Updated Results from a Bayesian Analysis of the Squid Resource *Loligo reynaudii*

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Introduction

A Bayesian assessment of the squid resource to take full account of model parameter value uncertainty was first undertaken in 2005. At that time the data were not informative enough to allow for precise estimation of all parameters, hindering convergence of the MCMC chain, and as a result the stock recruitment steepness parameter, *h*, was fixed. Twelve models were therefore considered, each for a different fixed value of *h* (ranging from 0.4 to 0.95 in units of 0.05), and results were then integrated over the models using Deviance Information Criterion (DIC) weighting.

For the purposes of in-season advice regarding the additional closed season for 2010, an updated analysis was conducted for one of the 12 models, namely that which assumed h=0.7. This model incorporated updated data where certain series e.g. jig catches, had been revised based on more reliable data sources. In addition, further years' data were also available (the previous analysis included data to 2002; the updated analysis includes data to 2008). The results indicated that there has been above-average recruitment in recent years and that the catch-effort curve derived from projections reflects a markedly more optimistic appraisal compared to that derived from the analysis of the past data.

A further advancement on the most recently reported results has been to allow for the estimation of *h* given that 6 more years of data are now included in the assessment. The adoption of such a model would remove the need to integrate results over twelve separate models. The results from this model are presented here.

The model

The model specifications are provided in Appendix A. The following prior distributions have been selected for the estimable parameters:

$\ell n X \sim U(0\ ;\ 99.726)$, where initial recruitment, $R_0 = \exp(\ell n X)$. The upper bound for $\ell n X$ is ten times the joint posterior mode value for $\ell n X$.
$h \sim U(0.5\ ;\ 1.0)$ and there is also a penalty function on h of the form $(h\text{-}0.499)/(0.001+h\text{-}0.499)$.
$\eta \sim U(0.01\ ;\ 0.99)$
$g \sim N(1.2\ ;\ 0.1^2)$
Stock-recruitment residuals $\xi_y \sim N(0\ ;\ \sigma^2_R)$

Model convergence proved problematic for values of *h* below 0.5, even though these were marginally preferred by the data. However, since values of *h* below 0.5 are rarely found in fish populations, it was decided to place an effective lower bound of 0.5 on *h.* The penalty function added to adjust the uniform prior on *h* values above 0.5 is to preclude a maximum likelihood estimate exactly on the boundary which leads to problems in calculating a Hessian and hence initiating MCMC in ADMB.

Results

For the Bayesian posterior computations a MCMC chain of 600 million samples was run. A burn-in of 150 million was discarded and the remaining chain was thinned by selecting one in every 5000 samples to reduce autocorrelation. 5000 samples selected randomly, with replacement from the chain were used to perform stochastic projections 10 years into the future under various constant effort scenarios. The assumptions made relating to effort in the projections are as follows:

- The proportion of annual jig effort expended in each period is equivalent to the average observed over the last 3 years for which data are available, which is 0.32:0.68 for the Jan-Mar and Apr-Dec periods.
- Future trawl effort is constant and is equivalent to the average standardized effort in the trawl fishery over the last 5 years for which data are available.
- The proportion of annual trawl effort expended in each period is equivalent to the average observed over the last 5 years for which data are available, and is 0.19:0.81 for Jan-Mar:Apr-Dec.

The parameter estimates at the joint posterior mode are shown in Table 1. The begin-year biomass time series (B_y^*) is shown in Figure 1 and the stock-recruitment residuals are shown in Figure 2, indicating that in recent years there has been above-average recruitment. The fit to the stock-recruitment relationship is shown in Figure 3. Also shown in Figure 3 is the replacement line; this reflects an exact balance between additions from recruitment and losses to mortality, and intersects the stock-recruitment curve at *K* in the absence of fishing mortality.

The diagnostics from the tests of Geweke (1992), Raftery and Lewis (1992) and Heidelberger and Welch (1983) were monitored for instances of non-convergence in the MCMC (these tests are used to show when convergence has not occurred rather than to prove that convergence to the posterior mode has occurred (Gamerman, 1997)). The diagnostics related to burn-in and thinning indicated that both were sufficient. Eight of the 38 stock-recruitment residuals failed the Heidelberger and Welch (1983) half-width test, indicating that a longer chain is ideally required.

The median average annual catch, together with 90% probability intervals obtained from the projections is shown in Figure 4, and is compared with that obtained from the *h*=0.7 model used to advise on the additional closed season for 2010.

Given that the results for the model that allows for the estimation of all parameters are similar to those for the *h*=0.7 model used to advise on the additional closed season (Figure 4), and that previously the results for *h*=0.7 utilizing the old data were similar to those when integrating over all twelve models (Figure 5), it is suggested that the model that allows for the estimation of all parameters is adopted for assessing the squid resource as a basis for providing management advice at this time.

References

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Table 1: Parameter estimates at the joint posterior mode.

Model parameters	Estimate
R_{1971}	24629
B^*_{1971}	34412
B^*_{2009}	16115
B^*_{2009}/B^*_{1971}	0.468
h	0.512
eta	0.350
g	1.258
stock-recruit residuals	
sigmaR (input)	0.3
sigmaR (estimated)	0.23
CPUE jig Jan-Mar	
q	0.002833
sigma	0.300
CPUE jig Apr-Dec	
q	0.001571
sigma	0.226
CPUE trawl Jan-Mar	
q	0.000551
sigma	0.245
CPUE trawl Apr-Dec	
q	0.000135
sigma	0.249
Survey Autumn	
q	1.15744
sigma	0.436
Survey spring	
q	0.67964
sigma	0.278
-lnL values	
jig A-D	-6.394
trawl J-M	-7.059
Trawl A-D	-6.430
autumn	6.281
spring	-1.223
S/R residuals	0.782
penalties	-1.143
total	-15.185

Figure 1: Begin-year biomass (B_y^*) time series.

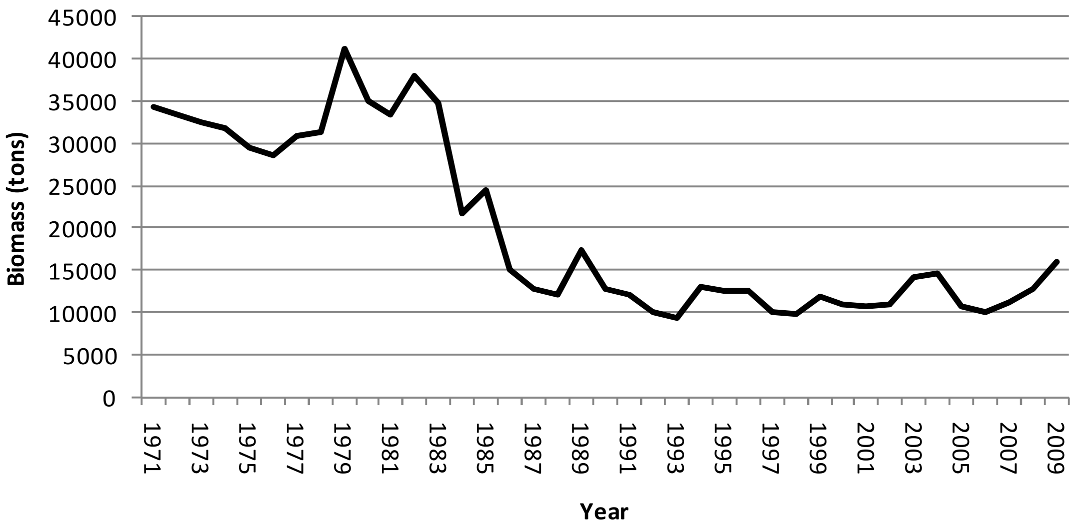


Figure 2: Estimated stock-recruitment residuals.

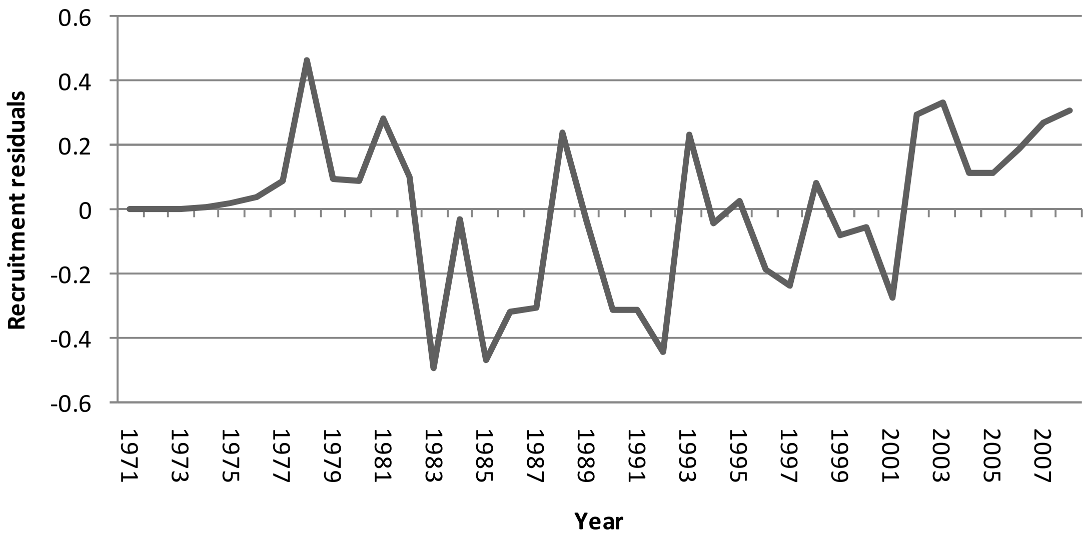


Figure 3: Model predicted stock-recruitment relationship and associated replacement line. The data points shown are the posterior mode estimates from the stock recruitment values each year, and the straight line through the origin is the replacement line.

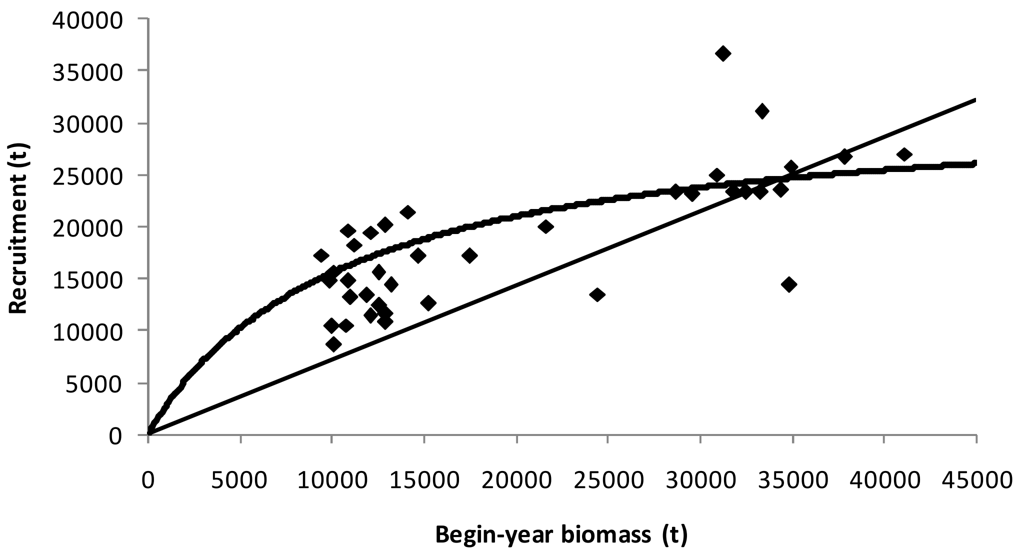


Figure 4: Median average annual catch (tons) for fixed levels of future effort, expressed in terms of man-days, derived from i) the model that estimates *h* and ii) the model where *h* was fixed at 0.7. The 90% probability intervals are also shown. The arrow indicates the current level of effort (300 000 man-days).

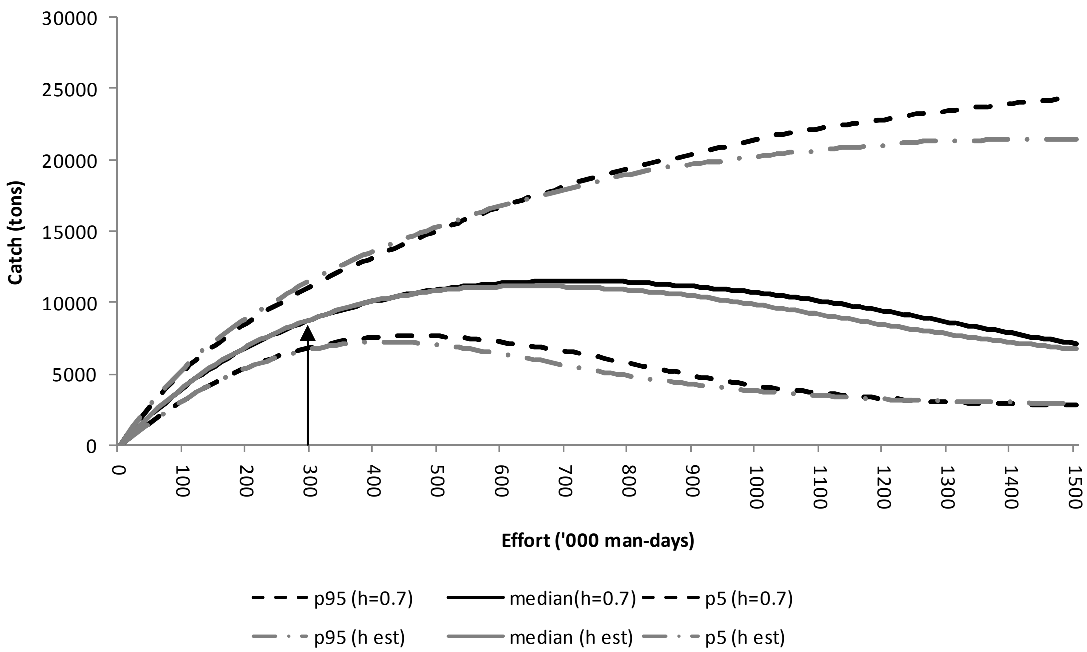
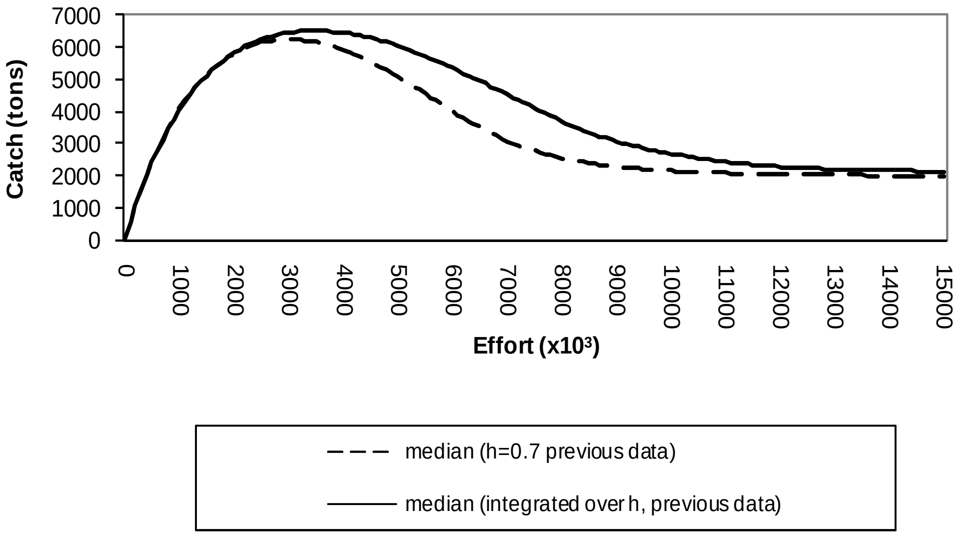


Figure 5: Comparison of median average annual catch (tons) for the previous models (utilizing data to 2002, prior to data updates and revisions) where results were integrated over *h* (ranging from 0.4-0.95) and the model for *h*=0.7 (utilizing the same data). Units of effort are in terms of man-hours (10 man-hours=1 man-day).



APPENDIX A: The biomass dynamics model specifications and projection-related catch equations and rules

The population model splits a year into two time periods, January-March and April-December, to better reflect the dynamics of the stock and the two fisheries (jig and trawl) that exploit it. Hardly any recruitment takes place in the January – March period, and jig and trawl catches are disproportionately divided between this and the April-December period (Roel and Butterworth, 2000). The biomass time series is estimated by projecting the assumed pristine biomass at the start of the period B_0^* (= B_{1971}^* = *K*) forward given the historic annual catches.

The biomass dynamics for the two periods are given by:

$$B_y = B_y^* e^{-g/4} - C_y^{jig\ J-M} - C_y^{trawl\ J-M} \qquad \text{A.1}$$

$$B_{y+1}^* = B_y e^{-3g/4} + R_y - C_y^{jig\ A-D} - C_y^{trawl\ A-D} \qquad \text{A.2}$$

where B_y^* is the biomass in year *y* at the start of January,

B_y is the biomass in year *y* at the start of April,

$C_y^{jig\ J-M}$ is the jig catch taken in year *y* between January and March,

$C_y^{jig\ A-D}$ is the jig catch taken in year *y* between April and December,

$C_y^{trawl\ J-M}$ is the trawl catch taken in year *y* between January and March,

$C_y^{trawl\ A-D}$ is the trawl catch taken in year *y* between April and December, and

g is a composite parameter that accounts for natural mortality, emigration and growth.

R_y is the recruitment in year *y*:

$$R_y = \frac{\alpha\beta_y^*(1-\eta F_{y-1}^{jig})}{\beta + B_y^*} e^{(\xi_y - \frac{\sigma_R^2}{2})} \qquad \text{A.3}$$

where:

$$F_y^{jig} = \frac{C_y^{jig\ A-D}}{B_y e^{-3g/4} + R_y} \qquad \text{A.4}$$

η is an estimable parameter and controls the extent to which recruitment is affected by jig fishing mortality. ξ_y is the process error reflecting fluctuation about the expected recruitment for year *y*, drawn from $N(0,\sigma_R^2)$. These residuals are treated as estimable parameters in the model fitting process (σ_R is assumed to be 0.3 on input). The estimated residuals may be used to calculate an estimated $\hat{\sigma}_R = \sqrt{\frac{1}{n}\sum_y \xi_y^2}$ on output. The $\frac{\sigma_R^2}{2}$ term is to correct for bias given the skewness of the log-normal distribution.

α and β are stock-recruit relationship parameters. In order to work with estimable parameters that are more meaningful biologically, the stock-recruit relationship is re-parameterized in terms of pre-exploitation equilibrium biomass, *K*, and the "steepness", *h*, of the stock-recruitment relationship ("steepness" being the fraction of pristine recruitment that results when biomass drops to 20% of its pristine level):

$$hR_0 = R(0.2K) \qquad \text{A.5}$$

from which it follows that:

$$h = \frac{0.2(\beta + K)}{\beta + 0.2K} \qquad \text{A.6}$$

and hence:

$$\alpha = \frac{4hR_0}{5h-1} \qquad \text{A.7}$$

and

$$\beta = \frac{K(1-h)}{5h-1} \qquad \text{A.8}$$

The likelihood is calculated assuming that the abundance indices are log-normally distributed about their expected values:

$$I_y^i = \hat{I}_y^i e^{\varepsilon_y^i} \quad \text{or} \quad \varepsilon_y^i = \ell n(I_y^i) - \ell n(\hat{I}_y^i) \qquad \text{A.9}$$

where

I_y^i is the abundance index for year *y* and series *i*, $\hat{I}_y^i = \hat{q}^i \overline{B}_y$ is the corresponding model estimate ($\hat{q}^i$ being the catchability coefficient corresponding to series *i* and $\overline{B}_y$ the average biomass during a given period in year *y*), and ε_y^i is the observation error corresponding to series *i* in year *y*.

For the January-March trawl index,

$$\overline{B}_y = \frac{B_y^* + B_y^* e^{-g/4} - C_y^{jig\ J-M} - C_y^{trawl\ J-M}}{2} \qquad \text{A.10}$$

For the April-December jig and trawl indices,

$$\overline{B}_y = \frac{B_y + R_y + B_{y+1}^*}{2} \qquad \text{A.11}$$

For the autumn survey biomass index,

$$\overline{B}_y = B_y + 0.5R_y \qquad \text{A.12}$$

For the spring survey biomass index

$$\overline{B}_y = B_y + R_y \qquad \text{A.13}$$

The contribution of each abundance index to the negative log-likelihood function (after the removal of constants) is given by:

$$-\ell n L_i = n\,\ell n \sigma^{*i} + \frac{1}{2(\sigma^{*i})^2} \sum_{y=1}^{n_i} (\varepsilon_y^i)^2 \qquad \text{A.14}$$

where $\hat{\sigma}^{*i} = \sqrt{(\hat{\sigma}^i)^2 + C^2}$ A.15

$$\hat{\sigma}^i = \sqrt{\frac{1}{n_i} \sum_y (\varepsilon_y^i)^2} \qquad \text{A.16}$$

and *C*=0.2. The introduction of the *C* factor is to ensure that no abundance index receives unrealistically high weight in the fitting process.

The contribution of the stock-recruitment residuals to the negative log-likelihood function is given by:

$$-\ell nL = \sum_{y}[\ell n\sigma_{R} + \frac{1}{2\sigma_{R}^{2}}\xi_{y}^{2}] \qquad \text{A.17}$$

This is a penalty term, being the equivalent in a frequentist framework of what would reflect a normal prior in a Bayesian context.

The derivation of future catches given variability about the catch-effort relationship

The catch-effort relationship $(\frac{C}{E}) = q\overline{B}e^{\varepsilon}$, may be re-arranged to yield $C = qE\overline{B}e^{\varepsilon}$. Substituting equation A.10 for $\overline{B}$ will yield the future catches made in the January-March period for the trawl and jig fisheries respectively. Ignoring the *y* subscripts, these are thus:

$$C^{trawl,J-M} = \frac{q_{trawl,J-M}E_{trawl,J-M}e^{\xi^{trawl,J-M}}B^{*}(1+e^{\frac{-g}{4}})}{(2+q_{jig,J-M}E_{jig,J-M}e^{\xi^{jig,J-M}}+q_{trawl,J-M}E_{trawl,J-M}e^{\xi^{trawl,J-M}})} \qquad \text{A.18}$$

$$C^{jig,J-M} = \frac{q_{jig,J-M}E_{jig,J-M}e^{\xi^{jig,J-M}}B^{*}(1+e^{\frac{-g}{4}})}{(2+q_{jig,J-M}E_{jig,J-M}e^{\xi^{jig,J-M}}+q_{trawl,J-M}E_{trawl,J-M}e^{\xi^{trawl,J-M}})} \qquad \text{A.19}$$

Similarly, for the second period (April-December), substituting equation A.11 for $\overline{B}$ will yield the future catches made in the trawl and jig fisheries respectively:

$$C^{trawl,A-D} = \frac{q_{trawl,A-D}E_{trawl,A-D}e^{\varepsilon_{trawl,A-D}}\{B(1+e^{\frac{-3g}{4}})+2R\}}{(2+q_{jig,A-D}E_{jig,A-D}e^{\varepsilon_{jig,A-Dy}}+q_{trawl,A-D}E_{trawl,A-D}e^{\varepsilon_{trawl,A-D}})} \qquad \text{A.20}$$

$$C^{jig,A-D} = \frac{q_{jig,A-D}E_{jig,A-D}e^{\varepsilon_{jig,A-D}}\{B(1+e^{\frac{-3g}{4}})+2R\}}{(2+q_{jig,A-D}E_{jig,A-D}e^{\varepsilon_{jig,A-Dy}}+q_{trawl,A-D}E_{trawl,A-D}e^{\varepsilon_{trawl,A-D}})} \qquad \text{A.21}$$

$\varepsilon_i \sim N(0,(\hat{\sigma}^{*i})^2)$, *i* denoting each index of abundance.

Rules for projections

If the estimated biomass in the second period was less than $0.05(B^* \times e^{\frac{-g}{4}})$ then the first period catches were set to $0.95p(B^* \times e^{\frac{-g}{4}})$ and the second period biomass to $0.05(B^* \times e^{\frac{-g}{4}})$. Similarly, if the estimated biomass in the first period of the following year was less than $0.05(B \times e^{\frac{-3g}{4}} + R)$ then the second period catches from the previous year were set to $0.95p(B \times e^{\frac{-3g}{4}} + R)$ and the first period biomass to $0.05(B \times e^{\frac{-3g}{4}} + R)$. *p* apportions the catches in the correct ratio for each period and each fishing type.