# A preliminary evaluation of the potential use of pelagic survey data in setting the horse mackerel PUCL 

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#### Abstract

Summary The existing population model for the horse-mackerel resource is extended to take commercial catch-at-length information from the mid-water trawl fishery and demersal surveys into account to allow the estimation of recruitment variations, which are correlated with the results for horse-mackerel abundance (considered to primarily reflect recruits) from the November pelagic surveys. Correlation is better with results for the West coast only from these surveys, rather than with those for the assessment area as a whole. At this stage the conclusion is either that these November survey estimates are (for whatever reason) a relatively weak predictor of incoming horse-mackerel recruitment strength, or that there are problems with the modelling of the demersal/midwater data which require further attention.


## 1. Introduction

The South African horse mackerel (Trachurus trachurus capensis) fishery consists of a demersal/midwater fishery concentrated on the South coast and a pelagic purse-seine fishery concentrated on the West coast. Adult horse mackerel are taken as by-catch by the demersal trawl fleet and as a targeted catch by the midwater trawl fleet. Juvenile horse mackerel are taken as by-catch by the pelagic purse-seine fleet. Since 2000, a Precautionary Upper Catch Limit (PUCL) for juvenile horse mackerel of 5000t has been in place for the pelagic purse-seine fishery.

The November 2010 pelagic acoustic survey biomass estimate indicated a substantial increase in horse mackerel on the West coast. Subsequently, in the current 2011 fishing season, large by-catches of juvenile horse mackerel have become problematic for industry. Therefore, in March 2011, the Demersal Scientific Working Group (DSWG) agreed to an ad
hoc increase of 5000t to the PUCL for the current season. This was considered a once-off adjustment to sustain industry pending further analyses of horse mackerel data.

This paper is a preliminary evaluation of the November pelagic survey biomass index as a predictor of juvenile abundance and hence its potential usefulness for adjusting the PUCL.

## 2. Method

An age-structured production model (ASPM) is used to model the South African horse mackerel fishery. The model assumes one combined stock (West coast plus South coast). For the most part it is unchanged from the 2007 assessment model (Johnston and Butterworth, 2007). Key differences are that:

- updated catch and survey biomass series are incorporated;
- length-frequency data from demersal surveys on the South coast are incorporated;
- fluctuations about expected recruitment are estimated for 1983-2008; and
- parameters for the demersal selectivity function, which decreases for ages greater than 5 years, are estimated.

The ASPM and its associated likelihood function components are described in full in Appendix A.

Once fitted to data, the model's recruitment estimates are compared to and correlated with November acoustic survey biomass estimates for both the West coast and the entire assessment area. The West coast is considered separately as it has a high proportion of juvenile horse mackerel.

## 3. Input data and model assumptions

### 3.1 Historical catch

The historical catch records for both the demersal (strictly demersal and midwater) and pelagic fisheries for 1949-2009 are reported in Table 1.

### 3.1 Demersal survey biomass

Biomass estimates and their associated CVs based on the autumn and spring demersal surveys are reported in Table 2.

### 3.2 Demersal survey length-frequencies

Demersal survey length-frequency data were provided by Fairweather (pers. commn).

### 3.3 Pelagic survey biomass

Model estimated recruitments for each year are correlated with biomass estimates from the November pelagic acoustic survey of the previous year for both the West Coast and the entire assessment area. The survey biomass estimates are reported in Table 3.

### 3.4 Demersal selectivity

For the 2007 assessment, selectivity corresponding to the demersal fleet was not estimated, but input based on length-frequency distributions. The addition of length-frequency data into this model makes it possible to estimate demersal selectivity. Experimentation shows a function of the form used in the 2007 assessment (increasing linearly to $S_{a}=1$ ) to provide a good fit for the data. The data also indicates that selectivity decreases for large horse mackerel. Therefore, a selectivity function of the following form is used:

$$
S_{a}^{d}= \begin{cases}a / a_{k i n k} & a<a_{\text {kink }} \\ 1 & a_{\text {kink }}<a \leq 5 \\ e^{-\mu(a-5)} & a>5\end{cases}
$$

where
$a_{k i n k} \quad$ is the age at which selectivity plateaus; and
$\mu \quad$ reflects the rate at which selectivity of the demersal fleet decreases for mackerel older than 5 years.

Both the $a_{\text {kink }}$ and $\mu$ parameters are estimated when fitting the model.

### 3.5 Recruitment fluctuations

It is assumed that recruitment fluctuates about its expected values for the years 1983-2008. Estimation of these fluctuations is possible because of the availability of length-frequency data for the years in question.

### 3.6 Model variants

As was the case for the 2007 assessment, four model variants are considered corresponding to four combinations of values for the "steepness" of the stock-recruitment curve, $h$, and the catchability coefficient of the autumn demersal survey, $q_{2}$ :

- Model 1: $q_{2}=0.5 ; h=0.6$
- Model 2: $q_{2}=1.0 ; h=0.6$
- Model 3: $q_{2}=0.5 ; h=0.9$
- Model 4: $q_{2}=1.0 ; h=0.9$


## 4. Results

Table 5 reports the various model estimates for each of the four models considered, as well as the correlation coefficients for regressions between predicted recruitment and biomass estimates based on pelagic surveys (the November 2010 pelagic survey is omitted from these regressions as there is as yet no corresponding recruitment estimate from the model).

The demersal selectivity functions estimated by each model are shown in Figure 1. Exploitable biomass, catch-at-length and mean catch-at-length residuals are shown in Figures 2,3 and 4 respectively. Figures 5 and 6 provide graphical representations of the correlation between predicted recruitment and pelagic survey biomass estimates for the

West coast, while Figures 6 and 7 do the same for the correlation between predicted recruitment and pelagic survey biomass estimates for the entire assessment area.

## 5. Discussion

An encouraging feature of the updated model results is the indication of recent increases in exploitable biomass (Fig. 2). It must be said, however, that the fit to the catch at length data is not entirely satisfactory as there is clear evidence of systematic effects in the residual patterns (Fig. 3).

Table 5 indicates that a somewhat more robust positive correlation between the estimates of recruitment from the model and the pelagic survey results for the West coast than for the whole assessment area. However, as evident from Fig. 5, the pelagic survey results show much greater variability than do the assessment results. Some such damping effect is to be expected, as the length distribution data will tend to smooth out evidence for different cohort sizes, but nevertheless the low values for correlation are disappointing.

At this stage the conclusion is either that the November survey estimates are (for whatever reason) a relatively weak predictor of incoming horse-mackerel recruitment strength, or that there are problems with the modelling of the demersal data which require further attention.

## References

Johnston, S.J. and Butterworth, D.S. 2007. The South African horse mackerel assessment for 2007 using an age-structured production model, with future biomass projections. MCM document, 2007:WG-Dem:HM:10

Coetzee, J, 2011. Request to the Demersal Scientific Working Group to consider an immediate ad hoc adjustment to the Horse Mackerel Precautionary Upper Catch Limit. DAFF document, FISHERIES/2011/SWG-PEL/15

Table 1: Annual landings (MT) of horse mackerel for demersal (Johnston and Butterwork, 2007; Fairweather, pers. commn) and pelagic (Coetzee, pers. commn) fisheries.

| Year | Demersal + Midwater | Pelagic |
| :---: | :---: | :---: |
| 1949 | - | 3360 |
| 1950 | 445 | 49900 |
| 1951 | 1105 | 98900 |
| 1952 | 1226 | 102600 |
| 1953 | 1456 | 85200 |
| 1954 | 2550 | 118100 |
| 1955 | 1926 | 78800 |
| 1956 | 1334 | 45800 |
| 1957 | 959 | 84600 |
| 1958 | 2073 | 56400 |
| 1959 | 2075 | 17700 |
| 1960 | 3712 | 62900 |
| 1961 | 3627 | 38900 |
| 1962 | 3079 | 66700 |
| 1963 | 1401 | 23300 |
| 1964 | 9522 | 24400 |
| 1965 | 7017 | 55000 |
| 1966 | 7596 | 26300 |
| 1967 | 6189 | 8800 |
| 1968 | 9116 | 1400 |
| 1969 | 12252 | 26800 |
| 1970 | 17872 | 7900 |
| 1971 | 33348 | 2200 |
| 1972 | 20556 | 1300 |
| 1973 | 35315 | 1600 |
| 1974 | 36654 | 2500 |
| 1975 | 69845 | 1600 |
| 1976 | 34814 | 400 |
| 1977 | 68816 | 1900 |
| 1978 | 35375 | 3600 |
| 1979 | 60068 | 4300 |
| 1980 | 42627 | 400 |
| 1981 | 33883 | 6100 |
| 1982 | 33091 | 1100 |
| 1983 | 41507 | 2100 |
| 1984 | 38817 | 2800 |
| 1985 | 31280 | 700 |
| 1986 | 35812 | 500 |
| 1987 | 41972 | 2834 |
| 1988 | 34333 | 6403 |
| 1989 | 34163 | 25872 |
| 1990 | 43646 | 7645 |
| 1991 | 23974 | 582 |
| 1992 | 23276 | 2057 |
| 1993 | 18426 | 11651 |
| 1994 | 8479 | 8207 |
| 1995 | 6702 | 1986 |
| 1996 | 9707 | 18920 |
| 1997 | 11332 | 12654 |
| 1998 | 13882 | 26680 |
| 1999 | 10174 | 2057 |
| 2000 | 24639 | 4503 |
| 2001 | 28044 | 915 |
| 2002 | 15961 | 8148 |
| 2003 | 28872 | 1012 |
| 2004 | 32087 | 2048 |
| 2005 | 34285 | 5627 |
| 2006 | 22190 | 4824 |
| 2007 | 29841 | 1903 |
| 2008 | 28221 | 2280 |
| 2009 | 33124 | 2087 |

Table 2: Swept area survey biomass estimates (MT) for the spring and autumn biomass series (Fairweather, pers. commn). Shaded data indicate surveys that were not performed by the Africana or that did not extend beyond 200 m and, therefore, are not incorporated in the model.

| Year | Autumn |  | Spring |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Abundance | CV | Abundance | CV |
| 1986 |  |  | 97363 | 0.13 |
| 1987 |  |  | 332973 | 0.14 |
| 1988 | 159074 | 0.29 |  |  |
| 1989 | 138203 | 0.54 |  |  |
| 1990 | 122746 | 0.28 | 551217 | 0.22 |
| 1991 | 352187 | 0.23 | 575014 | 0.17 |
| 1992 | 422209 | 0.23 | 477289 | 0.27 |
| 1993 | 435281 | 0.20 | 307167 | 0.16 |
| 1994 | 340719 | 0.26 | 337586 | 0.16 |
| 1995 | 195129 | 0.24 | 276369 | 0.23 |
| 1996 | 261770 | 0.23 |  |  |
| 1997 | 241017 | 0.23 |  |  |
| 1998 |  |  |  |  |
| 1999 | 330631 | 0.24 |  |  |
| 2000 | 322417 | 0.33 |  | 0.18 |
| 2001 |  |  | 316721 |  |
| 2002 |  |  |  | 0.20 |
| 2003 | 146723 | 0.24 | 231362 | 0.19 |
| 2004 | 195733 | 0.32 | 366499 | 0.19 |
| 2005 | 175042 | 0.21 |  | 0.19 |
| 2006 | 386566 | 0.20 | 350279 | 473216 |
| 2007 | 243582 | 0.40 |  |  |
| 2008 | 279857 | 0.27 | 300000 | 0.17 |
| 2009 | 337160 | 0.24 |  |  |
| 2010 | 271795 | 0.37 |  |  |

Table 3: Biomass estimates (MT) based on the November pelagic acoustic survey (Coetzee, 2011). Note that the year shown is the year after the November in which the survey took place, for consistency with the year to which the model's estimate of recruitment refers.

| Year | West Coast <br> only | Assessment <br> area |
| :---: | :---: | :---: |
| 1998 | 22983.76 | 23268.57 |
| 1999 | 1830.10 | 20386.85 |
| 2000 | 1040.93 | 5124 |
| 2001 | 849.33 | 196063.39 |
| 2002 | 5963.47 | 52909.27 |
| 2003 | 4257.04 | 15286.42 |
| 2004 | 10324.18 | 21470.2 |
| 2005 | 939.32 | 43143.31 |
| 2006 | 8136.34 | 12447.64 |
| 2007 | 11959.86 | 49800.1 |
| 2008 | 683.71 | 976.79 |
| 2009 | 1659.76 | 11660.24 |
| 2010 | 6292.55 | 12821.33 |
| 2011 | 51982.64 | 112192.41 |

Table 4: Selectivity (Johnston and Butterworth, 2007) and weight-at-age vectors. Note that, as was the case for the 2007 assessment, there are three pelagic selectivity vectors for three different periods.

| $a$ | $S_{a}^{p}$ <br> $1948-1962$ | $S_{a}^{p}$ <br> $1963-1967$ | $S_{a}^{p}$ <br> $1968+$ | $w_{a+\frac{1}{2}}(\mathrm{~g})^{*}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0.00 | 0.14 | 0.28 | 8.74 |
| 1 | 0.00 | 0.50 | 1.00 | 43.80 |
| 2 | 0.30 | 0.40 | 0.50 | 106.83 |
| 3 | 1.00 | 0.50 | 0.00 | 191.20 |
| 4 | 0.50 | 0.25 | 0.00 | 288.41 |
| 5 | 0.50 | 0.25 | 0.00 | 390.88 |
| 6 | 0.25 | 0.13 | 0.00 | 492.73 |
| 7 | 0 | 0.00 | 0.00 | 589.96 |
| 8 | 0 | 0.00 | 0.00 | 680.06 |
| 9 | 0 | 0.00 | 0.00 | 761.75 |
| $10+$ | 0 | 0.00 | 0.00 | 834.57 |

Table 5: Summary of results. Under the 'Negative log-likelihoods' heading: ' S - R ' refers to the contribution from stockrecruitment residuals, 'abund' refers to the contribution from the demersal survey biomass indicies and 'CAL' refers to the contribution from the demersal length-frequency data. Under the 'Regressions' heading: ' $\boldsymbol{r}$ ' refers to the Pearson correlation coefficient of the regression between model estimated recruitment and pelagic survey biomass estimates, ' $\boldsymbol{r}$ $(\log )$ ' refers to the Pearson correlation coefficient of the regression between log-transformed values and ' $\boldsymbol{s}$ (log)' refers to the slope of the regression line for the log-transformed values.

|  |  | Model 1 |  | Model 2 | Model 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | Model 4



Fig 1: Demersal selectivity functions.


Fig 2: Model fits to West coast demersal survey biomass estimates. Note that the values of the spring survey biomass series relative those of the autumn survey biomass series changes with each model, as the value of each series' associated catchability coefficient changes with each model.


Fig 3: Bubble plot of catch-at-length residuals. Positive residuals are dark and negative residuals are light. Only one residual plot is shown as there is little difference between models. The residuals shown are for model 1.

Model 1


Model 3


Model 2


Model 4


Fig 4: Bar graph of mean catch-at-length proportions averaged over years.


Fig 5: Normalised time series of biomass estimates for the West coast based on the November pelagic surveys and model estimated recruitment. The series have been normalised by dividing each series by its mean.


Fig 6: Regressions between biomass estimates for the West coast based on the November pelagic surveys and estimated recruitment for each model.


Fig 7: Normalised time series of biomass estimates for the entire assessment area based on the November pelagic surveys and model estimated recruitment. The series have been normalised by dividing each series by its mean.


Fig 8: Regressions between estimates for the entire assessment area based on the November pelagic surveys and estimated recruitment for each model.

## Appendix A

## Mathematical details of the age-structured production model (ASPM) applied

## A. 1 Dynamics

The dynamics of the population are described using the following deterministic equations:

$$
\begin{align*}
& N_{y+1,0}=R_{y+1}  \tag{A.1}\\
& N_{y+1, a+1}=\left(N_{y, a} e^{-\frac{M_{a}}{2}}-C_{y, a}\right) e^{-\frac{M_{a}}{2}} \quad 0 \leq a \leq m-2  \tag{A.2}\\
& N_{y+1, m}=\left(N_{y, m} e^{-\frac{M_{m}}{2}}-C_{y, m}\right) e^{-\frac{M_{m}}{2}}+\left(N_{y, m-1} e^{-\frac{M_{m-1}}{2}}-C_{y, m-1}\right) e^{-\frac{M_{m-1}}{2}} \tag{A.3}
\end{align*}
$$

where
$N_{y, a} \quad$ is the number of horse mackerel of age $a$ at the start of year $y$,
$C_{y, a} \quad$ is the total number of horse mackerel of age $a$ taken by the fishery, i.e. by the pelagic and demersal (plus midwater) fleets combined, in year $y$,
$R_{y} \quad$ is the number of recruits at the start of year $y$ (see below),
$M_{a} \quad$ is the natural mortality rate for fish of age $a$, and
$m \quad$ is the minimum age of the plus-group ( $m=10$ for this paper).
The approximation of the fishery as a pulse catch in the middle of the season is considered of sufficient accuracy for present purposes.

The total number of horse mackerel of age $a$ caught each year $\left(C_{y, a}\right)$ is given by:

$$
\begin{equation*}
C_{y, a}=\sum_{f} C_{y, a}^{f} \tag{A.4}
\end{equation*}
$$

where $f$ indicates the fishery/fleet concerned and is either $p$ (pelagic) or $d$ (demersal).
The annual catch by mass ( $C_{y}^{f}$ ) for fleet $f$ is given by:

$$
\begin{align*}
C_{y}^{f} & =\sum_{a=0}^{m} w_{a+1 / 2} C_{y, a}^{f} \\
& =\sum_{a=0}^{m} w_{a+1 / 2} S_{a}^{f} F_{y}^{f} N_{y, a} e^{-u_{a} / 2} \tag{A.5}
\end{align*}
$$

where $S_{a}^{f}$ is the fishing selectivity-at-age for fleet $f$. [Note that the pelagic selectivity is assumed to change over time - see Table 4]. $F_{y}^{f}$ is the fleet-specific fishing mortality for a fully selected age class in year $y$, and $w_{a+1 / 2}$ denotes the mid-year mass of a horse mackerel of age $a$.

The fleet-specific exploitable component of abundance is computed in terms of exploitable biomass at mid-year:

$$
\begin{equation*}
B_{y}^{f}=\sum_{a=0}^{m} w_{a+\frac{1}{2}} S_{a}^{f} N_{y, a} e^{-\mu_{a} / 2} \tag{A.6}
\end{equation*}
$$

or numbers:

$$
\begin{equation*}
N_{y}^{f}=\sum_{a=0}^{m} S_{a}^{f} N_{y, a} e^{-u_{a} / 2} \tag{A.7}
\end{equation*}
$$

The proportion of the resource harvested each year $\left(F_{y}^{f}\right)$ by fleet $f$ is therefore given by:

$$
\begin{equation*}
F_{y}^{f}=C_{y}^{f} / B_{y}^{f} \tag{A.8}
\end{equation*}
$$

and

$$
\begin{equation*}
C_{y, a}^{f}=S_{a}^{f} F_{y}^{f} N_{y, a} e^{-M_{a} / 2} \tag{A.9}
\end{equation*}
$$

## A. 2 Spawning biomass - recruitment relationship

The spawning biomass in year $y$ is given by:

$$
\begin{equation*}
B_{y}^{s p}=\sum_{a=a_{m}}^{m} w_{a} N_{y, a} \tag{A.10}
\end{equation*}
$$

where $a_{m}$ is the age corresponding to $100 \%$ sexual maturity, which is assumed here to be described by a knife-edge function of age.

The number of recruits at the start of fishing year $y$ is related to the spawner stock size by a Beverton-Holt stock-recruitment relationship:

$$
\begin{equation*}
R\left(B_{y}^{s p}\right)=\frac{\alpha B_{y}^{s p}}{\beta+B_{y}^{s y}} e^{\varsigma_{y}} \tag{A.11}
\end{equation*}
$$

where
$\alpha$ and $\beta$ are spawner biomass-recruitment parameters, and
$S_{y} \quad$ are stock-recruitment residuals reflecting fluctuations about the expected recruitment in year $y$.

In order to work with estimable parameters that are more biologically meaningful, the stock-recruit relationship is re-parameterised in terms of the pre-exploitation equilibrium spawning biomass, $K^{s p}$, and the "steepness" of the stock-recruit relationship, where "steepness" is the fraction of pristine recruitment ( $R_{0}$ ) that results when spawning biomass drops to $20 \%$ of its pristine level, i.e.:

$$
\begin{equation*}
h R_{0}=R\left(0.2 K^{s p}\right) \tag{A.12}
\end{equation*}
$$

from which it follows that:

$$
\begin{equation*}
h=0.2\left[\beta+K^{s p}\right] /\left[\beta+0.2 K^{s p}\right] \tag{A.13}
\end{equation*}
$$

and hence:

$$
\begin{equation*}
\alpha=\frac{4 h R_{0}}{5 h-1} \tag{A.14}
\end{equation*}
$$

and:

$$
\begin{equation*}
\beta=\frac{K^{s p}(1-h)}{5 h-1} \tag{A.15}
\end{equation*}
$$

Given a value for the pre-exploitation spawning biomass $K^{s p}$ of horse mackerel, together with the assumption of an initial equilibrium age structure, pristine recruitment can be determined by:

$$
\begin{equation*}
R_{0}=K^{s p} /\left[\sum_{a=a_{m}}^{m-1} w_{a} e^{-\sum_{a=0}^{a-1} M_{a^{\prime}}}+w_{m} e^{-\sum_{a=0}^{m-1} M_{a^{\prime}}} /\left(1-e^{-M_{m}}\right)\right] \tag{A.16}
\end{equation*}
$$

Numbers-at-age for subsequent years are then computed by means of equations (A.1)(A.11).

## A. 3 Estimable and input parameters

The estimable parameters are:

- $K^{s p}$, pristine spawning biomass;
- $q_{1}$, catchability coefficient of the spring demersal survey;
- $a_{\text {kink }}$, position of the kink in the demersal selectivity function;
- $\mu$, rate of decay of the demersal selectivity function after age 5 ; and
- $\varsigma_{y}$, fluctuations about expected recruitment for years 1983-2008 ([TODO: Can't read Doug's correction clearly] limited to these years there is associated information only for years where length distribution information is available).

The input parameters take the same values as in the 2007 assessment and are as follows:

- $M$, natural mortality, is equal to $0.3 \mathrm{yr}^{-1}$;
- $S_{a}^{p}$, selectivity at age values used for the pelagic fleet, are reported in Table 4;
- $w_{a+\frac{1}{2}}$, mid-year mass of a horse mackerel of age $a$, is reported in Table 4;
- $a_{m}$, age of sexual maturity, is 3 years;
- $q_{2}$, catchability coefficient of the autumn demersal survey, is considered to be either 1 or 0.5;
- $h$, the "steepness" of the stock-recruit curve, is considered to be either 0.6 or 0.9.


## A. 4 Likelihood functions

The model is fitted to survey biomass and length-frequency data. Stock-recruitment residuals also contribute to the negative log-likelihood function.

## A.4.1 Survey biomass

The model is fitted to two series of survey biomass data (Table 2). The associated likelihood contribution is calculated by assuming that the observed abundance index is log-normally distributed about its expected value:

$$
\begin{equation*}
I_{y}^{s}=\hat{I}_{y}^{s} e^{\varepsilon_{y}^{s}} \quad \text { or } \quad \varepsilon_{y}^{s}=\ln \left(I_{y}^{s}\right)-\ln \left(\hat{I}_{y}^{s}\right) \tag{A.17}
\end{equation*}
$$

where
$I_{y}^{s} \quad$ is the survey biomass data for year $y$ for survey $s(s=1$ (spring) or 2 (autumn)),
$\hat{I}_{y}^{s} \quad=q_{s} B_{y}^{f}$ is the corresponding model estimated value, where $B_{y}^{f}$ is the model value for exploitable resource biomass at mid-year corresponding to the demersal fleet, given by equation (A.6), and
$q_{s} \quad$ is a constant of proportionality (the demersal catchability coefficient).
The negative of the log-likelihood function (after removal of constants) is given then by:

$$
\begin{equation*}
-\ln L=\sum_{s} \sum_{y}\left\lfloor\ln \sigma_{y}^{s}+\left(\varepsilon_{y}^{s}\right)^{2} / 2\left(\sigma_{y}^{s}\right)^{2}\right] \tag{A.18}
\end{equation*}
$$

The standard deviations are calculated from the CVs reported in Table 2 by the following formula:

$$
\begin{equation*}
\sigma_{y}^{s}=\sqrt{\ln \left(1+C V_{s, y}^{2}\right)} \tag{A.19}
\end{equation*}
$$

## A.4.2 Length-frequency

Model estimated demersal catch-at-length proportions are fitted to demersal survey lengthfrequency data.

The demersal catch-at-age model estimates (equation A.9) are converted to catch-at-length estimates using an age-length key:

$$
\begin{equation*}
\hat{C}_{y, l}=\sum_{a=0}^{m} A_{l, a} C_{y, a} \tag{A.20}
\end{equation*}
$$

where $A_{l, a}$ is the proportion of fish of age $a$ that are of length $l$, which is calculated by assuming that lengths at a given age $a$ are normally distributed according to $N\left(l(a),(\beta l(a))^{2}\right)$, where $l(a)$ is the mean length of a mackerel of age $a$ and $\beta$ is a constant taken to be equal to 0.075 (for which reasonable fits to the data were obtained).

The contribution of catch-at-length data to the negative of the log-likelihood function when assuming a log-normal error distribution and when making an adjustment to effectively weight in proportion to sample size is given by:

$$
\begin{equation*}
-\ln L=\sum_{y} \sum_{l}\left\lfloor\ln \left(\sigma_{c a l} / \sqrt{p_{y, l}}\right)+p_{y, l}\left(\ln p_{y, l}-\ln \hat{p}_{y, l}\right)^{2} / 2 \sigma_{c a l}^{2}\right\rfloor \tag{A.21}
\end{equation*}
$$

Where
$p_{y, l} \quad$ is the observed proportion of fish caught in year $y$ that are of length $l$,
$\hat{p}_{y, l} \quad=\hat{C}_{y, l} / \sum_{l} \hat{C}_{y, l}$ is the model predicted proportion of fish caught in year $y$ that are of length $l$, and
$\sigma_{c a l}$ is the standard deviation associated with catch-at-length data, estimated in the fitting procedure by:

$$
\begin{equation*}
\sigma_{c a l}=\sqrt{\left[\sum_{y} \sum_{l} p_{y, l}\left(\ln p_{y, l}-\ln \hat{p}_{y, l}\right)^{2} / \sum_{y} \sum_{l} 1\right]} \tag{A.22}
\end{equation*}
$$

Note that allowance is made for a minus group (fish 19 cm and smaller) and a plus group (fish 40 cm and larger), and length classes are specified with intervals of 2 cm .

## A.4.3 Stock-recruitment residuals

It is assumed that these residuals are log-normally distributed and are not serially correlated. Therefore, the contribution to the negative log-likelihood function is given by:

$$
\begin{equation*}
-\ln L=\sum_{y} \frac{\varsigma_{y}^{2}}{2 \sigma_{R}^{2}} \tag{A.23}
\end{equation*}
$$

where $\sigma_{R}$ is the standard deviation of the log residuals, which is assumed to be equal to 0.3.

