-t_{0}\right)}\right), \vartheta_{a}^{2}\right) \quad 0 \leq a \leq 4$

[^3]\[

$$
\begin{array}{ll}
L_{a, 2}^{\text {May }} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(a+11.5 / 12-t_{0}\right)}\right), \vartheta_{a}^{2}\right) & 0 \leq a \leq 4 \\
L_{a, 3}^{\text {May }} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(a+8.5 / 12-t_{0}\right)}\right), \vartheta_{a}^{2}\right) & 0 \leq a \leq 4 \\
L_{5+}^{\text {May }} \sim N\left(L_{\infty}\left(1-e^{-\kappa\left(6-t_{0}\right)}\right), \vartheta_{a}^{2}\right) 4 &
\end{array}
$$
\]

## Fitting the Model to Observed Data (Likelihood)

The survey observations are assumed to be log-normally distributed, and sampling CVs (squared) of the untransformed survey observations are used to approximate the "sampling" component of the total variance of the corresponding log-distributions. The commercial proportions at length from the raised length frequencies are assumed to be lognormally distributed. Thus we have:

$$
\begin{align*}
& -\ln L=\frac{1}{2} \sum_{y=y 1}^{y n-1}\left\{\frac{\left(\ln B_{N o v, y}^{R H}-\ln \left(k_{N}^{R H} \hat{B}_{N o v, y}^{R H}\right)^{2}\right.}{\left(\sigma_{y, N}^{R H}\right)^{2}+\left(\lambda_{N}^{R H}\right)^{2}}+\ln \left[2 \pi\left(\left(\sigma_{y, N}^{R H}\right)^{2}+\left(\lambda_{N}^{R H}\right)^{2}\right)\right]\right\} \\
& +\frac{1}{2} \sum_{y=y l+1}^{y n}\left\{\frac{\left(\ln B_{M a y, y}^{R H}-\ln \left(k^{\text {coverage }} k_{r}^{R H} \hat{B}_{M a y, y}^{R H}\right)\right)^{2}}{\left(\sigma_{y, r}^{R H}\right)^{2}+\left(\lambda_{r}^{R H}\right)^{2}}+\ln \left[2 \pi\left(\left(\sigma_{y, r}^{R H}\right)^{2}+\left(\lambda_{r}^{R H}\right)^{2}\right)\right)\right\} \\
& +w_{c o m, \text { min }} \sum_{y=y l+1}^{y n}\left\{\frac{p_{y, l \text { min }}^{c o m}\left(\ln p_{y, l \text { min }}^{c o m}-\ln \hat{p}_{y, l \text { min }}^{c o m}\right)^{2}}{2\left(\sigma_{c o m, \text { min }}\right)^{2}}+\ln \left(\frac{\sigma_{c o m, \text { min }}}{\sqrt{p_{y, \text { min }}^{c o m}}}\right)\right\} \\
& +w_{c o m} \sum_{y=y l+l}^{y n} \sum_{l=l \min +1}^{l \max }\left\{\frac{p_{y, l}^{c o m}\left(\ln p_{y, l}^{c o m}-\ln \hat{p}_{y, l}^{c o m}\right)^{2}}{2\left(\sigma_{c o m}\right)^{2}}+\ln \left(\frac{\sigma_{\text {com }}}{\sqrt{p_{y, l}^{c o m}}}\right)\right\} \\
& +w_{N o v, \text { min }} \sum_{y=y 1}^{y n-1}\left\{\frac{p_{y, l \min N}^{N o v}\left(\ln p_{y, l \min N}^{N o v}-\ln \hat{p}_{y, l \text { min } N}^{N o v}\right)^{2}}{2\left(\sigma_{N o v, \text { min }}^{N}\right)^{2}}+\ln \left(\frac{\sigma_{N o v, \text { min }}}{\sqrt{p_{y, l \min N}^{N o n}}}\right)\right\} \\
& +w_{N o v} \sum_{y=y l l=l}^{v n-1} \sum_{\min N+1}^{l \max N}\left\{\frac{p_{y, l}^{N o v}\left(\ln p_{y, l}^{N o v}-\ln \hat{p}_{y, l}^{N o v}\right)^{2}}{2\left(\sigma_{N o v}\right)^{2}}+\ln \left(\frac{\sigma_{N_{o o v}}}{\sqrt{p_{y, l}^{N o v}}}\right)\right\} \\
& +w_{M a y, \text { min }} \sum_{y=y l+1}^{y n}\left\{\frac{p_{y, l \min M}^{M a y}\left(\ln p_{y, l \min M}^{M a y}-\ln \hat{p}_{y, l \min M}^{M a y}\right)^{2}}{2\left(\sigma_{M a y, \text { min }}\right)^{2}}+\ln \left(\frac{\sigma_{M a y, \text { min }}}{\sqrt{p_{y, l \min M}^{M a y}}}\right)\right\} \\
& +w_{\text {May }} \sum_{y=y l+1}^{y n} \sum_{l=l}^{l \min M+1} \max _{\max }\left\{\frac{p_{y, l}^{\text {May }}\left(\ln p_{y, l}^{\text {May }}-\ln \hat{p}_{y, l}^{\text {May }}\right)^{2}}{2\left(\sigma_{\text {May }}\right)^{2}}+\ln \left(\frac{\sigma_{\text {May }}}{\sqrt{p_{y, l}^{\text {May }}}}\right)\right\} \tag{A.17}
\end{align*}
$$

where
$B_{N o v, y}^{R H} \quad$ is the acoustic survey estimate (in thousand tons) of round herring biomass from the
November survey in year $y$, with associated $\mathrm{CV} \sigma_{y, N}^{R H}$ and constant of proportionality (multiplicative bias ${ }^{5}$ ) $k_{N}^{R H}$;

[^4]$B_{\text {May }, y}^{R H}$

$\left(\lambda_{N / r}^{R H}\right)$
$k^{\text {coverage }}$
is the acoustic survey estimate (in thousands of tons) of round herring biomass from the May survey in year $y$, with associated CV $\sigma_{y, r}^{R H}$ and constant of proportionality $k_{r}^{R H}{ }^{6}$;
is the additional variance (over and above the survey sampling $\mathrm{CV} \sigma_{y, N / r}^{R H}$ that reflects survey inter-transect variance) associated with the November/recruit surveys (see Appendix C); is the multiplicative bias associated with the May survey denoting the proportion of the biomass for the fully selected age $($ age $=1)$ that is covered by the survey, in comparison to the proportion of the biomass surveyed in November - note that this is not incorporated in $k_{r}^{R H}$; is the observed proportion (by number) of the commercial catch in length group $l$ during year $y$ (June $y-1$ to May $y$ );
is the observed proportion (by number) of length group $l$ fish during the November/May survey in year $y$;
is the weighting applied to the commercial proportion at length 13 cm (the minus group); is the weighting applied to the remainder of the commercial proportion at length data; is the weighting applied to the November/May proportion at length $11 / 13 \mathrm{~cm}$ (the minus group);
is the weighting applied to the remainder of the November/May proportion at length data; is the standard deviation associated with the proportion-at-length 13 cm (minus group) data in the commercial catch, which is estimated in the fitting procedure by:

$$
\sigma_{c o m, \min }=\sqrt{\sum_{y=y l+1}^{y n} p_{y, l \min }^{c o m}\left(\ln p_{y, l \min }^{c o m}-\ln \hat{p}_{y, l \min }^{c o m}\right)^{2} / \sum_{y=y 1+1}^{y n} 1} .
$$

is the standard deviation associated with the remaining proportion-at-length data in the commercial catch, which is estimated in the fitting procedure by:

$$
\sigma_{c o m}=\sqrt{\sum_{y=y 1+1}^{y n} \sum_{l=l \min +1}^{l \max } p_{y, l}^{c o m}\left(\ln p_{y, l}^{c o m}-\ln \hat{p}_{y, l}^{c o m}\right)^{2} / \sum_{y=y 1+1}^{y n} \sum_{l=l \min +1}^{l \max } 1} .
$$

is the standard deviation associated with the proportion-at-length 11 cm (minus group) data in the November survey data, which is estimated in the fitting procedure by:

$$
\sigma_{N o v, \min }=\sqrt{\sum_{y=y 1}^{y n-1} p_{y, l \min N}^{N o v}\left(\ln p_{y, l \min N}^{N o v}-\ln \hat{p}_{y, l \min N}^{N o v}\right)^{2} / \sum_{y=y 1}^{y n-1} 1} .
$$

is the standard deviation associated with the remaining proportion-at-length data in the November survey data, which is estimated in the fitting procedure by:

[^5]$$
\sigma_{N o v}=\sqrt{\sum_{y=y l l=l}^{v n-1} \sum_{\text {min }}^{l \max N} p_{y, l}^{N o v}\left(\ln p_{y, l}^{N o v}-\ln \hat{p}_{y, l}^{N o v}\right)^{2} / \sum_{y=y l l=l}^{v n-1} \sum_{\text {min }}^{l \max N} 1} .
$$
$\sigma_{\text {May,min }} \quad$ is the standard deviation associated with the proportion-at-length 13 cm (minus group) data in the May survey data, which is estimated in the fitting procedure by:
$$
\sigma_{M a y, \min }=\sqrt{\sum_{y=y l+1}^{y n} p_{y, l \min M}^{M a y}\left(\ln p_{y, l \min M}^{M a y}-\ln \hat{p}_{y, l \min M}^{M a y}\right)^{2} / \sum_{y=y l+1}^{y n} 1}
$$
$\sigma_{\text {Nov }} \quad$ is the standard deviation associated with the remaining proportion-at-length data in the November survey data, which is estimated in the fitting procedure by:
$$
\sigma_{M a y}=\sqrt{\sum_{y=y l+1}^{y n} \sum_{l=l \min }^{l \max _{M+1}^{M}} p_{y, l}^{M a y}\left(\ln p_{y, l}^{M a y}-\ln \hat{p}_{y, l}^{M a y}\right)^{2} / \sum_{y=y l+l l=l \min }^{y n} \sum_{M+1}^{l \max M} 1} .
$$

The raw commercial catch data are in 0.5 cm length classes of caudal length, $\mathrm{L}_{\mathrm{c}}$.

## Fixed Parameters

The following parameters are fixed externally in this assessment:
Natural mortality: $M_{a}^{R H}=1.3,0 \leq a \leq 5+$.
There are 16 length classes in the commercial catch data, 18 in the November survey data and 11 in the May survey data. However, these length classes are not all independent as there are only about 5 age groups. Therefore dividing the length data contribution to the likelihood by 3 gives it a weighting close to the 5 age groups. Thus $w_{\text {com }}=w_{\mid \text {Nov }}=w_{\text {May }}=0.33$.

The assumption is made that $w_{c o m, \text { min }}=w_{\text {Nov, min }}=w_{M a y, \text { min }}=1$ as it represents a single age group.
It is assumed that age 1 fish are fully selected in the commercial catch, i.e. $S_{1}=1$.
Initial model fits with additional parameters indicated a negligible selectivity for ages $2+$ and thus these are fixed at 0, i.e. $S_{3}=S_{4}=S_{5+}=0$.

It is assumed that age 0 (March and June sub-cohorts) and age 1 fish are fully selected in the November survey,
i.e. $S_{0}^{N o v}=S_{1}^{\text {Nov }}=1$

It is assumed that age 0 (all sub-cohorts) fish are fully selected in the May survey, i.e. $S_{0}^{\text {May }}=1$
The multiplicative bias on the November survey estimate of abundance and on the May recruitment estimate were fixed at $k_{N}^{R H}=0.340$ and $k_{r}^{R H}=0.427$, respectively, corresponding to the means of normal distributions fitted to pdfs of all individual constant bias factors (Appendix C). Additional (inter-transect) variance on the November and May surveys were fixed at $\left(\lambda_{N}^{R H}\right)^{2}=0.076$ and $\left(\lambda_{r}^{R H}\right)^{2}=0.052$, respectively, corresponding to the means of normal distributions fitted to pdfs of all individual variable and random bias factors (Appendix C).

## Estimable Parameters and Prior Distributions

Annual recruitment: $\hat{N}_{y, 0}^{R H} \sim U(0,5000)$ billion, $y_{1} \leq y \leq y_{n}-1$
Initial numbers at age: $\hat{N}_{1987,1}^{R H} \sim U(0,500)$ and $\hat{N}_{1987, a}^{R H} \sim U(0,50), 2 \leq a \leq 5+$
Split of recruitment by sub-cohort: $p_{1}=q_{1} q_{2}, p_{2}=q_{2}-p_{1}$, with $q_{1}, q_{2} \sim U(0,1)$
With $\quad \hat{N} s u b_{1987, a, 1}^{R H}=p_{1} \times \hat{N}_{1987, a}^{R H}, 1 \leq a \leq 4$

$$
\begin{aligned}
& \hat{N} s u b_{1987, a, 2}^{R H}=p_{2} \times \hat{N}_{1987, a}^{R H}, 1 \leq a \leq 4 \\
& \hat{N} s u b_{1987, a, 3}^{R H}=\left(1-p_{1}-p_{2}\right) \times \hat{N}_{1987, a}^{R H}, 1 \leq a \leq 4
\end{aligned}
$$

And $\quad \hat{N} s u b_{y, 0,1}^{R H}=p_{1} \times \hat{N}_{y, 0}^{R H}$

$$
\begin{aligned}
& \hat{N} s u b_{y, 0,2}^{R H}=p_{2} \times \hat{N}_{y, 0}^{R H} \\
& \hat{N} s u b_{y, 0,3}^{R H}=\left(1-p_{1}-p_{2}\right) \times \hat{N}_{y, 0}^{R H}
\end{aligned}
$$

Selectivity at age: $S_{0}, S_{2} \sim U(0,1)$
November survey selectivity at age: $S_{a}^{\text {Nov }}=e^{-x(a-1)}, 2 \leq a \leq 5+$, with $x \sim U(0,5)$ estimated
May survey selectivity at age: $S_{a}^{M a y}=e^{-z(a-1)}, 1 \leq a \leq 5+$ with $z \sim U(0,10)$ estimated
May Survey coverage compared to November survey coverage: $k^{\text {coverage }} \sim U(0,2)$
Variance about the mean length at age $0: \vartheta_{0}^{2} \sim U(0,0.15)$
Variance about the mean length at age: $\vartheta_{a}^{2} \sim U(0,0.15), a=1, \ldots, 4$
Variance about the mean length at age $5+: \vartheta_{5+}^{2} \sim U(0,0.15)$

## APPENDIX B: Data and standard inputs used in the South African Round Herring Assessment

## November acoustic survey

A time series of estimates of annual biomass from November 1984 to November 2009 are available, together with CVs (Table B.1). The assumption is made that these estimates of abundance are comparable. Coetzee and Merkle (2009) compared visually survey effort and biomass, noting too the general correlation between increased survey estimated recruitment (which should be less influenced by offshore extensions of survey effort) and subsequent increased survey November biomass, and concluded that the increase in biomass for the duration of the time series was 'real' and not correlated to the increase in survey effort.

Although the November survey length frequencies indicate that some recruits ( $<12 \mathrm{~cm} \mathrm{~L}_{\mathrm{c}}$ ) are sampled by the survey, the numbers are low (Janet Coetzee pers. comm.). The weight of these recruits and their contribution to the total biomass would therefore be small. Thus the survey estimates of abundance are assumed to measure the relative $1+$ biomass.

## May recruit acoustic survey

A time series of estimates of annual recruitment numbers and biomass is available from May 1987 to May 2010, together with CVs (Table B.1). The assumption is made that these estimates of biomass and recruitment are comparable.

## Von Bertalanffy Growth Curve

The von Bertalanffy parameters are: $L_{\infty}=20.30 \mathrm{~L}_{\mathrm{c}}, \kappa=0.937, t_{0}=0.1$. The derivation of this growth curve is detailed in Durholtz et al. (2010).

## Weight at age

A length-weight relationship has been calculated from the 5 years of November survey data between 2005 and 2009 (Y. Geja and D. Durholtz pers. comm.):
$W=0.0084 \times L_{c}^{3.0883}$
where weight is in grams and caudal length $\left(\mathrm{L}_{\mathrm{c}}\right)$ in cms . This length-weight relationship was applied to the length-at-age calculated by the mean von Bertlanffy relationship assumed for the model to give the weight-atage values listed in Tables B.2a-d. The weight-at-age $5+$ was calculated as $w_{5+}=\frac{\sum_{a=5}^{8} w_{a} \operatorname{prop}_{a}}{\sum_{a=5}^{8} \operatorname{prop}_{a}}$, where $\operatorname{prop}_{a}=e^{-(a-5) M_{a d}}$ denotes the relative proportion at age, assuming a low fishing mortality on older ages.

## Commercial catch

Commercial catch raised length frequencies are available by month from 1987 onwards. The annual data listed in Table B. 3 is the sum of the months of June of the previous year to May of the reported year.

Table B.1. Time series of annual estimates of $1+$ biomass from the November acoustic survey (in tons), with CVs, and estimates of recruitment from the May acoustic survey (in billions), with CVs.

| Year | November survey |  | May survey up to Cape Infanta |  |
| :--- | ---: | ---: | ---: | ---: |
|  | Biomass | CV | Biomass | CV |
| 1984 | 80546 | 0.337 |  |  |
| 1985 | 253750 | 0.227 |  | 0.152 |
| 1986 | 349282 | 0.305 |  | 0.277 |
| 1987 | 545522 | 0.201 | 58214 | 0.267 |
| 1988 | 380531 | 0.323 | 18711 | 0.689 |
| 1989 | 881286 | 0.264 | 54286 | 0.235 |
| 1990 | 440117 | 0.181 | 33095 | 0.334 |
| 1991 | 642954 | 0.250 | 93830 | 0.225 |
| 1992 | 751462 | 0.170 | 126229 | 0.217 |
| 1993 | 523388 | 0.220 | 100967 | 0.548 |
| 1994 | 284887 | 0.213 | 62609 | 0.345 |
| 1995 | 586870 | 0.135 | 152197 | 0.224 |
| 1996 | 596511 | 0.156 | 378938 | 0.376 |
| 1997 | 624054 | 0.295 | 195492 | 0.217 |
| 1998 | 1247966 | 0.149 | 160525 | 0.424 |
| 1999 | 1398329 | 0.171 | 355087 | 0.247 |
| 2000 | 1420454 | 0.169 | 582579 | 0.296 |
| 2001 | 1045517 | 0.131 | 312982 | 0.212 |
| 2002 | 917853 | 0.189 | 406132 | 0.275 |
| 2003 | 1761631 | 0.108 | 337754 | 0.169 |
| 2004 | 1475464 | 0.100 | 415721 | 0.185 |
| 2005 | 1616260 | 0.130 | 436840 | 0.250 |
| 2006 | 1228446 | 0.106 | 301534 | 0.212 |
| 2007 | 1720865 | 0.153 | 257984 | 0.239 |
| 2008 | 1260460 | 0.118 | 562608 | 0.189 |
| 2009 | 1990831 | 0.108 | 260185 |  |
| 2010 |  |  | 278731 |  |

Table B.2a. The weight-at-age (in grams) corresponding to the pulse of commercial catch at 1 March, $w_{a, c}^{M a r}$, $a=0, \ldots, 4$ and $w_{5+}^{M a r}$.

| Age | March sub-cohort $(c=1)$ | June sub-cohort $(c=2)$ | September sub-cohort $(c=3)$ |
| :--- | :--- | :--- | :--- |
| 0 | 16.15 | 8.13 | 2.53 |
| 1 | 51.88 | 43.78 | 34.82 |
| 2 | 74.29 | 70.09 | 65.02 |
| 3 | 84.60 | 82.78 | 80.52 |
| 4 | 88.89 | 88.15 | 87.22 |
| $5+$ |  | 89.40 |  |

Table B.2b. The weight-at-age (in grams) corresponding to the May survey, $w_{a, c}^{M a y}, a=0, \ldots, 4$ and $w_{5+}^{M a y}$.

| Age | March sub-cohort $(c=1)$ | June sub-cohort $(c=2)$ | September sub-cohort $(c=3)$ |
| :--- | :--- | :--- | :--- |
| 0 | 23.81 | 14.70 | 7.00 |
| 1 | 57.86 | 50.60 | 42.34 |
| 2 | 77.20 | 73.64 | 69.31 |
| 3 | 85.83 | 84.32 | 82.44 |
| 4 | 89.38 | 88.77 | 88.01 |
| $5+$ |  | 89.40 |  |

Table B.2c. The weight-at-age (in grams) corresponding to the pulse of commercial catch taken prior to the November survey at 1 September, $w_{a, c}^{\text {Sep }}, a=0, \ldots, 4$ and $w_{5+}^{\text {Sep }}$.

| Age | March sub-cohort $(c=1)$ | June sub-cohort $(c=2)$ | September sub-cohort $(c=3)$ |
| :--- | :--- | :--- | :--- |
| 0 | 2.53 | 0.17 | N/A |
| 1 | 34.82 | 25.39 | 16.15 |
| 2 | 65.02 | 58.97 | 51.88 |
| 3 | 80.52 | 77.73 | 74.29 |
| 4 | 87.22 | 86.05 | 84.60 |
| $5+$ |  | 89.40 |  |

Table B.2d. The weight-at-age (in grams) corresponding to the November survey, $w_{a, c}^{N o v}, a=0, \ldots, 4$ and

$$
w_{5+}^{N o v} .
$$

| Age | March sub-cohort $(c=1)$ | June sub-cohort $(c=2)$ | September sub-cohort $(c=3)$ |
| :--- | :--- | :--- | :--- |
| 0 | 7.00 | 1.91 | N/A |
| 1 | 42.34 | 33.27 | 23.81 |
| 2 | 69.31 | 64.08 | 57.86 |
| 3 | 82.44 | 80.10 | 77.20 |
| 4 | 88.01 | 87.04 | 85.83 |
| $5+$ |  | 89.40 |  |

MCM/2010/SWG-PEL/46
Table B.3. The numbers at length (in thousands) in the commercial catch, and the corresponding catch (in tons). Note that the catch for year $y$ consists of the catch from $1^{\text {st }}$ June $y-1$ to $31^{\text {st }}$ May $y$. The catch (in tons)
from 1 June $y-1$ to mid-November $y-1$ is also tabled.

| Length Class ( $\mathrm{L}_{\mathrm{c}}$ in cm) | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Tonnage to mid-Nov | 293 | 156 | 273 | 225 | 245 | 1383 | 12357 | 5934 | 958 | 4701 | 6922 | 2095 |
| Tonnage | 64582 | 44600 | 46276 | 33550 | 47005 | 46054 | 60448 | 81819 | 43512 | 90107 | 57663 | 57336 |
| 3.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1210 | 0 | 0 | 0 | 0 |
| 4.0 | 0 | 0 | 188 | 136 | 344 | 933 | 0 | 4033 | 0 | 0 | 0 | 0 |
| 4.5 | 0 | 20 | 1590 | 3802 | 42 | 1396 | 3 | 5451 | 264 | 0 | 1027 | 1142 |
| 5.0 | 9 | 436 | 1560 | 18107 | 644 | 11656 | 3278 | 17966 | 5717 | 1402 | 2918 | 91752 |
| 5.5 | 132 | 1312 | 4112 | 24879 | 32578 | 72276 | 31932 | 13226 | 28412 | 2044 | 14124 | 3812 |
| 6.0 | 1228 | 20776 | 12596 | 37549 | 9808 | 43569 | 88839 | 82300 | 28940 | 3703 | 35206 | 30732 |
| 6.5 | 15795 | 21662 | 10209 | 63102 | 25452 | 29200 | 119539 | 113561 | 55497 | 6037 | 54445 | 21765 |
| 7.0 | 22868 | 127778 | 55153 | 57340 | 36893 | 35559 | 88063 | 88795 | 72965 | 36038 | 61051 | 59664 |
| 7.5 | 12226 | 22966 | 45484 | 45462 | 34696 | 26987 | 63975 | 65737 | 119077 | 148602 | 40950 | 88257 |
| 8.0 | 6729 | 2988 | 40204 | 33918 | 26483 | 22613 | 283301 | 68331 | 111041 | 132460 | 32035 | 156403 |
| 8.5 | 3058 | 4870 | 36379 | 20096 | 22533 | 28108 | 95722 | 66983 | 49106 | 83287 | 39678 | 124366 |
| 9.0 | 4708 | 9780 | 22763 | 9712 | 20849 | 26834 | 47427 | 50800 | 37392 | 132225 | 43534 | 76824 |
| 9.5 | 6509 | 12952 | 13304 | 8572 | 18682 | 28838 | 42161 | 47815 | 38429 | 105417 | 34327 | 54855 |
| 10.0 | 7500 | 11244 | 6010 | 3053 | 17083 | 25657 | 24391 | 39363 | 51578 | 101222 | 19499 | 49394 |
| 10.5 | 6659 | 4814 | 3893 | 384 | 18878 | 24017 | 12950 | 28011 | 41749 | 45901 | 22480 | 17651 |
| 11.0 | 5038 | 2168 | 1764 | 219 | 13731 | 19798 | 9684 | 13505 | 17607 | 43712 | 27557 | 18421 |
| 11.5 | 2526 | 1785 | 463 | 148 | 8272 | 17082 | 4497 | 5061 | 5926 | 27754 | 32206 | 7219 |
| 12.0 | 2346 | 1833 | 626 | 2097 | 6143 | 11738 | 4581 | 2605 | 1740 | 8444 | 39926 | 4293 |
| 12.5 | 1867 | 165 | 422 | 289 | 2630 | 9134 | 5238 | 2263 | 783 | 5476 | 44173 | 470 |
| 13.0 | 2267 | 927 | 1221 | 492 | 2109 | 11601 | 9803 | 3634 | 713 | 21051 | 27825 | 924 |
| 13.5 | 2235 | 1435 | 777 | 337 | 922 | 15123 | 27055 | 5910 | 938 | 16216 | 22024 | 1889 |
| 14.0 | 968 | 3114 | 3350 | 662 | 6293 | 23003 | 63718 | 13002 | 3907 | 44289 | 30556 | 4016 |
| 14.5 | 5979 | 2354 | 3910 | 1905 | 21313 | 48125 | 113041 | 26941 | 10425 | 46367 | 38148 | 18667 |
| 15.0 | 34697 | 8311 | 6808 | 5852 | 54216 | 110940 | 196053 | 82506 | 23207 | 61819 | 48845 | 36325 |
| 15.5 | 76580 | 34783 | 7315 | 16042 | 103608 | 188813 | 198565 | 181847 | 43286 | 75067 | 62061 | 62604 |
| 16.0 | 102302 | 49349 | 15379 | 36821 | 161932 | 226282 | 189852 | 296054 | 108218 | 134197 | 95323 | 115229 |
| 16.5 | 119760 | 106594 | 35935 | 59041 | 154602 | 138816 | 139083 | 286859 | 167712 | 190492 | 151824 | 142682 |
| 17.0 | 125465 | 151644 | 80222 | 84928 | 115861 | 57493 | 86520 | 217875 | 161957 | 269486 | 171789 | 153853 |
| 17.5 | 114880 | 141090 | 122301 | 87714 | 75105 | 27162 | 53894 | 146720 | 96829 | 252920 | 138899 | 133992 |
| 18.0 | 99058 | 87488 | 126588 | 74923 | 54165 | 15541 | 29239 | 96800 | 51393 | 178824 | 83154 | 87150 |
| 18.5 | 95477 | 52635 | 90429 | 51736 | 33495 | 10290 | 17069 | 59433 | 20568 | 87594 | 44962 | 52519 |
| 19.0 | 69588 | 26878 | 57948 | 33950 | 20346 | 8032 | 9292 | 35549 | 10703 | 35864 | 23275 | 33807 |
| 19.5 | 45410 | 14189 | 32785 | 15663 | 10240 | 3657 | 4045 | 19385 | 4770 | 11312 | 11404 | 16563 |
| 20.0 | 26329 | 8321 | 17370 | 8845 | 4508 | 1909 | 5103 | 8588 | 1965 | 5176 | 6800 | 8898 |
| 20.5 | 11553 | 4694 | 8119 | 2999 | 1921 | 383 | 779 | 3788 | 1167 | 2542 | 1297 | 7085 |
| 21.0 | 5192 | 2560 | 4315 | 1246 | 683 | 175 | 179 | 885 | 501 | 1944 | 1472 | 3990 |
| 21.5 | 2433 | 1293 | 1804 | 242 | 172 | 0 | 193 | 79 | 616 | 234 | 416 | 2133 |
| 22.0 | 806 | 568 | 889 | 156 | 121 | 0 | 23 | 162 | 132 | 0 | 570 | 195 |
| 22.5 | 358 | 37 | 202 | 144 | 33 | 0 | 97 | 0 | 186 | 0 | 0 | 0 |
| 23.0 | 124 | 3 | 28 | 0 | 12 | 0 | 670 | 0 | 0 | 0 | 0 | 65 |

Table B. 3 (continued).

| Length Class ( $\mathrm{L}_{\mathrm{c}}$ in cm) | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Tonnage to mid-Nov | 3216 | 4461 | 1890 | 739 | 8230 | 15231 | 3829 | 7526 | 1683 | 699 | 4247 |
| Tonnage | 36346 | 56703 | 56815 | 34941 | 40171 | 39444 | 37814 | 53650 | 66651 | 36989 | 89152 |
| 3.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4.0 | 0 | 275 | 0 | 4 | 196 | 0 | 0 | 74 | 0 | 0 | 257 |
| 4.5 | 1192 | 688 | 0 | 1523 | 3321 | 0 | 134 | 496 | 0 | 0 | 3376 |
| 5.0 | 11450 | 321 | 4 | 4438 | 14270 | 6207 | 6030 | 6577 | 712 | 378 | 18651 |
| 5.5 | 35039 | 125 | 11 | 19670 | 183961 | 33083 | 5678 | 41347 | 100 | 1087 | 99617 |
| 6.0 | 63035 | 6432 | 14 | 24780 | 26944 | 82217 | 8543 | 127469 | 315 | 6098 | 102628 |
| 6.5 | 84150 | 37148 | 547 | 40858 | 36528 | 108793 | 23529 | 183168 | 4417 | 16971 | 84844 |
| 7.0 | 78973 | 43545 | 2413 | 54890 | 40369 | 125373 | 40999 | 218400 | 11669 | 23122 | 224061 |
| 7.5 | 26290 | 41098 | 6409 | 48228 | 49446 | 167992 | 67744 | 172644 | 18771 | 19210 | 167132 |
| 8.0 | 29705 | 6405 | 5258 | 29945 | 56747 | 125025 | 68441 | 267471 | 11324 | 13098 | 111732 |
| 8.5 | 38184 | 30105 | 7231 | 23683 | 47262 | 88430 | 75416 | 235538 | 18666 | 6450 | 34107 |
| 9.0 | 24748 | 7949 | 6412 | 31420 | 30321 | 55699 | 57736 | 86507 | 6597 | 11662 | 27878 |
| 9.5 | 8632 | 26369 | 4196 | 40279 | 14053 | 82027 | 35880 | 30578 | 7562 | 6664 | 13358 |
| 10.0 | 5364 | 27965 | 16714 | 12369 | 5434 | 116156 | 15237 | 20868 | 8279 | 3714 | 10293 |
| 10.5 | 6427 | 23672 | 646 | 11671 | 4837 | 130120 | 9070 | 34572 | 2280 | 7932 | 7691 |
| 11.0 | 16943 | 9648 | 668 | 19239 | 5773 | 122409 | 7029 | 49602 | 2621 | 5503 | 6071 |
| 11.5 | 7215 | 9781 | 384 | 39320 | 12406 | 92964 | 6470 | 29637 | 1294 | 13035 | 10353 |
| 12.0 | 7965 | 27725 | 226 | 57984 | 15463 | 60157 | 4783 | 89153 | 6305 | 6364 | 6521 |
| 12.5 | 6192 | 20581 | 1106 | 58749 | 24343 | 65392 | 5530 | 89402 | 7637 | 12429 | 1667 |
| 13.0 | 14955 | 27998 | 9166 | 40492 | 55310 | 60594 | 8973 | 54335 | 46794 | 15098 | 5018 |
| 13.5 | 10318 | 61858 | 20629 | 25414 | 77530 | 73402 | 31605 | 48990 | 85095 | 5038 | 9613 |
| 14.0 | 26121 | 93757 | 35362 | 21438 | 100981 | 118835 | 71706 | 33521 | 140802 | 5940 | 19121 |
| 14.5 | 22166 | 113375 | 47090 | 19203 | 78765 | 125703 | 124276 | 73678 | 170200 | 17159 | 52565 |
| 15.0 | 36337 | 114009 | 99863 | 28821 | 75099 | 94089 | 105847 | 90036 | 204847 | 29836 | 85427 |
| 15.5 | 62425 | 81671 | 202682 | 35635 | 70288 | 60513 | 89128 | 95659 | 214324 | 46223 | 116385 |
| 16.0 | 89618 | 103865 | 253809 | 53999 | 73909 | 53355 | 74529 | 107805 | 187902 | 79515 | 170832 |
| 16.5 | 135751 | 112680 | 199878 | 73355 | 77908 | 37704 | 74136 | 90062 | 135926 | 94850 | 214739 |
| 17.0 | 116358 | 127691 | 122076 | 73376 | 66474 | 28054 | 64269 | 76487 | 96997 | 128300 | 241224 |
| 17.5 | 56383 | 112969 | 73704 | 60171 | 55146 | 15520 | 41053 | 66708 | 65951 | 88425 | 211322 |
| 18.0 | 26062 | 78831 | 37771 | 52317 | 27437 | 9804 | 25769 | 51371 | 43888 | 67805 | 179034 |
| 18.5 | 11273 | 37670 | 23546 | 35419 | 22215 | 7510 | 18671 | 35062 | 26616 | 28851 | 108929 |
| 19.0 | 4232 | 15546 | 9521 | 18854 | 13175 | 5020 | 11949 | 24049 | 15292 | 18296 | 64755 |
| 19.5 | 792 | 8283 | 6436 | 11327 | 5865 | 2615 | 11102 | 11869 | 8729 | 11344 | 32209 |
| 20.0 | 1067 | 2763 | 2095 | 4995 | 5780 | 2083 | 6318 | 6340 | 4847 | 2706 | 15368 |
| 20.5 | 414 | 607 | 418 | 2133 | 745 | 475 | 2794 | 2880 | 2181 | 1872 | 8788 |
| 21.0 | 203 | 93 | 515 | 1822 | 72 | 367 | 1216 | 2157 | 972 | 1739 | 5690 |
| 21.5 | 0 | 96 | 164 | 439 | 650 | 3 | 667 | 1647 | 315 | 294 | 1462 |
| 22.0 | 174 | 40 | 0 | 117 | 0 | 103 | 0 | 694 | 88 | 0 | 782 |
| 22.5 | 0 | 0 | 0 | 67 | 0 | 0 | 0 | 350 | 0 | 0 | 183 |
| 23.0 | 0 | 0 | 0 | 33 | 0 | 0 | 0 | 170 | 0 | 0 | 31 |

APPENDIX C: Calculating the bias in estimates of round herring abundance from the November and
May hydro-acoustic surveys

Probability density functions (pdfs) for the overall biases in the November and May surveys, $k_{N}^{R H}$ and $k_{r}^{R H}$, were calculated by drawing ten thousand samples from the individual pdfs for each source of constant error, together with the median values of the individual pdfs of each source of variable and random error (see Tables C. 1 and C. 2 with reasons given by Janet Coetzee in the Annex). Pdfs of the inter-transect variance, ( $\left.\lambda_{N}^{R H}\right)^{2}$ and $\left(\lambda_{r}^{\text {RH }}\right)^{2}$, were then calculated by drawing ten thousand samples from the individual pdfs for each source of variable and random error. The resultant pdfs on the model predicted biomass (i.e. the inverse of the pdfs calculated using the errors provided), together with normal distributions fitted to these pdfs are given in Figures C. 1 to C. 4 .

Table C.1. Individual error factors for November hydro-acoustic surveys of round herring biomass, where the values define trapezium form pdfs. Note that these error factors apply to the observed biomass, i.e. they reflect the inverse of the multiplicative bias (applied to predicted biomass) in this document.

| Error | Minimum | Likely <br> (lower) | Likely <br> (midpoint) | Likely <br> (upper) | Maximum | Nature |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Target Strength | 0.50 | 1.10 | 1.40 | 1.70 | 2.00 | Constant |
| Depth dependence <br> on target strength | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Variable |
| Calibration <br> (On-axis sensitivity) <br> (Beam factor) | 0.90 | 0.95 | 1.00 | 1.05 | 1.10 | Random $^{6}$ |
| Attenuation | 1.00 | 1.05 | 1.075 | 1.10 | 1.15 | Variable |
| Target Identification | 0.50 | 0.80 | 1.00 | 1.20 | 1.50 | Random |
| Weather Effects | 1.00 | 1.10 | 1.20 | 1.30 | 1.50 | Variable |

[^6]Table C.2. Individual error factors for May hydro-acoustic surveys of round herring recruitment, where the values define trapezium form pdfs. Note that these error factors apply to the observed recruitment, i.e. they reflect the inverse of the multiplicative bias (applied to predicted recruitment) in this document.

| Error | Minimum | Likely <br> (lower) | Likely <br> (midpoint) | Likely <br> (upper) | Maximum | Nature |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Target Strength | 0.50 | 1.10 | 1.40 | 1.70 | 2.00 | Constant |
| Depth dependence <br> on target strength | 1.00 | 1.125 | 1.25 | 1.375 | 1.50 | Variable |
| Calibration |  | 0.90 | 0.95 | 1.00 | 1.05 | 1.10 |
| (On-axis sensitivity) | 0.75 | 0.90 | 1.00 | 1.10 | 1.25 | Random |
| (Beam factor) | 1.00 | 1.025 | 1.05 | 1.075 | 1.10 | Variable |
| Attenuation | 0.60 | 0.85 | 1.00 | 1.15 | 1.40 | Random |
| Target Identification | 1.00 | 1.005 | 1.10 | 1.15 | 1.50 | Variable |
| Weather Effects |  |  |  |  |  |  |

Multiplicative bias in the estimate of round herring abundance from the November survey


Figure C.1. The probability density function for the overall bias in the estimate of round herring abundance from the November survey, calculated by drawing 10000 samples from the individual probability distribution functions for each source of constant error, together with the median values of the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $k_{N}^{R H} \sim N\left(0.3404,0.083^{2}\right)$.


Figure C.2. The probability density function for the additional standard deviation in the estimate of round herring abundance from the November survey, calculated by drawing 10000 samples from the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $\lambda_{N}^{R H} \sim N\left(0.2763,0.954^{2}\right)$.

## Multiplicative bias in the estimate of round herring recruitment from the May survey



Figure C.3. The probability density function for the overall bias in the estimate of round herring recruitment from the May survey, calculated by drawing 10000 samples from the individual probability distribution functions for each source of constant error, together with the median values of the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $k_{r}^{R H} \sim N\left(0.4269,0.103^{2}\right)$.

Additional standard deviation in the estimate of round herring recruitment from the May survey


Figure C.4. The probability density function for the additional standard deviation in the estimate of round herring recruitment from the May survey, calculated by drawing 10000 samples from the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $\lambda_{r}^{R H} \sim N\left(0.2284,0.966^{2}\right)$.

# ANNEX: Estimating the likely ranges of individual error factors of round herring abundance from the November and May hydro-acoustic surveys 

## Janet Coetzee

A BENEFIT workshop held in December 2000 summarised the most likely sources of error relevant to acoustic estimates of fish biomass (Anon 2000) and estimated their likely ranges based on expert opinion and available data. Although these error factors pertained mainly to surveys of anchovy and sardine, they are also pertinent to acoustic surveys of round herring although the effect of the individual errors may differ. Consequently, an initial attempt has been made to update the parameter estimates that define the likely, and minimum and maximum ranges of these errors for round herring. Where necessary, new errors factors have been added and their effects estimated based on available knowledge. It is likely that these may be improved as more data become available. Rationale for the derivation of parameters describing each error factor is provided below and should be read in conjunction with those published in the Survey errors workshop report.

## November surveys

- Target Strength

The TS of round herring is unknown and no published data on round herring TS exists. Currently the target strength of round herring is assumed to be the same as that currently used for similar sized sardine (Barange et al. 1996). A general published TS for clupeoids (Foote 1987) would result in a biomass that is 1.38 times higher. Preliminary unpublished data suggests that the TS of round herring should be higher than that of anchovy, but lower than that of sardine. Some recent published TS data for anchovy (Sawada et al. 2009), however, suggests that the TS of anchovy is much higher than previously thought, and we have therefore opted for a relatively high minimum of 0.5 . It is unlikely that the maximum error associated with TS can be higher than 2 , given the generally similar size and morphology of round herring and sardine and similar acoustic signature at high density. A study is currently being initiated to estimate the TS of round herring from in-situ data and it is likely that the effect of this bias on estimate of round herring biomass will be more accurately determined in the near future.

- Depth dependence on Target Strength

This error was not considered important for anchovy and sardine at the time that the Survey errors workshop was held and is therefore an additional error that has been considered important in the context of round herring biomass estimation. Published findings for herring (Ona 2003) suggests a strong depth dependence on target strength, with halving of TS between the surface and a depth of 200 m with the steepest decrease in TS in the first few (upper 50) meters of the water column. Round herring are close to the surface at night only, migrating to deeper water before dawn and staying close to the bottom during the day. At dusk they again migrate up in the water column. Additionally, our surveys are conducted by day and night, so round herring should be deep for approximately $50 \%$ of the time and the max error (factor of 2 ) should therefore be applicable for half of the acoustic intervals only. But, during migration (up and down in the water column) the tilt angle will be
substantially increased and lead to a reduction in TS. For this reason, we have opted to use the max error range and not only applied it for half the time.

## - Calibration

Calibration errors are likely to be similar to those of anchovy and sardine and have therefore not been changed from those agreed on at the Survey errors workshop.

- Attenuation

The effect of attenuation on round herring estimates has not been determined. This error factor is, however, most likely substantially less than that of denser schooling sardine (which averages at around 1.15 for November surveys). We have therefore opted to use 1.15 for the maximum error and 1 (cannot be less) for the minimum error and a symmetrical distribution around the likely value of 1.075. It is possible to estimate the effect of attenuation on round herring biomass estimates using a similar method to that used for quantifying attenuation effects in dense schools of sardine.

- Target Identification

The same parameters estimated for the minimum and maximum Target Identification error for anchovy have been applied, but the likely range has been increased. This is to account for larger overestimation of round herring (when the assumption is made that deep targets during the day are most likely to be round herring, but could possibly include horse mackerel). Conversely, diving behaviour at dawn may result in (larger relative to other pelagic species) under-sampling of round herring in some trawls, and consequent underestimation of biomass.

- Weather effects

Weather effects are likely to play a larger role when fish are deeper (vessel pitch and roll effects are amplified at depth) and as such we have opted for a slightly wider likely range compared to that for anchovy, but have suggested that the maximum effect is the same. Again it is likely that the effect of this error may be more accurately estimated in the future.

## May surveys

The rationale for the derivation of parameters describing error factors that differ in range from that applicable to November surveys, is provided below; these should again be read in conjunction with those published in the aforementioned Survey errors workshop report:

- Depth dependence on Target Strength

Juvenile round herring tend to be distributed closer inshore during the recruit survey than adults during the November survey and therefore the maximum error is likely to be lower. However, given that the largest reduction in TS occurs within the first 50 m of the water column, it is still considered to be an important source
of negative bias. Further analyses on the mean depth of round herring recruits during the May survey may improve estimates of the likely effect.

- Attenuation

As for sardine recruits, attenuation biases are likely to be smaller than those applicable to denser schooling adults. No information on the likely reduction in this effect for round herring recruits is available, but we have assumed that the maximum error is $10 \%$ and that the distribution of the error is symmetrical around the likely midpoint of $5 \%$.

- Target Identification

The range of this error should be smaller than that for November surveys, given the closer inshore distribution and smaller overlap between round herring recruits and adult horse mackerel. Similarly the under-sampling of round herring during trawling is likely to be less than that for adults because slower swimming juvenile round herring are less likely to avoid capture than adults. The maximum and minimum ranges have therefore been reduced, although the distribution is still centred on 1 (equal chance of under- or over estimation).

- Weather effects

The maximum range for this error is assumed to be similar for May and November surveys, although the likely range has been halved to account for the more inshore distribution of recruits relative to adults, and consequent reduction in mean depth distribution during the day.

## Annex References:

Anon 2000 - Survey Errors Workshop. Benguela Environment and Fisheries Interaction and Training programme report. 4-7 December, Breakwater Lodge, Cape Town.
Barange M, Hampton I, Soule M 1996 - Empirical determination of in situ target strengths of three loosely aggregated pelagic fish species. ICES Journal of Marine Science 53:225-232
Foote KG 1987 - Fish target strengths for use in echo integrator surveys. Journal of the Acoustical Society of America. 82:981-987.

Ona E 2003 - An expanded target-strength relationship for herring. ICES Journal of Marine Science 60:493499.

K Sawada, H Takahashi, K Abe, T Ichii, K Watanabe, Y Takao 2009-Target-strength, length, and tilt-angle measurements of Pacific saury (Cololabis saira) and Japanese anchovy (Engraulis japonicus) using an acoustic-optical system. ICES Journal of Marine Science: 66:1212-1218.


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[^1]:    ${ }^{1}$ It is assumed that the current year's March sub-cohort would be too small to be picked up in the survey.

[^2]:    ${ }^{2}$ Age 6 is used here to account for the greater average age of the plus group.

[^3]:    ${ }^{3}$ Age 6 is used here to account for the greater average age of the plus group.

[^4]:    ${ }^{4}$ Age 6 is used here to account for the greater average age of the plus group.

[^5]:    ${ }^{5}$ This includes an estimate of all bias associated with the survey, including the bias introduced due to the use of a target strength for a species other than round herring (see Appendix C).

[^6]:    ${ }^{6}$ Note that for the purposes of this simulation, 'random' and 'variable' factors are treated in the same manner.

