Assessment of the South African sardine resource using data from 1984-2010: further work towards a base case single stock hypothesis

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Introduction

Further work towards choosing a base case single sardine stock hypothesis operating model has been undertaken. This work extends that reported in de Moor and Butterworth (2011).

Population Dynamics Model

The updated generalised operating model for the South African sardine resource, applying to both the single and two stock hypotheses, is detailed in Appendix A. The data used in this assessment are listed in de Moor *et al*. (2011). The model is now fit to survey proportion-at-length data in addition to the other survey and commercial data previously used.

Some changes have occurred in the use of prior distributions on the multiplicative bias parameters relating to the November and May hydroacoustic surveys. The prior distribution formerly developed for the bias on the November survey, $k^S_{j,N}$, has now been correctly termed a bias on the hydroacoustic survey, k^S_{ac}, (see Appendix B). The assumption is made that full coverage of the sardine abundance is obtained during the November survey. The May survey is subject to the same estimated hydroacoustic bias parameter as well as an additional parameter which models the coverage of the sardine recruit abundance in May as a ratio of the 1+ biomass covered in November, k^S_{cov}. Given that not all of the recruitment is assumed to be available to the survey by mid-May, this ratio is constrained by a maximum of 1.

Random effects about juvenile natural mortality have now also been included in the model. Initial results estimated similar annual autocorrelation coefficients in annual residuals of adult and juvenile natural mortality. As a result a single autocorrelation coefficient is now estimated for both adult and juvenile natural mortality.

The results presented in this document assume a hockey stick stock recruitment curve, with parameters differing between peak (2000-2004) and non-peak years. These results are also based on the assumption

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of $\overline{M}_{ad}^{S}=0.6$ and $\overline{M}_{j}^{S}=1.0$. Note that $\overline{M}_{ad}^{S}$ and $\overline{M}_{j}^{S}$ are the central values (prior median) about which adult and juvenile natural mortality varies over time.

Results and Discussion

A likelihood profile on the standard deviation in the annual residuals about adult natural mortality, σ_{ad}, has been plotted (results not shown). Unlike for anchovy, the likelihood profile is monotonic with a better (smaller) objective function value obtained for smaller σ_{ad} values. This is primarily a result of the large contribution to the likelihood from the log-prior on the residuals about adult natural mortality. In contrast, the fit to the hydroacoustic survey estimates of November 1+ biomass improve for higher σ_{ad} values. Due to the need to fit the latter time series well, and since an alternative of no random effects on natural mortality will be considered as a robustness test, and possibly an alternative hypothesis, the range for σ_{ad} has been constrained to [0.2, 0.5] as for anchovy.

The updated population model fits to the time series of abundance estimates of November 1+ biomass and May recruitment are shown in Figures 1 and 2, respectively. Figures 3 and 5 show the fits to the time series of survey and commercial proportion-at-age data, respectively. The model estimated survey and commercial selectivities at age are plotted in Figures 4 and 6, respectively. Figure 7 shows the residuals from the fit to the November survey proportion-at-length data and Figure 8 shows the residuals from the fit to the quarterly commercial proportion-at-length data. The model predicted November spawner biomass and recruitment at the posterior mode are plotted in Figure 9, together with the model estimated hockey-stick stock-recruitment curve and constant recruitment from 2000 to 2004.

Finally, the model estimated annual juvenile and adult natural mortality are plotted in Figure 10 together with the estimated residuals. The autocorrelation between these residuals is estimated to be relatively high with the standard deviation in these residuals on the lower bound of the uniform prior distribution (Table 1).

The estimated parameter values and other key outputs are listed in Table 1 together with the individual contributions to the objective function at the posterior mode. Note that these results presented may not be at the posterior mode as a non positive definite Hessian results. The corresponding values excluding random effects about juvenile natural mortality from de Moor and Butterworth (2011) are also shown.

Summary and Future Work

This document has detailed further work towards a base case single sardine stock hypothesis. The effect of alternative stock recruitment relationships and alternative values for median natural mortality still need to be tested before a decision on the choice of a base case can be made and robustness testing to that base case undertaken.

References

de Moor, C.L., and Butterworth, D.S. 2011. Assessment of the South African sardine resource using data from 1984-2010: further results of a single stock hypothesis. Department of Agriculture, Forestry and Fisheries Document FISHERIES/2011/SWG-PEL/44. 34pp

de Moor, C.L., Coetzee, J., Durholtz D.,Merkle, D., and van der Westhuizen, J.J., 2011. A final record of the generation of data used in the 2011 sardine and anchovy assessments. Department of Agriculture, Forestry and Fisheries Document FISHEREIS/2011/SWG-PEL/51. 31pp.

Table 1. Key model parameter values and model outputs estimated at the joint posterior mode for the model given in this document, including random effects on $\overline{M}_j^S$ as well as that excluding random effects on $\overline{M}_j^S$ from de Moor and Butterworth (2011) $\overline{M}_j^S$. Values fixed on input are given in **bold**. Numbers are reported in billions and biomass in thousands of tonnes.

Parameter	Including random effects on $\overline{M}_j^S$	Excluding random effects on $\overline{M}_j^S$	Parameter	Including random effects on $\overline{M}_j^S$	Excluding random effects on $\overline{M}_j^S$
Objective function	36.84	58.87	$S_{q,1}$,	0.09 from '84-06, then 0.20 from '07-10	0.25 from '84-06, then 0.47 from '07-10
$-\ln L^{Nov}$	7.33	9.45	$S_{q,2}$,	0.33	0.52
$-\ln L^{rec}$	24.40	27.39	S_3	0.63	0.49
$-\ln L^{sur\ propa}$	0.17	0.78	S_4	**1.00**	**1.00**
$-\ln L^{com\ propa}$	1.52	4.21	S_{5+}	2.00 from '84-06, then 0.90 from '07-10	2 from '84-06, then 0.85 from '07-10
$-\ln L^{coml\ propl\ \min}$	1.93	1.97	N_{1983}^S	2.52	2.60
$-\ln L^{coml\ propl}$	15.12	15.03	$\overline{B}_{Nov}^{S}$ [1]	615	612
$-\ln L^{sur\ propl\ \min}$	0.23		K_{normal}^S	2037	1645
$-\ln L^{surl\ propl}$	1.74		K_{peak}^S	2267	2126
$-\ln(priors)$	-15.61	0.05	a^S	33.8	27.3
$\overline{M}_j^S$	**1.0**	**1.0**	b^S	696.0	579.9
$\overline{M}_{ad}^S$	**0.6**	**0.6**	c^S	37.6	35.2
$k_{j,N}^S = k_{ac}^S$	0.75	0.73	σ_r^S	0.40	0.40[1]
$k_{j,r}^S$	0.67	0.72	$\sigma_{r,peak}^S$	1.08	0.84
$k_{j,r}^S / k_{j,N}^S$	0.90	0.99	η_{2009}^S	0.700	0.463
$S_{j,1}^{survey}$	1.10	0.90	s_{cor}^S	0.448	0.474
$S_{j,2}^{survey}$	0.90	0.90	L_∞	19.5	20.2
$S_{j,3}^{survey}$	0.90	1.10	κ	0.59	0.57
$S_{j,4}^{survey}$	**1.00**	**1.00**	t_0	**-1.7**	**-1.7**
$S_{j,5+}^{survey}$	0.90	1.10	ϑ_1	0.26	0.19
$(\lambda_N^S)^2$	0.016	0.030	ϑ_2	0.15	0.08
$(\lambda_r^S)^2$	0.309	0.424	ϑ_3	0.08	0.06

[1] OMP-04 and OMP-08 were developed using Risk defined as "the probability that 1+ sardine biomass falls below the average 1+ sardine biomass between November 1991 and November 1994 at least once during the projection period of 20 years".

Table 1 (continued).

Parameter	Including random effects on $\overline{M}_j^S$	Excluding random effects on $\overline{M}_j^S$	Parameter	Including random effects on $\overline{M}_j^S$	Excluding random effects on $\overline{M}_j^S$
σ_j	0.20		ϑ_4	0.06	0.06
σ_{ad}	0.20	0.20	ϑ_{5+}	0.06	0. 06
p	0.67	0.67			

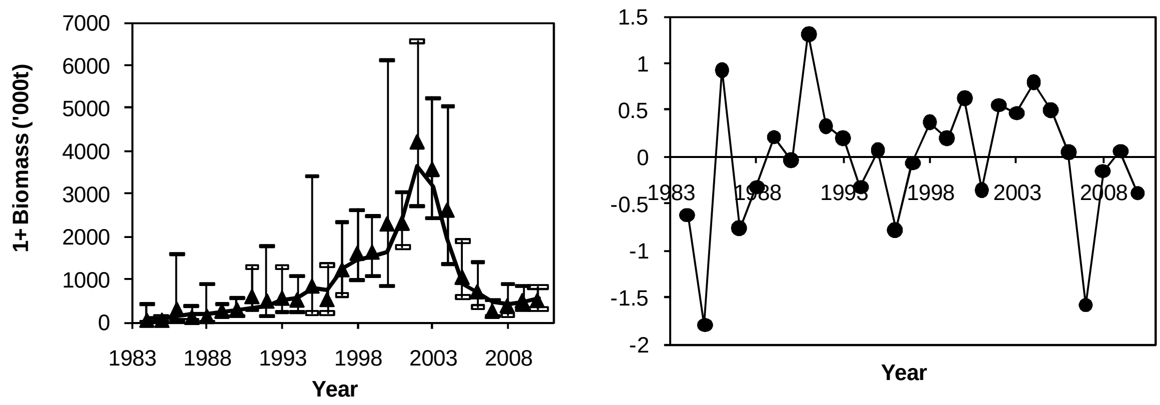

Figure 1. Acoustic survey observed and model predicted November sardine 1+ biomass from 1984 to 2010. The observed indices are shown with 95% confidence intervals. The standardised residuals from the fits are given in the right hand plot.

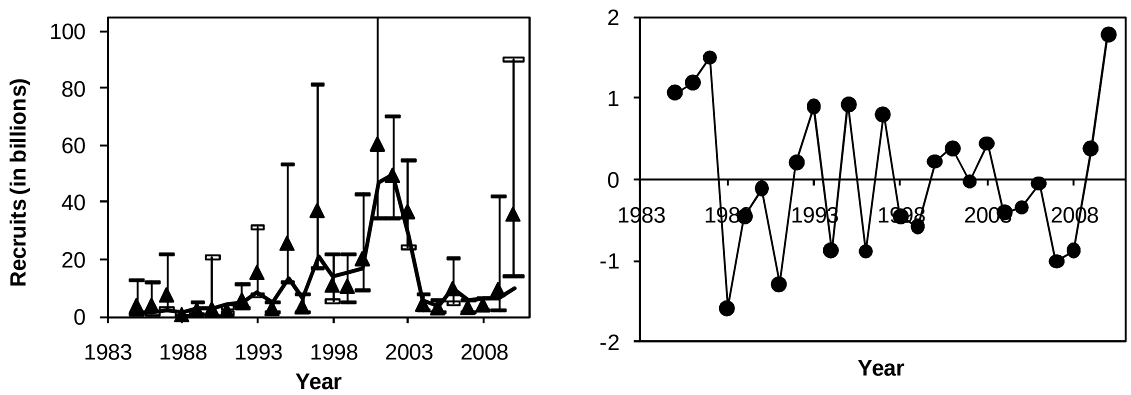

Figure 2. Observed and model predicted sardine recruitment numbers from May 1985 to May 2010. The observed indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plot.

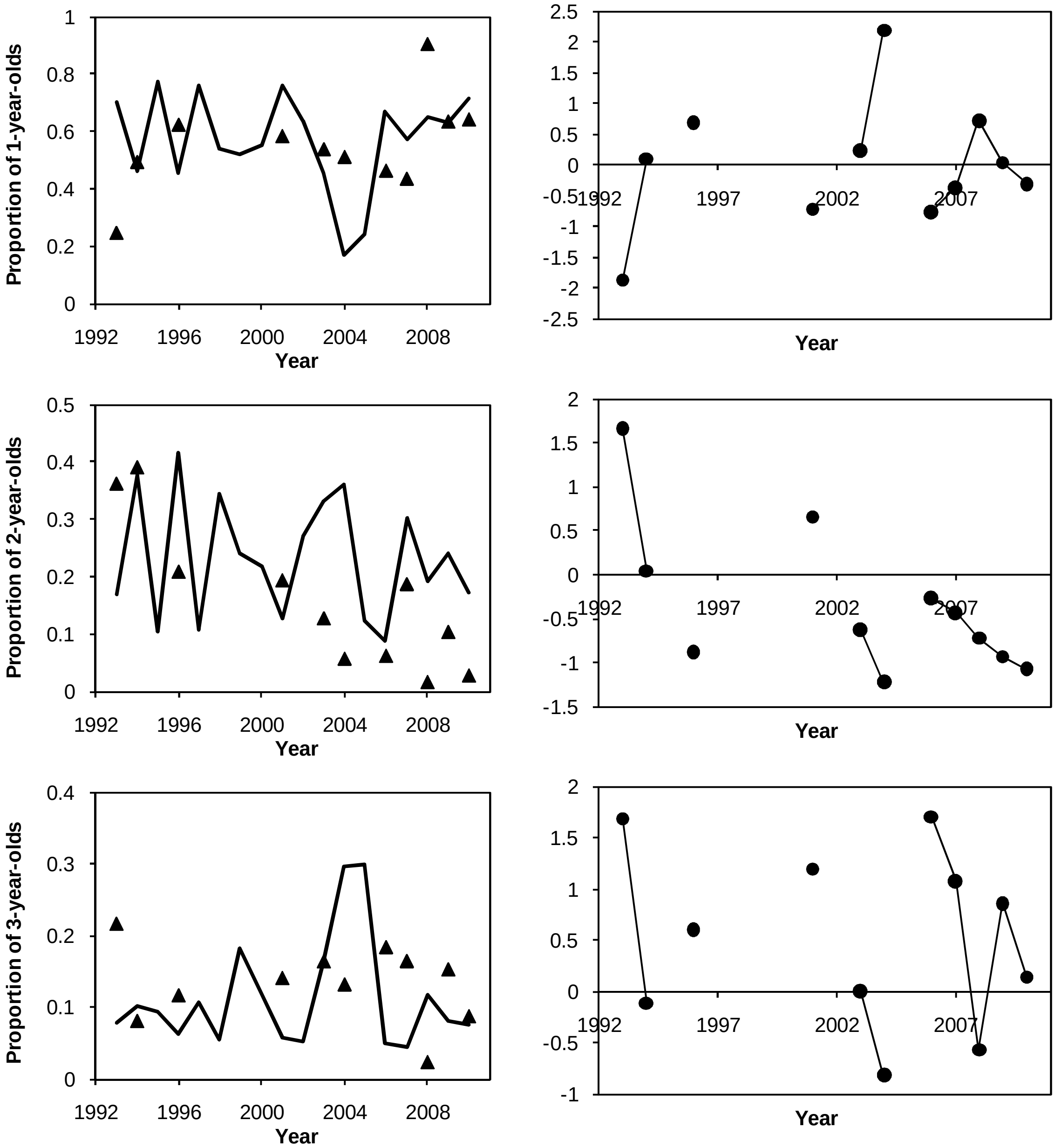

Figure 3. Observed and model predicted sardine proportion-at-ages 1 (at the top) to 5+ (lowest plot) associated with the November surveys from 1993 to 2010. The residuals from the fits are given in the right hand plots.

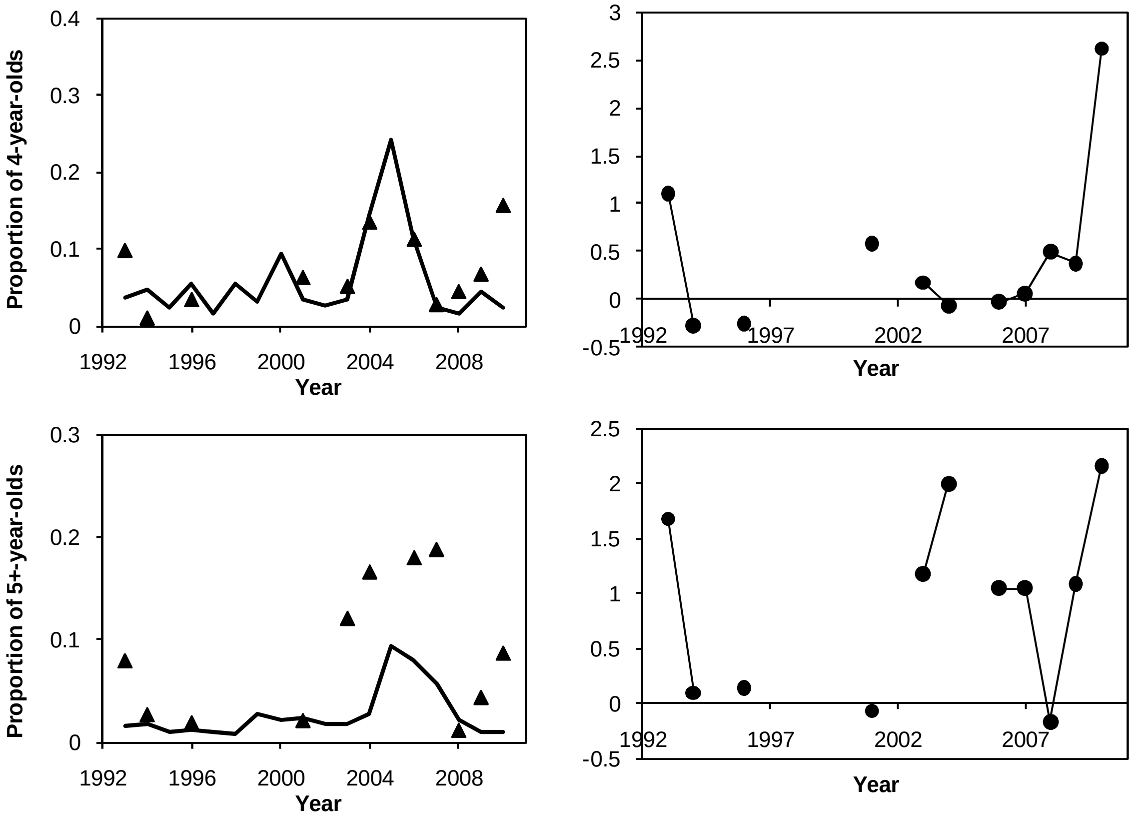

Figure 3 (continued).

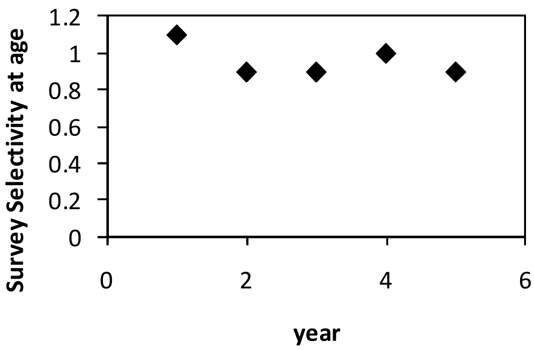

Figure 4. The model estimated November survey selectivity at age.

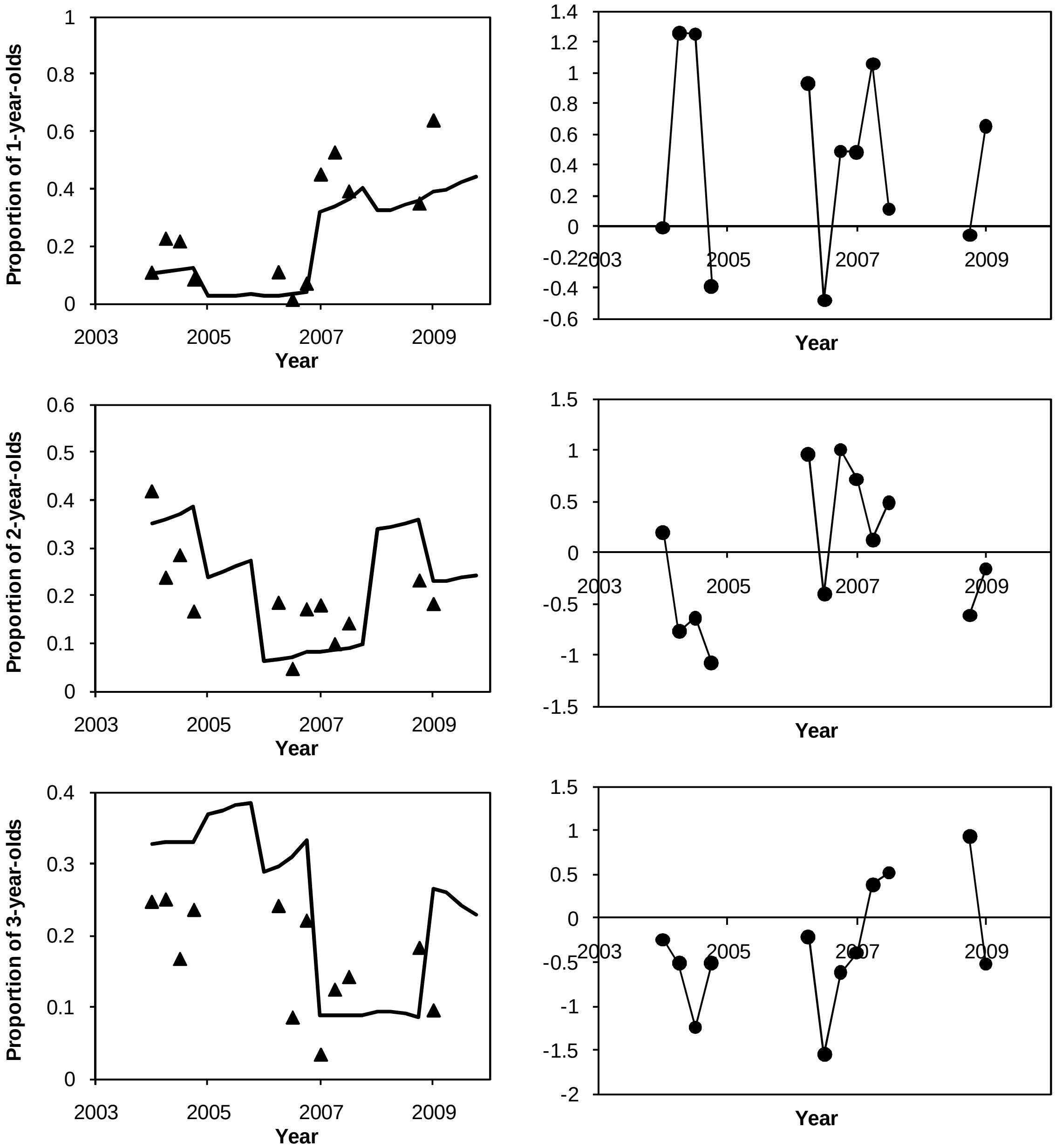

Figure 5. Observed and model predicted sardine proportion-at-ages 1 (at the top) to 5+ (lowest plot) associated with the quarterly commercial catch from 2004 to 2009. The residuals from the fits are given in the right hand plots.

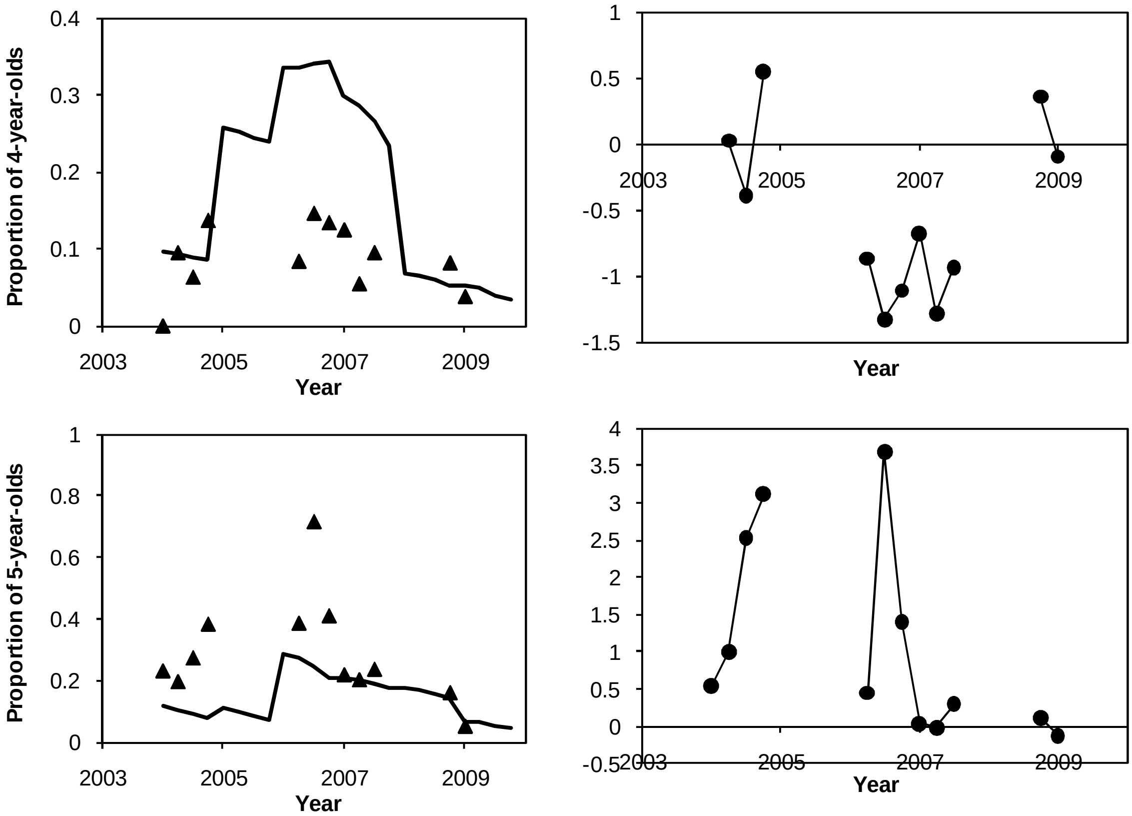

Figure 5 (continued).

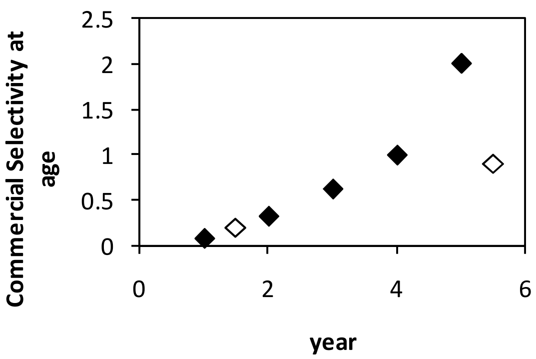

Figure 6. The model estimated commercial selectivity at age. The open diamonds represent the selectivity at ages 1 and 5+ estimated from 2007 to 2010, while the solid diamonds for these ages represent the selectivity from 1984 to 2006.

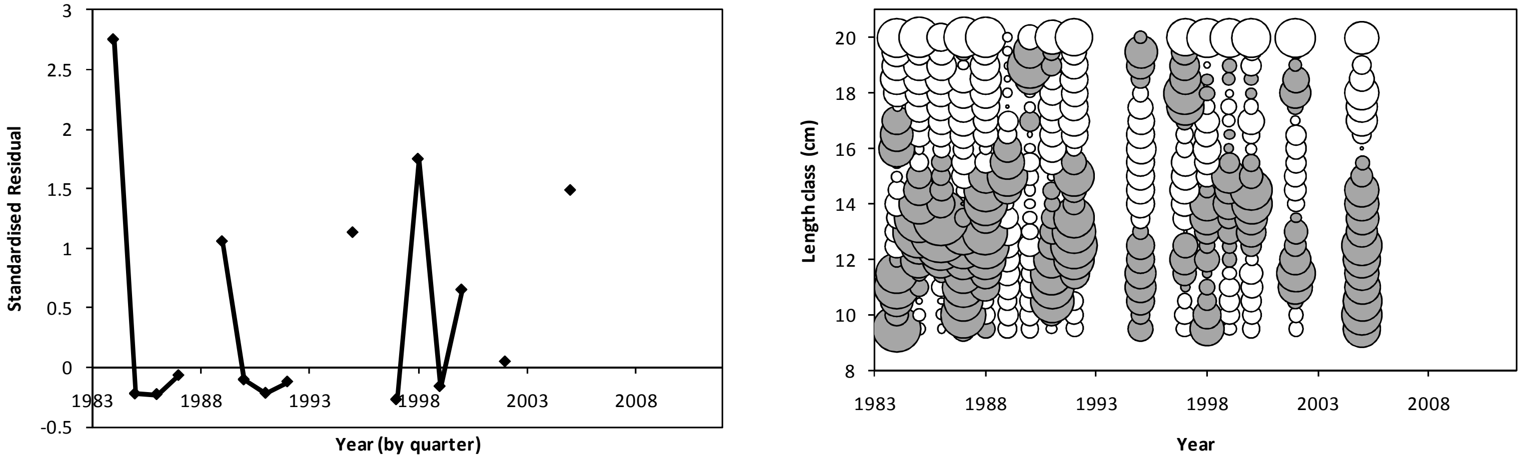

Figure 7. Residuals from the fit of the model predicted proportion-at-length in the November survey to the observed proportions. The left panels show the residuals for the minus length class (9cm) and the right panels show the residuals for the remaining length classes.

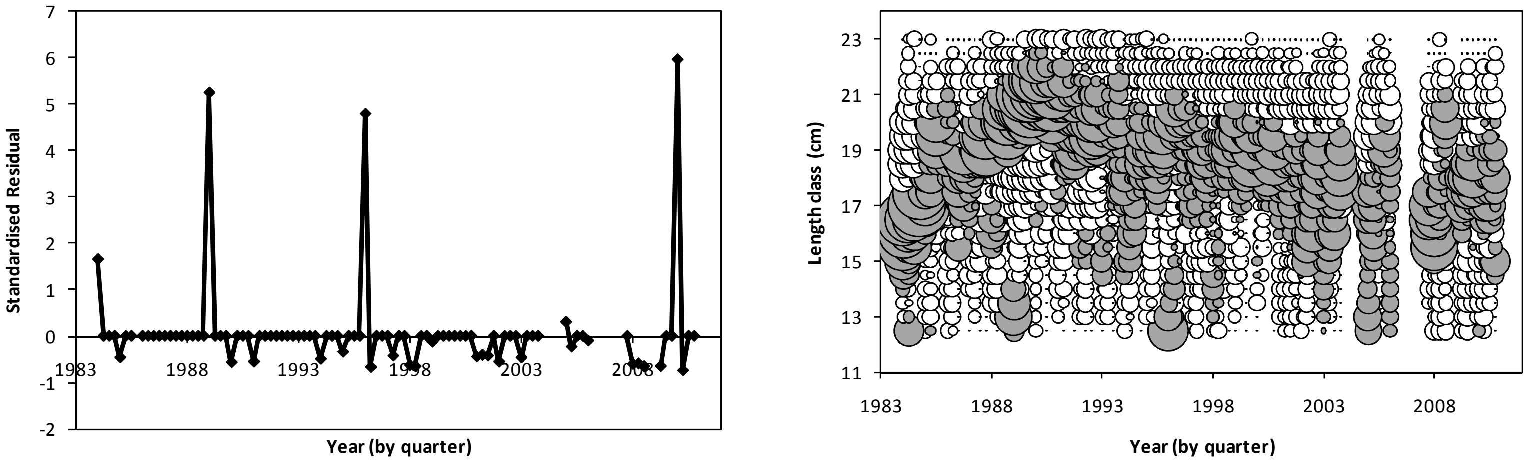

Figure 8. Residuals from the fit of the model predicted proportion-at-length in the commercial catch to the observed proportions. The left panels show the residuals for the minus length class (12cm) and the right panels show the residuals for the remaining length classes.

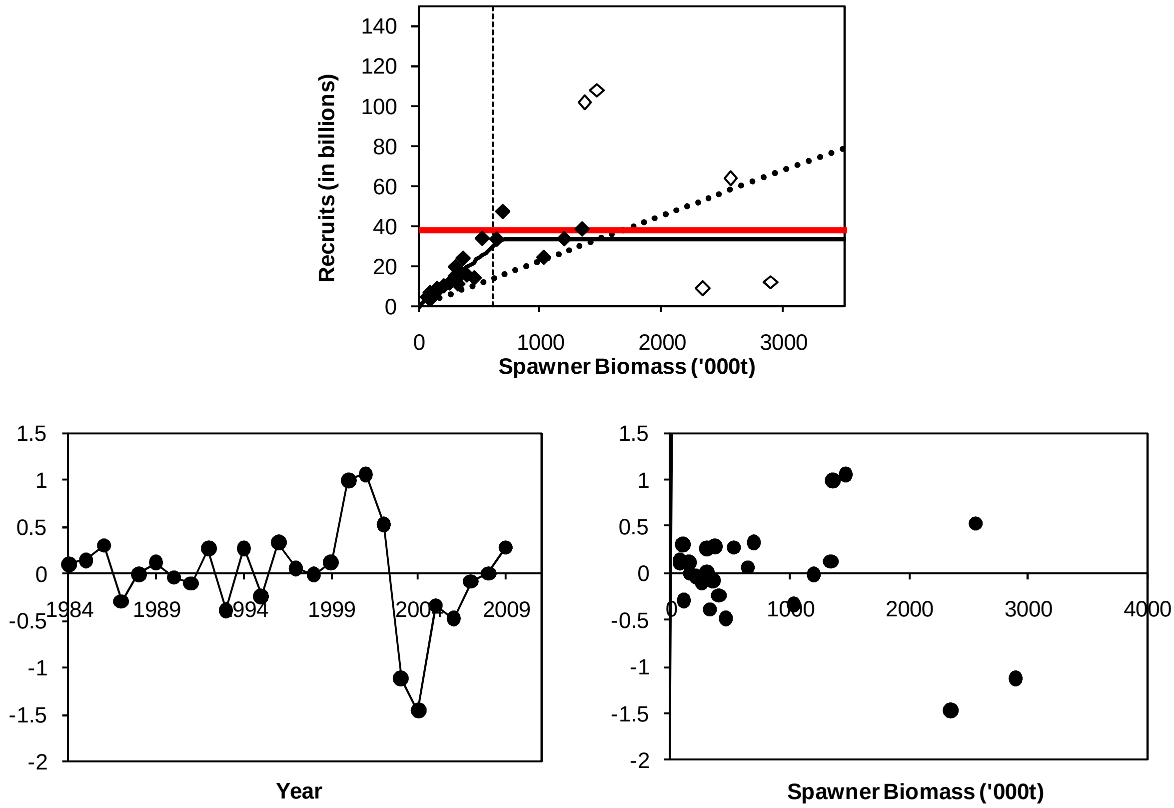

Figure 9. Model predicted sardine recruitment (in November) plotted against spawner biomass from November 1984 to November 2009, with the 'hockey-stick' stock recruitment curve and the constant recruitment between 2000 and 2004 also shown, for $\overline{M}_{ad}^{S} = 0.8$ (top left panel) and $\overline{M}_{ad}^{S} = 0.6$ (top right panel). The open diamonds denote the 2000 to 2004 November spawner biomass and recruitment. The vertical thin dashed line indicates the average 1991 to 1994 spawner biomass (used in the definition of risk in OMP-04 and OMP-08). The dotted line indicates the replacement line for normal recruitment. The standardised residuals from the fits are given in the lower plots, against year and against spawner biomass for $\overline{M}_{ad}^{S} = 0.8$ (black) and $\overline{M}_{ad}^{S} = 0.6$ (red).

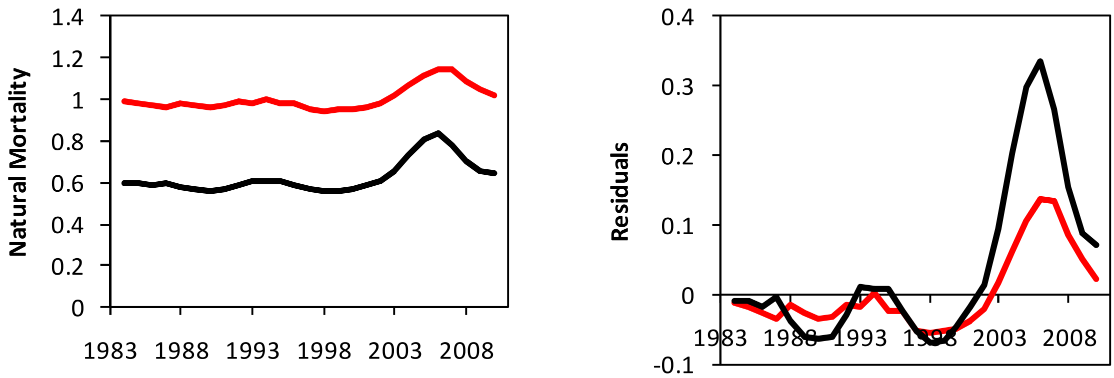

Figure 10. Model estimated annual juvenile (red) and adult (black) natural mortality for $\overline{M}_{j}^{S} = 1.0$ and $\overline{M}_{ad}^{S} = 0.6$. The random effects are plotted in the right hand panel.

Appendix A: Bayesian age-structured operating model for the South African sardine resource

Base Case Model Assumptions

1) All fish have a theoretical birthdate of 1 November.
2) Sardine spawn for the first time when they turn two years old.
3) A plus group of age five is assumed.
4) Two surveys are held each year: the first takes place in November (known as the November survey) and surveys the adult (1+) stock; the second is in May/June (known as the recruit survey) and surveys juvenile (0-year-old) sardine (also called recruits).
5) The November survey provides a relative index of abundance of unknown bias.
6) The recruit survey provides a relative index of abundance of unknown bias.
7) The survey strategy is such that it results in surveys of invariant bias over time.
8) Pulse fishing occurs four times a year, in the middle of each quarter after the birthdate.
9) Natural mortality is year-invariant for juvenile and adult fish, and age-invariant for adult fish.

Population Dynamics

The basic dynamic equations for sardine, based on Pope's approximation (Pope, 1984), are as follows, where $y_1 = 1984$ and $y_n = 2010$. The numbers-at-age are modelled at 1 November each year.

Catch is taken at four intervals during the year where $q=1$ is from November $y-1$ to January y, $q=2$ from February to April y, $q=3$ from May to July y and $q=4$ from August to October y:

Numbers-at-age at 1 November

$$N^S_{j,y,a} = \left(\left(\left(\left((N^S_{j,y-1,a-1} e^{-M^S_{a-1,y}/8} - C^S_{j,y,1,a-1}) e^{-M^S_{a-1,y}/4}\right) - C^S_{j,y,2,a-1}\right) e^{-M^S_{a-1,y}/4} - C^S_{j,y,3,a-1}\right) e^{-M^S_{a-1,y}/4} - C^S_{j,y,4,a-1}\right) e^{-M^S_{a-1,y}/8}$$

$$y = y_1, \ldots, y_n,\; a = 1, \ldots, 4$$

$$N^S_{j,y,5+} = \left(\left(\left(\left((N^S_{j,y-1,4} e^{-M^S_{4,y}/8} - C^S_{j,y,1,4}) e^{-M^S_{4,y}/4}\right) - C^S_{j,y,2,4}\right) e^{-M^S_{4,y}/4} - C^S_{j,y,3,4}\right) e^{-M^S_{4,y}/4} - C^S_{j,y,4,4}\right) e^{-M^S_{4,y}/8}$$
$$+ \left(\left(\left(\left((N^S_{j,y-1,5+} e^{-M^S_{5+,y}/8} - C^S_{j,y,1,5+}) e^{-M^S_{5+,y}/4}\right) - C^S_{j,y,2,5+}\right) e^{-M^S_{5+,y}/4} - C^S_{j,y,3,5+}\right) e^{-M^S_{5+,y}/4} - C^S_{j,y,4,5+}\right) e^{-M^S_{5+,y}/8}$$

$$y = y_1, \ldots, y_n \qquad \text{(A.1)}$$

where

$N^S_{j,y,a}$ is the model predicted number (in billions) of sardine of age *a* at the beginning of November in year y of stock j;

$C^S_{j,y,a,q}$ is the model predicted number (in billions) of sardine of age *a* of stock j caught during quarter q of year y;

$M^S_{a,y}$ is the rate of natural mortality (in year^{-1}) of sardine of age a in year y.

Movement

In the two stock hypothesis, movement of west stock ($j=1$) recruits to the east stock ($j=2$) at the beginning of November, i.e. when the recruits turn age 1, is modelled as follows:

$$N^S_{1,y,1} = \left(1 - move_y\right) N^{S*}_{1,y,1}$$

$$N^S_{2,y,1} = move_y N^{S*}_{2,y,1} \qquad y = y_1,\ldots,y_n$$

where $N^{S*}_{j,y,1}$ is simply the numbers-at-age 1 given by equation (A.1) prior to movement, and

$move_y$ is the proportion of west stock recruits which migrate to the east stock at the beginning of November of year y.

Biomass associated with the November survey

$$B^S_{j,y} = k^S_{j,N} \sum_{a=1}^{5+} N^S_{j,y,a} w^S_{j,y,a} \qquad y = y_1,\ldots,y_n \qquad \text{(A.2)}$$

where

$B^S_{j,y}$ is the model predicted biomass (in thousand tonnes) of adult sardine of stock j at the beginning of November in year *y*, associated with the November survey;

$k^S_{j,N}$ is the constant of proportionality (multiplicative bias) associated with the November survey of stock j; and

$w^S_{j,y,a}$ is the mean mass (in grams) of sardine of age *a* of stock j sampled during the November survey of year *y* (de Moor et al. 2011).

The multiplicative bias in the November survey is assumed to be equal to that resulting from the acoustic survey only; hence it is assumed that the full distribution of sardine is covered by the survey, i.e.

$$k^S_{j,N} = k^S_{ac}$$

where

k^S_{ac} is the multiplicative bias associated with the acoustic survey (see Appendix B).

Sardine are assumed to mature at age two and thus the spawning stock biomass is:

$$SSB^S_{j,y} = \sum_{a=2}^{5+} N^S_{j,y,a} w^S_{j,y,a} \qquad y = y_1,\ldots,y_n \qquad \text{(A.3)}$$

Proportion at age and length associated with the November survey

The model predicted proportion-at-age in the survey is:

$$p^S_{j,y,a} = \frac{S^{survey}_{j,a} N^S_{j,y,a}}{\sum_{a=1}^{5+} S^{survey}_{j,a} N^S_{j,y,a}} \qquad y = y_1, \ldots, y_n \;,\; a = 1,...,5+ \qquad \text{(A.4)}$$

where

$S^{survey}_{j,a}$ is the survey selectivity at age a in the November survey for stock j.

The model predicted proportion-at-length associated with the November survey is:

$$p^S_{j,y,l\min} = \sum_{a=1}^{5+} \sum_{l=1}^{l\min} A^{sur}_{j,a,l} p^S_{j,y,a} \qquad y = y_1, \ldots, y_n \qquad \text{(A.5)}$$

$$p^S_{j,y,l} = \sum_{a=1}^{5+} A^{sur}_{j,a,l} p^S_{j,y,a} \qquad y = y_1, \ldots, y_n \,,\; l = l\min + 1,...,l\max \qquad \text{(A.6)}$$

where

$A^{sur}_{j,a,l}$ is the proportion of sardine of age a in stock j that fall in the length group l in November.

The matrix $A^{sur}_{j,a,l}$ is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A^{com}_{j,a,l} \sim N\left(L_{j,\infty}\left(1 - e^{-\kappa_j\left(a - t_{0,j}\right)}\right), \vartheta_{j,a}\right) \qquad \text{(A.7)}$$

where

$L_{j,\infty}$ denotes the maximum length of sardine of stock j;

κ_j denotes the annual growth rate of sardine of stock j;

$t_{0,j}$ denotes the age at which the growth rate is zero of sardine of stock j; and

$\vartheta_{j,a}$ denotes the variance about the meal length for age a of stock j.

Catch

Sardine are landed by three major fisheries: the sardine-directed fishery (*fleet*=1), the red-eye-directed fishery (*fleet*=2), and the anchovy-directed fishery (*fleet*=3). Landings from the former two fisheries comprise mainly adult sardine while bycatch from the anchovy-directed fishery is primarily juvenile sardine. The assumption is made that all sardine smaller than a pre-determined cut-off length are 0-year-olds:

$$C^S_{j,y,1,0} = \sum_{fleet=1}^{3} \sum_{m=1}^{1} \sum_{l<lcut_{y,m}} C^{RLF,fleet}_{j,y,m,l}$$

$$C^S_{j,y,2,0} = \sum_{fleet=1}^{3} \sum_{m=2}^{4} \sum_{l<lcut_{y,m}} C^{RLF,fleet}_{j,y,m,l}$$

$$C^{S}_{j,y,3,0} = \sum_{fleet=1}^{3}\sum_{m=5}^{7}\sum_{l<lcut_{y,m}} C^{RLF,\,fleet}_{j,y,m,l}$$

$$C^{S}_{j,y,4,0} = \sum_{fleet=1}^{3}\sum_{m=8}^{10}\sum_{l<lcut_{y,m}} C^{RLF,\,fleet}_{j,y,m,l} \qquad y = y_1,\ldots,y_n \qquad \text{(A.8)}$$

where

$C^{RLF}_{j,y,m,l}$ is the number of fish in length class l landed in month m of year y of stock j (the 'raised length frequency'); and

$lcut_{y,m}$ is the cut off length for recruits in month m of year y (see de Moor et al. (2011) for details).

The remaining sardine bycatch from the anchovy-directed fishery are assumed to be 1-year olds, while the remaining directed sardine and redeye bycatch are split between ages using a model estimated selectivity:

$$C^{bycatch}_{j,y,1,1} = \sum_{m=11}^{1}\sum_{l>=lcut_{y,m}} C^{RLF,\,fleet=3}_{j,y,m,l}$$

$$C^{bycatch}_{j,y,2,1} = \sum_{m=2}^{4}\sum_{l>=lcut_{y,m}} C^{RLF,\,fleet=3}_{j,y,m,l}$$

$$C^{bycatch}_{j,y,3,1} = \sum_{m=5}^{7}\sum_{l>=lcut_{y,m}} C^{RLF,\,fleet=3}_{j,y,m,l}$$

$$C^{bycatch}_{j,y,4,1} = \sum_{m=8}^{10}\sum_{l>=lcut_{y,m}} C^{RLF,\,fleet=3}_{j,y,m,l}$$

$$C^{bycatch}_{j,y,q,a} = 0 \qquad y = y_1,\ldots,y_n,\ q = 1,\ldots,4,\ a = 2,\ldots,5+$$

$$C^{S}_{j,y,1,a} = \left(N^{S}_{j,y-1,a}e^{-M^{S}_{a}/8} - C^{bycatch}_{j,y,1,a}\right)S_{j,y,1,a}F_{j,y,1}$$

$$C^{S}_{j,y,2,a} = \left(\left(N^{S}_{j,y-1,a}e^{-M^{S}_{a}/8} - C^{bycatch}_{j,y,1,a} - C^{S}_{j,y,1,a}\right)e^{-M^{S}_{a}/4} - C^{bycatch}_{j,y,2,a}\right)S_{j,y,2,a}F_{j,y,2}$$

$$C^{S}_{j,y,3,a} = \left(\left(\left(N^{S}_{j,y-1,a}e^{-M^{S}_{a}/8} - C^{bycatch}_{j,y,1,a} - C^{S}_{j,y,1,a}\right)e^{-M^{S}_{a}/4} - C^{bycatch}_{j,y,2,a} - C^{S}_{j,y,2,a}\right)e^{-M^{S}_{a}/4} - C^{bycatch}_{j,y,3,a}\right)S_{j,y,3,a}F_{j,y,3}$$

$$C^{S}_{j,y,4,a} =$$
$$\left(\left(\left(\left(N^{S}_{j,y-1,a}e^{-M^{S}_{a}/8} - C^{bycatch}_{j,y,1,a} - C^{S}_{j,y,1,a}\right)e^{-M^{S}_{a}/4} - C^{bycatch}_{j,y,2,a} - C^{S}_{j,y,2,a}\right)e^{-M^{S}_{a}/4} - C^{bycatch}_{j,y,3,a} - C^{S}_{j,y,3,a}\right)e^{-M^{S}_{a}/4} - C^{bycatch}_{j,y,4,a}\right)S_{j,y,4,a}F_{j,y,4}$$

$$y = y_1,\ldots,y_n,\ a = 0,\ldots,5+ \qquad \text{(A.9)}$$

$S_{j,y,q,a}$ is the commercial selectivity at age a during quarter q of year y of stock j; and

$F_{j,y,q}$ is the fished proportion in quarter q of year y for a fully selected age class a of stock j, by the directed and redeye bycatch fisheries.

In the equations above the difference in the year subscript between the catch-at-age and initial numbers-at-age is because these numbers-at-age pertain to November of the previous year.

The fished proportion from the directed and redeye bycatch fisheries is estimated by:

$$F_{j,y,1} = \frac{\sum_{fleet=1}^{2}\sum_{m=11}^{12}\sum_{l>=lcut_{y,m}} C_{j,y-1,m,l}^{RLF,fleet} \sum_{fleet=1}^{2}\sum_{l>=lcut_{y,m}} C_{j,y,1,l}^{RLF,fleet}}{\sum_{a=1}^{5+}\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch}\right) S_{j,y,1,a}}$$

$$F_{j,y,2} = \frac{\sum_{fleet=1}^{2}\sum_{m=2}^{4}\sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet}}{\sum_{a=0}^{5+}\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch}\right) S_{j,y,2,a}}$$

$$F_{j,y,3} = \frac{\sum_{fleet=1}^{2}\sum_{m=5}^{7}\sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet}}{\sum_{a=0}^{5+}\left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch} - C_{j,y,2,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,3,a}^{bycatch}\right) S_{j,y,3,a}}$$

$$F_{j,y,4} =$$

$$\frac{\sum_{fleet=1}^{2}\sum_{m=8}^{10}\sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet}}{\sum_{a=0}^{5+}\left(\left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch} - C_{j,y,2,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,3,a}^{bycatch} - C_{j,y,3,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,4,a}^{bycatch}\right) S_{j,y,4,a}} \qquad \text{(A.10)}$$

A penalty is imposed within the model to ensure that $F_{j,y,q} < 0.95$.

Recruitment

For the base case assessment of a single stock hypothesis, a Hockey Stick stock-recruitment curve is currently assumed for all years outside of the "peak" years, during which a constant recruitment (i.e. in respect of distribution median and independent of spawning biomass) is assumed. Recruitment at the beginning of November is assumed to fluctuate lognormally about the stock-recruitment curve. For the single stock hypothesis a change in the maximum recruitment is assumed during the peak years:

$$N_{j,y,0}^{S} = \begin{cases} a_j^S e^{\varepsilon_{j,y}^S} & \text{,if SSB}_{j,y}^S \ge b_j^S \\ \frac{a_j^S}{b_j^S} SSB_{j,y}^S e^{\varepsilon_{j,y}^S} & \text{,if SSB}_{j,y}^S < b_j^S \end{cases} \qquad y = 1984,...,1999,2005,...,2010$$

$$N_{j,y,0}^{S} = c^S e^{\varepsilon_{j,y}^S} \qquad y = 2000,...,2004 \quad \text{(A.11)}$$

while for the two stock hypothesis, the same curve is taken to apply to all years, but differ by stocks:

$$N^S_{j,y,0} = \begin{cases} a^S_j e^{\varepsilon^S_{j,y}} & \text{, if } SSB^S_{j,y} \geq b^S_j \\ \dfrac{a^S_j}{b^S_j} SSB^S_{j,y} e^{\varepsilon^S_{j,y}} & \text{, if } SSB^S_{j,y} < b^S_j \end{cases} \qquad y = y_1, \ldots, y_n$$

where

a^S_j is the maximum recruitment of stock j (in billions) (i.e. median of the distribution in question);

b^S_j is the spawner biomass above which there should be no recruitment failure risk in the hockey stick model for stock j;

c^S is the constant recruitment (distribution median) during the "peak" years of 2000 to 2004; and

$\varepsilon^S_{j,y}$ is the annual lognormal deviation of sardine recruitment.

Number of recruits at the time of the recruit survey

The number of recruits at the time of the recruit survey is calculated taking into account the recruit catch during quarters 1 and 2 (November to April) and an estimate of the recruit catch between 1 May and the start of the survey:

$$N^S_{j,y,r} = k^S_{j,r}\left(\left((N^S_{j,y-1,0}e^{-M^S_0/8} - C^S_{j,y,1,0})e^{-M^S_0/4} - C^S_{j,y,2,0}\right)e^{-0.5t^S_y \times M^S_0/12} - \tilde{C}^S_{j,y,0bs}\right)e^{-0.5t^S_y \times M^S_0/12}$$

$$y = y_1, \ldots, y_n \qquad \text{(A.12)}$$

where

$N^S_{j,y,r}$ is the model predicted number (in billions) of juvenile sardine of stock j at the time of the recruit survey in year y;

$k^S_{j,r}$ is the constant of proportionality (multiplicative bias) associated with the recruit survey;

$\tilde{C}^S_{j,y,0bs}$ is the number (in billions) of juvenile sardine of stock j caught between 1 May and the day before the start of the recruit survey (see de Moor et al. 2011); and

t^S_y is the time lapsed (in months) between 1 May and the start of the recruit survey in year y (see de Moor et al. 2011).

The multiplicative bias in the recruit survey is assumed to be equal to that resulting from the acoustic survey as well as the proportion of the recruit abundance which the survey covers in comparison to the November survey. In addition, for the two stock hypothesis, the proportion of the east stock recruit abundance covered compared to that of the west stock abundance is also required. Thus

$k^S_{1,r} = k^S_{\text{cov}} \times k^S_{ac}$

and for the two stock hypothesis, $k^S_{2,r} = k^S_{\text{cov}\,E} \times k^S_{\text{cov}} \times k^S_{ac}$

where

k^S_{cov} is the multiplicative bias associated with the coverage of the recruits by the recruit survey in comparison to the 1+ biomass by the November survey; and

$k^S_{\text{cov}\,E}$ is the multiplicative bias associated with the coverage of the east stock recruits by the recruit survey in comparison to the west stock recruits during the same survey.

Proportion at age and length associated with the commercial catch

The model predicted proportion-at-age in the commercial catch from the directed and redeye bycatch fisheries is:

$$p^{com,S}_{j,y,q,a} = \frac{C^S_{j,y,q,a}}{\sum_{a=1}^{5+} C^S_{j,y,q,a}} \qquad y = y_1, \ldots, y_n\ ,\ q = 1,...,4\ ,\ a = 1,...,5+ \quad \text{(A.13)}$$

The model predicted proportion-at-length in the commercial catch from the directed and redeye bycatch fisheries is:

$$p^{coml,S}_{j,y,q,l\min} = \sum_{a=1}^{5+} \sum_{l=1}^{l\min} A^{com}_{j,q,a,l}\, p^{com,S}_{j,y,q,a} \qquad y = y_1, \ldots, y_n\ ,\ q = 1,...,4 \quad \text{(A.14)}$$

$$p^{coml,S}_{j,y,q,l} = \sum_{a=1}^{5+} A^{com}_{j,q,a,l}\, p^{com,S}_{j,y,q,a} \qquad y = y_1, \ldots, y_n\ ,\ q = 1,...,4\ ,\ l = l\min + 1,...,l\max \quad \text{(A.15)}$$

where

$A^{com}_{j,q,a,l}$ is the proportion of sardine of age a in stock j that fall in the length group l in quarter q .

The matrix $A^{com}_{j,q,a,l}$ is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A^{com}_{j,1,a,l} \sim N\left(L_{j,\infty}\left(1 - e^{-\kappa_j\left(a+1/8-t_{0,j}\right)}\right), \vartheta_{j,a}\right) \quad \text{(A.16)}$$

$$A^{com}_{j,2,a,l} \sim N\left(L_{j,\infty}\left(1 - e^{-\kappa_j\left(a+3/8-t_{0,j}\right)}\right), \vartheta_{j,a}\right) \quad \text{(A.17)}$$

$$A^{com}_{j,3,a,l} \sim N\left(L_{j,\infty}\left(1 - e^{-\kappa_j\left(a+5/8-t_{0,j}\right)}\right), \vartheta_{j,a}\right) \quad \text{(A.18)}$$

$$A^{com}_{j,4,a,l} \sim N\left(L_{j,\infty}\left(1 - e^{-\kappa_j\left(a+7/8-t_{0,j}\right)}\right), \vartheta_{j,a}\right) \quad \text{(A.19)}$$

Fitting the Model to Observed Data (Likelihood)

The survey observations are assumed to be lognormally distributed. The standard errors of the log-distributions for the survey observations of adult biomass and recruitment numbers are approximated by the CVs of the untransformed distributions and a further additional variance parameter. The estimated proportions-at-age are also assumed to be lognormally distributed, with variance inversely proportional to the number of samples used to calculate the ALK and the observed proportion, while the variance for the proportions-at-length are inversely proportional to the observed proportion. Thus the negative log-likelihood function is given by:

$$\begin{aligned}-\ln L = &-\ln L^{Nov} - \ln L^{rec} - \ln L^{sur\ propa} - \ln L^{com\ propa} \\ &- \ln L^{sur\ propl\min} - \ln L^{sur\ propl} - \ln L^{com\ propl\min} - \ln L^{com\ propl}\end{aligned} \qquad \text{(A.20)}$$

where

$$-\ln L^{Nov} = \tfrac{1}{2}\sum_j \sum_{y=y1}^{yn}\left\{\frac{\left(\ln\left(\hat{B}^S_{j,y}\right) - \ln(B^S_{j,y})\right)^2}{(\sigma^S_{j,y,Nov})^2 + (\lambda^S_{j,N})^2} + \ln\left[2\pi\left((\sigma^S_{j,y,Nov})^2 + (\lambda^S_{j,N})^2\right)\right]\right\}$$

$$-\ln L^{rec} = \tfrac{1}{2}\sum_j \sum_{y=y1+1}^{yn}\left\{\frac{\left(\ln\left(\hat{N}^S_{j,y,r}\right) - \ln(N^S_{j,y,r})\right)^2}{(\sigma^S_{j,y,rec})^2 + (\lambda^S_{j,r})^2} + \ln\left[2\pi\left((\sigma^S_{j,y,rec})^2 + (\lambda^S_{j,r})^2\right)\right]\right\}$$

$$-\ln L^{sur\ propa} = w^{survey}_{propa}\ \tfrac{1}{2}\sum_j \sum_{ys} \sum_{a=1}^{5}\left\{\frac{n_{s,y}\,\hat{p}^S_{j,y,a}\left(\ln\left(\hat{p}^S_{j,y,a}\right) - \ln\left(p^S_{j,y,a}\right)\right)^2}{(\sigma^S_p)^2} + \ln\left[2\pi(\sigma^S_p)^2/(n_{s,y}\,\hat{p}^S_{j,y,a})\right]\right\}$$

$$-\ln L^{com\ propa} = w^{com}_{propa}\sum_{yc}\sum_{qc}\sum_{a=1}^{5}\left\{\frac{n^{com}_{y,q}\,\hat{p}^{com,S}_{1,y,q,a}\left(\ln\left(\hat{p}^{com,S}_{1,y,q,a}\right) - \ln\left(p^{com,S}_{1,y,q,a}\right)\right)^2}{(\sigma^S_{com})^2} + \ln\left[2\pi(\sigma^S_{com})^2/(n^{com}_{y,q}\,\hat{p}^{com,S}_{1,y,q,a})\right]\right\}$$

$$-\ln L^{sur\ propl\min} = w^{sur}_{propl\min}\sum_j \sum_{y=y1}^{yn}\left\{\frac{\hat{p}^S_{j,y,l\min}\left(\ln\left(\hat{p}^S_{j,y,l\min}\right) - \ln\left(p^S_{j,y,l\min}\right)\right)^2}{2\left(\sigma^S_{j,l\min}\right)^2} + \ln\left(\frac{\sigma^S_{j,l\min}}{\sqrt{\hat{p}^S_{j,y,l\min}}}\right)\right\}$$ [2]

$$-\ln L^{sur\ propl} = w^{sur}_{propl}\sum_j \sum_{y=y1}^{yn}\sum_{l=l\min+1}^{l\max}\left\{\frac{\hat{p}^S_{j,y,l}\left(\ln\left(\hat{p}^S_{j,y,l}\right) - \ln\left(p^S_{j,y,l}\right)\right)^2}{2\left(\sigma^S_{j,l}\right)^2} + \ln\left(\frac{\sigma^S_{j,l}}{\sqrt{\hat{p}^S_{j,y,l}}}\right)\right\}$$ [3]

$$-\ln L^{com\ propl\min} = w^{com}_{propl\min}\sum_j \sum_{y=y1}^{yn}\sum_{q=1}^{4}\left\{\frac{\hat{p}^{coml,S}_{j,y,q,l\min}\left(\ln\left(\hat{p}^{coml,S}_{j,y,q,l\min}\right) - \ln\left(p^{coml}_{j,y,q,l\min}\right)\right)^2}{2\left(\sigma^S_{j,coml\min}\right)^2} + \ln\left(\frac{\sigma^S_{j,coml\min}}{\sqrt{\hat{p}^{coml,S}_{j,y,q,l\min}}}\right)\right\}$$ [4]

$$-\ln L^{com\ propl} = w^{com}_{propl}\sum_j \sum_{y=y1}^{yn}\sum_{q=1}^{4}\sum_{l=l\min+1}^{l\max}\left\{\frac{\hat{p}^{coml,S}_{j,y,q,l}\left(\ln\left(\hat{p}^{coml,S}_{j,y,q,l}\right) - \ln\left(p^{coml}_{j,y,q,l}\right)\right)^2}{2\left(\sigma^S_{j,coml}\right)^2} + \ln\left(\frac{\sigma^S_{j,coml}}{\sqrt{\hat{p}^{coml,S}_{j,y,q,l}}}\right)\right\}$$ [5] (A.21)

Here

$\hat{B}^S_{j,y}$ is the acoustic survey estimate (in thousands of tonnes) of adult sardine biomass of stock j from the November survey in year y, with associated CV $\sigma^S_{j,y,Nov}$;

$\hat{N}^S_{j,y,r}$ is the acoustic survey estimate (in billions) of sardine recruitment numbers of stock j from the recruit survey in year y, with associated CV $\sigma^S_{j,y,rec}$; and

[2] Note that the years over which the sum occurs excludes those for which survey proportion-at-age data are used.
[3] Although strictly there may be bias in the proportions of length-at-age data, no bias is assumed in this assessment. The effect of such a bias is assumed to be small.

$(\lambda^S_{j,N/r})^2$ is the additional variance (over and above the survey sampling CV $\sigma^S_{y,Nov/rec}$ that reflects survey inter-transect variance) associated with the November/recruit surveys of stock j;

$\hat{p}^S_{j,y,a}$ is an estimate of the proportion (by number) of sardine of age a in stock j in the November survey of year y;

$n_{s,y}$ is the number of fish from the November survey trawls in year y used to compile the age-length key for calculating $\hat{p}^S_{j,y,a}$;

$(\sigma^S_p)^2$ is the overall variance-related parameter for the log-transformed survey proportion-at-age observations, $\hat{p}^S_{j,y,a}$ [note variance = $(\sigma^S_p)^2/(n_y \hat{p}^S_{j,y,a})$];

ys denotes the years for which ALKs are available to calculate proportion-at-age in the November survey ('93, '94, '96, '01, '03, '04, '06-'10);

w^{survey}_{propa} is the weighting applied to the survey proportion-at-age data;

$\hat{p}^{com,S}_{y,q,a}$ [5] is an estimate of the proportion (by number) of single-stock or "west stock" sardine of age a in the commercial catch of quarter q of year y (calculated using the raised length frequencies of the directed and redeye-bycatch fisheries – see de Moor et al. 2011);

$n^{com}_{y,q}$ is the number of fish from the commercial trawls in quarter q of year y used to compile the age-length key for calculating $\hat{p}^{com,S}_{y,q,a}$;

$(\sigma^S_{com})^2$ is the overall variance-related parameter for the log-transformed commercial proportion-at-age observations, $\hat{p}^{com,S}_{y,q,a}$ [note variance = $(\sigma^S_{com})^2/(n^{com}_{y,q}\hat{p}^{com,S}_{y,q,a})$];

yc/qc denotes the years/quarters for which ALKs are available to calculate quarterly proportions-at-age in the commercial catch ('04 Q1-4, '06 Q2-4, '07 Q1-3, '08 Q4, '09 Q1);

w^{com}_{propa} is the weighting applied to the commercial proportion-at-age data;

$\hat{p}^S_{j,y,l}$ is the observed proportion (by number) of sardine in length group l in the November survey of year y;

$w^{sur}_{propl\min}$ is the weighting applied to the survey proportion at length data for the minus length class;

w^{sur}_{propl} is the weighting applied to the remaining survey proportion at length data;

$\sigma^S_{l\min}$ is the variance-related parameter for the log-transformed survey proportion-at-length data of the minus length class, which is estimated in the fitting procedure by the closed form solution:

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[4] Note that the years and quarters over which the sum occurs excludes those for which quarterly commercial proportion-at-age data are used.

$$\sigma^S_{j,l\min} = \sqrt{\sum_{y=y1}^{yn} \hat{p}^S_{j,y,l\min}\left(\ln \hat{p}^S_{j,y,l\min} - \ln p^S_{j,y,l\min}\right)^2 \Big/ \sum_{y=y1}^{yn} 1}$$ [6]; and

σ^S_l is the variance-related parameter for the log-transformed survey proportion-at-length data, which is estimated in the fitting procedure by the closed form solution:

$$\sigma^S_{j,l} = \sqrt{\sum_{y=y1}^{yn} \sum_{l=l\min+1}^{l\max} \hat{p}^S_{j,y,l}\left(\ln \hat{p}^S_{j,y,l} - \ln p^S_{j,y,l}\right)^2 \Big/ \sum_{y=y1}^{yn} \sum_{l=l\min+1}^{l\max} 1}$$ [7].

$\hat{p}^{coml,S}_{j,y,q,l}$ is the observed proportion (by number) of the directed and redeye bycatch commercial catch in length group l of during quarter q ($q=1$ for Nov-Jan, $q=2$ for Feb-Apr, $q=3$ for May-Jul, $q=4$ for Aug-Oct) of year y;

$w^{com}_{propl\min}$ is the weighting applied to the commercial proportion at length data for the minus length class;

w^{com}_{propl} is the weighting applied to the remaining commercial proportion at length data;

$\sigma^S_{coml\min}$ is the variance-related parameter for the log-transformed commercial proportion-at-length data of the minus length class, which is estimated in the fitting procedure by the closed form solution:

$$\sigma^S_{j,coml\min} = \sqrt{\sum_{y=y1}^{yn}\sum_{q=1}^{4} \hat{p}^{coml,S}_{j,y,q,l\min}\left(\ln \hat{p}^{coml,S}_{j,y,q,l\min} - \ln p^{coml}_{j,y,q,l\min}\right)^2 \Big/ \sum_{y=y1}^{yn}\sum_{q=1}^{4} 1}$$ [8]; and

σ^S_{coml} is the variance-related parameter for the log-transformed commercial proportion-at-length data, which is estimated in the fitting procedure by the closed form solution:

$$\sigma^S_{j,coml} = \sqrt{\sum_{y=y1}^{yn}\sum_{q=1}^{4}\sum_{l=l\min+1}^{l\max} \hat{p}^{coml,S}_{j,y,q,l}\left(\ln \hat{p}^{coml,S}_{j,y,q,l} - \ln p^{coml}_{j,y,q,l}\right)^2 \Big/ \sum_{y=y1}^{yn}\sum_{q=1}^{4}\sum_{l=l\min+1}^{l\max} 1}$$ [9].

<u>Fixed Parameters</u>

Six parameters were fixed externally in this model:

$M^S_{j,y} = \overline{M}^S_{ju} = 1$ for all years. Adult natural mortality varies around a median of $\overline{M}^S_{ad} = 0.8$ as follows:

$$M^S_{a,y} = \overline{M}^S_{ad} e^{\varepsilon_{ad,y}} \text{ for } a = 1,\ldots,5+ \text{ with } \varepsilon^{ad}_y = p\varepsilon^{ad}_{y-1} + \sqrt{1-p^2}\,\eta^{ad}_y \qquad \text{(A.8)}$$

where $\eta^{ad}_y \sim N\left(0, \sigma^2_{ad}\right)$ and

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[5] This is not stock-dependent as only ALKs for the “west” coast are available.
[6] Note that the years and quarters over which the sum occurs excludes those for which quarterly commercial proportion-at-age data are used.
[7] Note that the years and quarters over which the sum occurs excludes those for which quarterly commercial proportion-at-age data are used.
[8] Note that the years and quarters over which the sum occurs excludes those for which quarterly commercial proportion-at-age data are used.
[9] Note that the years and quarters over which the sum occurs excludes those for which quarterly commercial proportion-at-age data are used.

σ_{ad} - is the standard deviation in the annual residuals about adult natural mortality; and

p - is the annual autocorrelation coefficient.

Sardine of age 4 are taken to be fully selected in both the survey and commercial trawls:

$S_{j,y,q,4} = 1$, for $y = y_1, \ldots, y_n$, $q = 1,...,4$ and $S_{j,4}^{survey} = 1$.

$\kappa \times L_\infty = 11.4957$ and $t_0 = -1.7087$ obtained from fitting a von Bertalanffy growth curve to the available ageing data (Durholtz and Mtengwane pers. commn.)

The weighting on the commercial proportion-at-length should be a quarter of that on the survey proportion-at-age as there could be up to four observations per year. However, as the survey proportion-at-age data appears to conflict with $-\ln L^{Nov}$, we have $w_{propa}^{survey} = 0.001$ and $w_{propa}^{com} = 0.01$.

$w_{propl\,\min}^{com} = w_{propl}^{com} = 0.0025$ since there are 23 length classes and 5 ages this is a quarter of w_{propa}^{com}.

The survey proportion-at-length data are not currently included in the assessment, thus $w_{propl\,\min}^{com} = w_{propl}^{com} = 0$.

Estimable Parameters and Prior Distributions

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve. For the single stock hypothesis, the variance about the stock recruitment curve is assumed to differ between peak and non-peak years, i.e.the prior pdfs for the recruitment residuals are given by:

$$\varepsilon_{j,y}^S \sim N\left(0, \left(\sigma_r^S\right)^2\right) \quad , \quad y = y_1, \ldots, 1999, 2005, \ldots, y_{n-1}$$

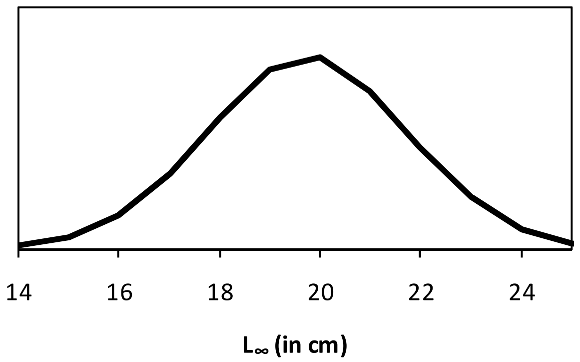
$$\varepsilon_{j,y}^S \sim N\left(0, \left(\sigma_{r,peak}^S\right)^2\right) \quad , \quad y = 2000, \ldots, 2004$$

while for the two stock hypothesis, the variance about the stock recruitment curves is assumed to differ between stocks, but not over years, i.e.

$$\varepsilon_{j,y}^S \sim N\left(0, \left(\sigma_{j,r}^S\right)^2\right) \quad , \quad y = y_1, \ldots, y_{n-1}$$

$k_{ac}^S \sim N\left(0.737, 0.077^2\right)$, see Appendix B

$L_{j,\infty} \sim N\left(19.7416, 2^2\right)$, where 19.7416 is the value of L_∞ obtained from fitting a von Bertalanffy growth curve to the available ageing data (Durholtz and Mtengwane pers. commn) and a standard deviation of 2 does not allocate too much probability to the 23.5-24 cm length class which is the largest observed historically in the November survey



The remaining estimable parameters are defined as having the following near non-informative prior distributions:

$k^{S}_{\mathrm{cov}} \sim U(0.3,1)$

$k^{S}_{\mathrm{cov}\,E} \sim U(0,1)$

$S^{survey}_{j,a} \sim U(0.9,1.1)$ [10], $a = 1,2,3,5+$

$S_{j,y,q,1} = \tilde{S}_{j,1a}$, $S_{j,y,q,5+} = \tilde{S}_{j,5+a}$, with $\tilde{S}_{j,1+a}, \tilde{S}_{j,1+a} \sim U(0,2)$ $\quad y = y_1, \ldots, 2006$

$S_{j,y,q,1} = \tilde{S}_{j,1b}$, $S_{j,y,q,5+} = \tilde{S}_{j,5+b}$, with $\tilde{S}_{j,5+a}, \tilde{S}_{j,5+b} \sim U(0,2)$ $\quad y = 2007, \ldots, y_n$

$S_{j,y,q,a} = \tilde{S}_{j,a}$, with $\tilde{S}_{j,a}, \sim U(0,2)$ $\quad y = y_1, \ldots, y_n$, $a = 2,3$

$\log(a^{S}_{j}) \sim U(0,8)$ (given the lack of *a priori* information on the magnitude of a^{S}, a log-scale was used)

$\log(c^{S}) \sim U(0,8)$

$b^{S}_{j} / K^{S}_{j} \sim U(0,1)$

For the single stock hypothesis: $(\sigma^{S}_{r})^{2} \sim U(0.16,10)$ and $(\sigma^{S}_{r,peak})^{2} \sim U(0.16,10)$

While for the two stock hypothesis: $(\sigma^{S}_{j,r})^{2} \sim U(0.16,10)$

$\log((\sigma^{S}_{p})^{2}) \sim U(-100,0.4)$

$\log((\sigma^{S}_{com})^{2}) \sim U(-100,0.4)$

$N_{1983} \sim U(0,50)$ billion

$$N^{S}_{j,1983,a} = N_{1983} e^{-\left(Finit - \overline{M}^{S}_{j} - (a-1)\overline{M}^{S}_{a}\right)}$$

$$N^{S}_{j,1983,5+} = N_{1983} \frac{e^{-\left(Finit - \overline{M}_{ju} - 4\overline{M}^{S}_{ad}\right)}}{1 - e^{-Finit - \overline{M}^{S}_{ad}}}, \text{ with } Finit = 0.01$$

[10] By design, surveys aim to achieve equal selectivity over all ages. Age 1 sardine distributed inshore may be under caught in comparison to other ages. On the other hand older, faster fish may be more able to avoid day-time trawls and thus be under represented in any day-time (about ½) trawl samples. It is, however, most likely that selectivity of ages 3 to 5+ is flat (Coetzee pers comm.).

$\vartheta_a \sim U(0.01,2)$

$\sigma_{ad} \sim U(0.20,0.5)$

$p \sim U(0,1)$

Further Outputs

Recruitment serial correlation:

$$s^S_{j,cor} = \frac{\sum_{y=y1}^{yn-2} \varepsilon_{j,y}\varepsilon_{j,y+1}}{\sqrt{\left(\sum_{y=y1}^{yn-2} \varepsilon^2_{j,y}\right)\left(\sum_{y=y1}^{yn-2} \varepsilon^2_{j,y+1}\right)}} \tag{A.22}$$

and the standardised recruitment residual value for 2009:

$$\eta^S_{j,2009} = \frac{\varepsilon^S_{j,2009}}{\sigma^S_{j,r}} \tag{A.23}$$

are also required as input into the OM.

For the single stock hypothesis, a separate carrying capacity, K^S_j (essentially the $B^S_{j,N}$ value where the replacement line and the stock recruit function intersect) is calculated representing the period of peak abundance (2000 – 2004) to that for the remaining years:

$$K^S_j = a^S_j \left[\sum_{a=1}^{4} \overline{w}^S_{j,a} e^{-\sum_{a'=0}^{a-1} M^S_{a'}} + \overline{w}^S_{j,5+} e^{-\sum_{a=0}^{4} M^S_a} \frac{1}{1-e^{-M^S_{5+}}} \right] \tag{A.24}$$

$$K^S_{peak} = c^S \left[\sum_{a=1}^{4} \overline{w}^S_{1,a} e^{-\sum_{a'=0}^{a-1} M^S_{a'}} + \overline{w}^S_{1,5+} e^{-\sum_{a=0}^{4} M^S_a} \frac{1}{1-e^{-M^S_{5+}}} \right] \tag{A.25}$$

(calculated assuming maximum recruitment in the absence of fishing), while for the two stock scenario a single carrying capacity is assumed for all years, differing by stocks. Here

$\overline{w}^S_{j,a}$ is the mean mass (in grams) of sardine of age *a* from stock j sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available (de Moor et al. 2011).

Note that we work with median rather than mean estimates of K^S_j and thus a bias correction factor for the log-normal distribution above is not used.

APPENDIX B: Calculating the bias in estimates of sardine from the May and November hydro-acoustic surveys

A probability density function (pdf) for the bias in the May and November survey that relate directly to the acoustic survey, rather than, for example the coverage of the stock, k_{ac}^{S}, was calculated by drawing ten thousand samples from the individual pdfs for each source of constant error, together with the median values of the individual pdfs of each source of variable error (see Table B.1, Anon. 2000). Constant error relates to a factor whose value is not known exactly, but whatever it is, it is the same for each year. In contrast variable errors relate to a factor whose true value will change from one year to the next. A pdf of the inter-transect variance, $(\lambda_{j,N}^{S})^2$, was then calculated by drawing ten thousand samples from the individual pdfs for each source of variable error. The resultant pdfs on the model predicted biomass (i.e. the inverse of the pdf calculated using the errors provided), together with normal distributions fitted to these pdfs are given in Figures B.1 and B.2. There may, however, still be systematic errors relating to the target strength that are unaccounted for in these pdfs. These are taken into account through sensitivity tests using alternative k_{ac}^{S} values.

Table B.1. Individual error factors for hydro-acoustic surveys of sardine biomass, where the values define trapezium form pdfs. Note that these error factors apply to the observed biomass, i.e. they reflect the inverse of the multiplicative bias (applied to the model predicted biomass) in this document.

Error	Minimum	Likely (lower)	Likely (midpoint)	Likely (upper)	Maximum	Nature
Calibration						
(On-axis sensitivity)	0.90	0.95	1.00	1.05	1.10	Variable[12]
(Beam factor)	0.75[11]	0.90	1.00	1.10	1.25	Constant
Surface Schooling	1.00	1.05	1.075	1.10	1.15	Variable
Target Identification	0.50	0.90	1.00	1.10	1.50	Variable[8]
Weather Effects	1.01	1.05	1.15	1.25	2.00	Variable

[11] This was originally reported as 0.8 in Anon 2000, but subsequently corrected (I. Hampton pers. Comm.).
[12] This was recorded in Anon. (2000) as random error denoting that it would be positive or negative rather than purely positive or negative.

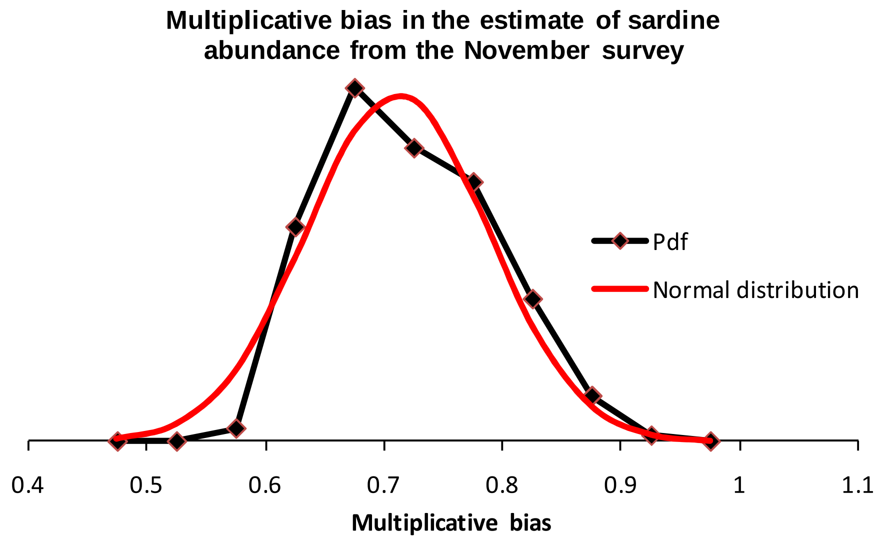


Figure B.1. The probability density function for the overall bias in the estimate of sardine abundance from the November survey, calculated by drawing 10 000 samples from the individual probability distribution functions for each source of constant error, together with the median values of the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $k_{ac}^{S} \sim N\left(0.737, 0.077^{2}\right)$.

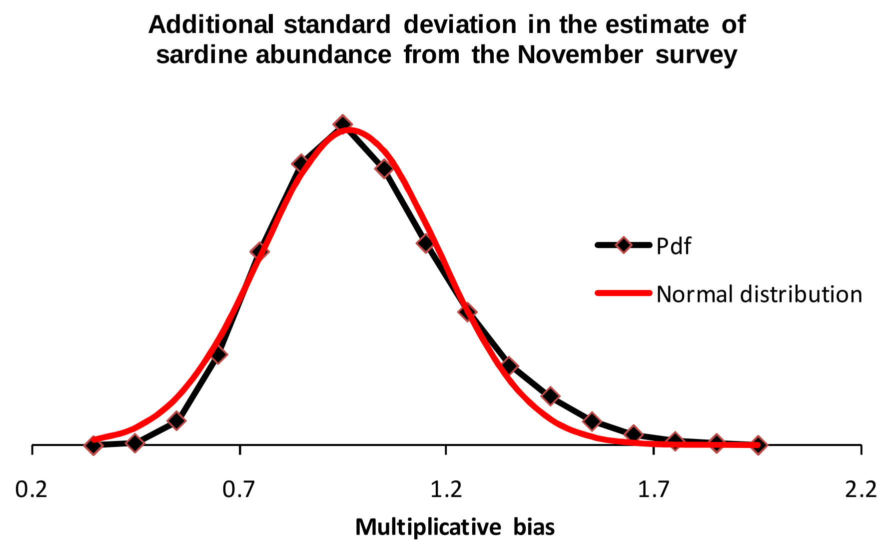


Figure B.2. The probability density function for the additional standard deviation in the estimate of sardine abundance from the November survey, calculated by drawing 10 000 samples from the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $\lambda_{j,N}^{S} \sim N\left(0.969, 0.215^{2}\right)$.

Appendix C: Glossary of parameters used in this document

Annual numbers and biomass:

$N^S_{j,y,a}$ - model predicted number (in billions) of sardine of age a at the beginning of November in year y of stock j

$B^S_{j,y}$ - model predicted biomass (in thousand tonnes) of adult sardine of stock j at the beginning of November in year y, associated with the November survey

$SSB^S_{j,y}$ - model predicted spawning stock biomass (in thousand tonnes) of stock j at the beginning of November in year y

$w^S_{j,y,a}$ - mean mass (in grams) of sardine of age a of stock j sampled during the November survey of year y

$N^S_{j,y,r}$ - model predicted number (in billions) of juvenile sardine of stock j at the time of the recruit survey in year y

t^S_y - time lapsed (in months) between 1 May and the start of the recruit survey in year y

$move_y$ - proportion of west stock recruits which migrate to the east stock at the beginning of November of year y

Natural mortality:

$M^S_{a,y}$ - rate of natural mortality (in year^{-1}) of sardine of age a in year y

$\overline{M}^S_{ju}$ - median juvenile rate of natural mortality (in year^{-1})

$\overline{M}^S_{ad}$ - median adult rate of natural mortality (in year^{-1})

ε^{ad}_y - annual residuals about adult natural mortality

η^{ad}_y - normally distributed error used in calculating ε^{ad}_y

σ_{ad} - standard deviation in the annual residuals about adult natural mortality

p - annual autocorrelation coefficient in annual residuals about adult natural mortality

Catch:

$C^S_{j,y,a,q}$ - model predicted umber (in billions) of sardine of age a of stock j caught during quarter q of year y

$C^{RLF}_{j,y,m,l}$ - number of fish in length class l landed in month m of year y of stock j (the 'raised length frequency')

$lcut_{y,m}$ - cut off length for recruits in month m of year y

$C^{bycatch}_{j,y,q,a}$ - the number of fish of age $a \geq 1$ from the anchovy-directed fishery in quarter q of year y

$S_{j,y,q,a}$ - commercial selectivity at age a during quarter q of year y of stock j

$F_{j,y,q}$ - fished proportion in quarter q of year y for a fully selected age class a of stock j, by the directed and redeye bycatch fisheries

$\tilde{C}^{S}_{j,y,0bs}$ - number (in billions) of juvenile sardine of stock j caught between 1 May and the day before the start of the recruit survey

<u>Proportions at age:</u>

$p^{S}_{j,y,a}$ - model predicted proportion-at-age a of stock j in the November survey of year y

$S^{survey}_{j,a}$ - survey selectivity at age a in the November survey for stock j

$p^{com,S}_{j,y,q,a}$ - model predicted proportion-at-age a of stock j in the directed and redeye bycatch commercial catch of quarter q of year y

<u>Recruitment:</u>

a^{S}_{j} - maximum recruitment of stock j (in billions)

b^{S}_{j} - spawner biomass above which there should be no recruitment failure risk in the hockey stick model for stock j

c^{S} - constant recruitment (distribution median) during the "peak" years of 2000 to 2004

$\varepsilon^{S}_{j,y}$ - annual lognormal deviation of sardine recruitment.

$\sigma^{S}_{j,r}$ - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve of stock j

$\sigma^{S}_{r,peak}$ - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve during peak years in the single stock hypothesis

<u>Proportions at length and growth curve:</u>

$p^{S}_{j,y,l}$ - model predicted proportion-at-length l of stock j associated with the November survey in year y

$A^{sur}_{j,a,l}$ - proportion of sardine of age a of stock j that fall in the length group l in November

$p^{coml,S}_{j,y,q,l}$ - model predicted proportion-at-length l of stock j in the directed and redeye bycatch commercial catch of quarter q of year y

$A^{com}_{j,q,a,l}$ - proportion of sardine of age a of stock j that fall in the length group l in quarter q

$L_{j,\infty}$ - maximum length of sardine of stock j

κ_j - annual growth rate of sardine of stock j

$t_{0,j}$ - age at which the growth rate is zero of sardine of stock j

$\vartheta_{j,a}$ - variance about the meal length for age a of sardine of stock j

Likelihoods:

$-\ln L^{Nov}$ - contribution to the negative log likelihood from the model fit to the November 1+ biomass data

$-\ln L^{rec}$ - contribution to the negative log likelihood from the model fit to the May recruit data

$-\ln L^{sur\ propa}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-age data

$-\ln L^{com\ propa}$ - contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-age data

$-\ln L^{sur\ propl\min}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data for the minus length class only

$-\ln L^{sur\ propl}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data for the minus length class only

$-\ln L^{com\ propl\min}$ - contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data for the minus length class only

$-\ln L^{com\ propl}$ - contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data for the remaining length classes

$\hat{B}^S_{j,y}$ - acoustic survey estimate (in thousands of tonnes) of adult sardine biomass of stock j from the November survey in year y

$\sigma^S_{j,y,Nov}$ - survey sampling CV associated with $\hat{B}^S_{j,y}$ that reflects survey inter-transect variance

$k^S_{j,N}$ - constant of proportionality (multiplicative bias) associated with the November survey of stock j

k^S_{ac} - multiplicative bias associated with the acoustic survey

$\hat{N}^S_{j,y,r}$ - acoustic survey estimate (in billions) of sardine recruitment numbers of stock j from the recruit survey in year y

$\sigma^S_{j,y,rec}$ - survey sampling CV associated with $\hat{N}^S_{j,y,r}$ that reflects survey inter-transect variance

$k^S_{j,r}$ - constant of proportionality (multiplicative bias) associated with the recruit survey of stock j

k^S_{cov} - multiplicative bias associated with the coverage of the recruits by the recruit survey in comparison to the 1+ biomass by the November survey

$k^S_{\text{cov}\,E}$ - multiplicative bias associated with the coverage of the east stock recruits by the recruit survey in comparison to the west stock recruits during the same survey

$(\lambda^S_{j,N/r})^2$ - additional variance (over and above $\sigma^S_{y,Nov/rec}$) associated with the November/recruit surveys of stock j

$\hat{p}^S_{j,y,a}$ - estimate of the proportion (by number) of sardine of age a in stock j in the November survey of year y

$n_{s,y}$ - number of fish from the November survey trawls in year y used to compile the age-length key for calculating $\hat{p}^S_{j,y,a}$

$(\sigma^S_p)^2$ - overall variance-related parameter for the log-transformed survey proportion-at-age observations, $\hat{p}^S_{j,y,a}$ [note variance = $(\sigma^S_p)^2/(n_y \hat{p}^S_{j,y,a})$]

ys - years for which ALKs are available to calculate proportion-at-age in the November survey ('93, '94, '96, '01, '03, '04, '06-'10);

w^{survey}_{propa} - weighting applied to the survey proportion-at-age data

$\hat{p}^{com,S}_{y,q,a}$ - estimate of the proportion (by number) of single-stock or "west stock" sardine of age a in the commercial catch of quarter q of year y

$n^{com}_{y,q}$ - number of fish from the commercial trawls in quarter q of year y used to compile the age-length key for calculating $\hat{p}^{com,S}_{y,q,a}$

$(\sigma^S_{com})^2$ - overall variance-related parameter for the log-transformed commercial proportion-at-age observations, $\hat{p}^{com,S}_{y,q,a}$ [note variance = $(\sigma^S_{com})^2/(n^{com}_{y,q}\hat{p}^{com,S}_{y,q,a})$]

yc/qc - years/quarters for which ALKs are available to calculate quarterly proportions-at-age in the commercial catch ('04 Q1-4, '06 Q2-4, '07 Q1-3, '08 Q4, '09 Q1);

w^{com}_{propa} - weighting applied to the commercial proportion-at-age data

$\hat{p}^{coml,S}_{j,y,q,l}$ - observed proportion (by number) of the directed and redeye bycatch commercial catch in length group l of during quarter q of year y;

$w^{com}_{propl\,\min}$ - weighting applied to the commercial proportion at length data for the minus length class

w^{com}_{propl} - weighting applied to the remaining commercial proportion at length data

$\sigma^S_{coml\,\min}$ - variance-related parameter for the log-transformed commercial proportion-at-length data of the minus length class

σ^S_{coml} - variance-related parameter for the log-transformed commercial proportion-at-length data

Other:

F_{init} - rate of fishing mortality assumed in the initial year

$s^{S}_{j,cor}$ - recruitment serial correlation for stock j

$\eta^{S}_{j,2009}$ - standardised recruitment residual value for 2009 for stock j

K^{S}_{j} - carrying capacity for stock j

K^{S}_{peak} - carrying capacity during peak years (only for single stock hypothesis)

$\overline{w}^{S}_{j,a}$ - mean mass (in grams) of sardine of age a from stock j sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available