# Full description of the Robben Island Penguin-Fish interaction model and base case Bayesian results 

William Robinson ${ }^{1}$ and Doug Butterworth MARAM (Marine Resource Assessment and Management Group)<br>Department of Mathematics and Applied Mathematics, University of Cape Town

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## Introduction

Work on the penguin-fish interaction model has progressed to the point where a reasonable base case for Robben Island has been achieved.

## Methodology

The Appendix describes the full model dynamics. A fundamental component of the model is the assumed relationship between pelagic fish abundance and penguin adult mortality. The statistical distribution assumed for the variability of mortality about the value suggested by the relationship was first described in document FISHERIES/2011/SWG-PEL/01.

Tag data for the full model period (1989-2010) have now been included using a multinomial likelihood following the procedure described in document MCM/2010/SWG-PEL/53. Over-dispersion has been estimated using the program MARK applied to these data in isolation.

Immigration of three year old birds has also been allowed over the period 1989-1999. This can now be estimated given tag-recapture data over this period which remove the confounding between immigration and survival rates that otherwise applies.

Table 1 lists the moult count data to which the model is fitted. Estimates of the direct impact of the two major oil spills are given in Table 2. Table 3 provides a composite list of model parameter values assumed on input.

Bayesian integration was executed using the MCMC algorithm implemented in AD Model Builder. A chain of length $5,500,000$ was run, discarding the first 500,000 and saving every 1000 th iteration. Thus a sample of 10,000 was used to estimate posterior distributions. Initial checks revealed nothing to suggest any problems with convergence.

## Results

Table 4 provides a composite list of model parameters which are estimated and the priors assumed for them. Parameter estimates at the joint posterior mode and the medians and $90 \%$ probability intervals of the Bayesian posterior distributions are given. Table 5 lists the time series of the $90 \%$ probability intervals of the Bayesian posterior distributions for the key parameters, namely penguin numbers, survival rates and random effects on mortality. Table 6 gives results for the parameters relating to the tag data analysis. The tables are followed by various illustrative plots.

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## Discussion

Generally the fits to the data and the residual patterns shown in Figures 2-4 appear acceptable. There are however two features that warrant further attention:
a) a tendency for observed re-sightings to exceed expected numbers after the year 2000, and
b) a PMLE of the power parameter $n$ in the mortality rate verses sardine biomass relationship that is on the boundary of the prior specified (see Table 4 and Figure 8).

These aspects will be investigated further, initially by checking whether they remain given alternative assumptions for some of the parameters fixed on input (see Table 3).

Note estimates of annual immigration over the 1989-1994 period with $90 \%$ probability intervals ranging from about 100 to 1200 (see Table 4). This range seems reasonably compatible with numbers of penguins counted at Dyer and Dassen Islands (the likely origins of these immigrants) over the period in question, though perhaps an upper bound should be introduced.

## Future work

Now that a satisfactory base case model has been attained, the following steps will be pursued:

1) checking sensitivity to alternative choices for the values of parameters fixed as input (see Table 3);
2) exploring alternative functional forms for the relationships between mortality and reproductive success with resource abundance;
3) exploring alternative assumptions (see document FISHERIES/2011/SWG-PEL/03) for the components of sardine and anchovy abundance upon which mortality and reproductive success might depend;
4) splitting reproductive success into its various components (MARAM IWS/DEC10/REP/1), for some of which further data are available;
5) linking the model with the updated OMP operating models to explore the consequences for future penguin abundance of alternative pelagic fish harvesting levels; and
6) extending the model first to some other Western Cape colonies in isolation, and then in combination.

## References

Moseley C, Wanless R, Altwegg R, Crawford R, Badenhorst A, Coetzee J. 2011. Hypotheses for relating anchovy and sardine abundance with African Penguin population processes. FISHERIES/2011/SWG-PEL/03.

Parma A, Punt AE, Stefansson G. 2010. International review panel report for the 2010 International Fisheries Stock Assessment workshop. MARAM IWS/DEC10/REP/1.

Robinson W, Butterworth DS. 2010. A proposed new method for obtaining penguin moult count estimates. MCM/2010/SWG-PEL/Island Closure Task Team/05.

Robinson W, Butterworth DS. 2010. Penguin survival estimates from tag data using a multinomial likelihood. MCM/2010/SWG-PEL/53.

## Appendix

## Description of the Robben Island Penguin-Fish interaction Model

## Basic dynamics

The model considers the number of female penguins $N_{y, a}$ at the start (1 January) of year $y$ of age $a$ at Robben Island. The initial population size (at the start of year 1988) $N_{0}$ and its age structure are:

$$
N_{y_{0}, a}= \begin{cases}N_{0} e^{-a \lambda} & \text { for } 1 \leq a<A  \tag{A1}\\ \frac{N_{0} e^{-a \lambda}}{1-e^{-\lambda}} & \text { for } a=A\end{cases}
$$

where $A$ is the plus-group age. Both $N_{0}$ and $\lambda$ are parameters whose values are estimated.

The following equations describe the population trajectory:

$$
N_{y+1, a}= \begin{cases}\frac{1}{2} H_{y}\left(\sum_{a=a}^{A} N_{y, a}\right) S_{y}^{4 / 12}\left(1-p_{y}^{\text {oil }}\right) & \text { if } a=1  \tag{A2}\\ N_{y, a-1}^{*} S_{y} & \text { if } a=2 \\ N_{y, a-1}^{*} S_{y}+I_{y} & \text { if } a=3 \\ N_{y, a-1}^{*} S_{y} & \text { if } a=4 \\ \left(N_{y, a-1}^{*}+N_{y, a}^{*}\right) S_{y} & \text { if } a=A\end{cases}
$$

with

$$
\begin{equation*}
N_{y, a}^{*}=N_{y, a}\left(1-\frac{N_{y}^{\mathrm{oil}}}{\left(\sum_{a=1}^{A} N_{y, a}\right) S_{y}^{m_{y} / 12}}\right) \tag{A3}
\end{equation*}
$$

where
$S_{y} \quad$ is the adult (post 1 January of first year of life) annual survival rate in year $y$ (taken to the power $4 / 12$ in the equation for $a=1$ since the peak of the breeding season is assumed to be on 1 May),
$H_{y} \quad$ is the annual reproductive success (number of chicks per mature female reaching 1 January of the year following birth, where $50 \%$ of these chicks are assumed to be female),
$a^{*} \quad$ is the age at which the penguins first attempt to breed (taken here to be $a^{*}=4$ ),
$p_{y}^{\text {oil }} \quad$ is the proportion of chicks estimated to have died as a result of an oil spill in year $y$,
$I_{y} \quad$ is the number of penguins (all assumed to be of age 3 ) immigrating to Robben Island in year $y$ (preliminary investigations suggest it appropriate to estimate this as three constant levels for the periods 1989-1991, 1992-1994 and 1995-1999),
$N_{y}^{\text {oil }} \quad$ is the number of juvenile and adult penguins estimated to have died as a result of oiling in year $y$, and
$m_{y} \quad$ is the month in which the oil spill occurred in year $y$.

See Table 2 for the values used for $p_{y}^{\text {oil }}, N_{y}^{\text {oil }}$ and $m_{y}$.

## Population model

Both the annual adult survival rate $S_{y}$ and the annual reproductive success $H_{y}$ are assumed to depend on some function of prey biomass (the deterministic effect), but to be influenced also by some noise (random effects). For reproductive success, the estimates are drawn from a beta distribution which ensures that biologically plausible bounds are respected. However, this approach does not work for the adult penguin survival rate at Robben Island, because a unimodal beta distribution can only have a very small variance when the mean is close to the boundary. This gives a high weight to years in which survival values want to go close to the maximum, which itself drives estimates towards the upper boundary. This is undesirable since it implies that when prey biomass is high, survival can only be very close to the maximum. Thus the alternative approach below was developed and has been found to work satisfactorily.

Adult survival depends on the normalized annual biomass levels $B_{S, y}$, where the time series $I_{S}$ is some function of the sardine and anchovy November spawner biomass and May recruit biomass survey results for particular spatial regions:

$$
\begin{equation*}
B_{S, y}=I_{S, y} / \max \left\{I_{S}\right\} \tag{A4}
\end{equation*}
$$

Similarly, reproductive success depends on a similar index of biomass level:

$$
\begin{equation*}
B_{H, y}=I_{H, y} / \operatorname{mean}\left\{I_{H}\right\} \tag{A5}
\end{equation*}
$$

In the base case implemented here, $B_{S}$ is taken to be the sardine November survey biomass west of Cape Agulhas (strata $\mathrm{A}-\mathrm{C}$ ), and $B_{H}$ is anchovy recruit survey biomass west of Cape Infanta.

## Adult survival

The rates of annual adult survival $S_{y}$ and annual natural mortality $M_{y}$ are related as

$$
\begin{equation*}
S_{y}=e^{-M_{y}} \tag{A6}
\end{equation*}
$$

where $M_{y}$ is modelled as follows:

$$
\begin{equation*}
M_{y}=M_{\min }+\bar{M}_{y} e^{X_{y}} \tag{A7}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{M}_{y}=M_{\min }^{*}+f_{S}\left(B_{S, y}\right) \tag{A8}
\end{equation*}
$$

and $X_{y}$ is distributed $N\left(0, \sigma_{y}^{2}\right)$ with

$$
\begin{equation*}
\sigma_{y}=\sqrt{e^{\tilde{\sigma}^{2} /\left(M_{\min }^{*}+f_{S}\left(B_{S, y}\right)\right)^{2}}-1} \tag{A9}
\end{equation*}
$$

Thus we have a log-normal random effect, but since the $\sigma_{y}$ depend on the biomass $B_{y}$, the $M_{y}$ distributions will all have exactly the same standard deviation. This is appropriate since data related to each year receives roughly equal weighting, and, when projecting, high biomass does not force low mortality.

The reason that the $M_{\text {min }}^{*}$ term is introduced is that when calculating $M_{y}$ the possibility that the term additional to $M_{\min }$ could go to zero when biomass is high is excluded. This would be a problem as then $\sigma_{y}$ would go infinite in equation (A9), rendering the associated data point redundant because of its infinite variance. The lower bound on achieved $M_{y}$ remains $M_{\min }$, but the lower bound on the median of its distribution is $M_{\text {min }}+M_{\text {min }}^{*}$. The base case model uses the following constant values: $\tilde{\sigma}=0.1, M_{\min }=0.04$ and $M_{\min }^{*}=0.02$.

The biomass-mortality relationship is defined as

$$
\begin{equation*}
f_{S}\left(B_{S, y}\right)=\left(a+\frac{B_{S, y}}{b}\right)^{-n} \tag{A10}
\end{equation*}
$$

Instead of $a$ and $b$, the expression can be re-parameterised in terms of the values of $\bar{M}_{y}$ at $B_{S}=0.1$ and $B_{S}=1$ in order to introduce estimable parameters which are more orthogonal (i.e. have low correlation, which assists with statistically stable estimation). If $U=\bar{M}\left(B_{S}=1\right)$ and $V=\bar{M}\left(B_{S}=0.1\right)-\bar{M}\left(B_{S}=1\right)$, then

$$
\begin{gather*}
b=\frac{0.9}{\left[U-M_{\min }^{*}\right]^{-1 / n}-\left[U+V-M_{\min }^{*}\right]^{-1 / n}}  \tag{A11}\\
a=\left[U-M_{\min }^{*}\right]^{-1 / n}-\frac{1}{b} \tag{A12}
\end{gather*}
$$

The estimable parameters are $U, V$ and $n$, where sensibly $V \geq 0$.

The prior added to the negative log likelihood for each year to reflect assumptions made above for the $X_{y}$ parameters is:

$$
\begin{equation*}
P_{S}=\frac{1}{2} \sum_{y}\left(\frac{X_{y}}{\sigma_{y}}\right)^{2} \tag{A13}
\end{equation*}
$$

An additional penalty term ("prior") ensures that the annual mortality rates are evenly distributed about the curve relating mortality and biomass, i.e. the sum of the residuals is forced to zero (this was found to aid estimation stability):

$$
\begin{equation*}
P_{\mathrm{B}-\mathrm{M}}=10^{5}\left\{\sum_{y}\left[M_{y}-\left(M_{\min }+\bar{M}_{y}\right)\right]\right\}^{2} \tag{A14}
\end{equation*}
$$

## Reproductive success

The parameters $\bar{H}_{y}^{*}=f_{H}\left(B_{H, y}\right)$ are estimated on the interval $[0,1]$. These are the transformed predicted annual reproductive success rates $\bar{H}_{y}$ which fall in the interval $\left[0, H_{\max }\right]$ :

$$
\begin{equation*}
\bar{H}_{y}=H_{\max } \bar{H}_{y}^{*} \tag{A15}
\end{equation*}
$$

The assumed relationship between $\bar{H}_{y}^{*}$ and the fish abundance index $B_{H, y}$ is currently assumed to be a constant given that earlier analyses have not provided any indication of dependence:

$$
\begin{equation*}
f_{H}\left(B_{H, y}\right)=h \tag{A16}
\end{equation*}
$$

but alternative functional forms may be considered in future.
The parameters $H_{y}^{*}$ are estimated for each year on the interval $[0,1]$ and then transformed to the range $\left[0, H_{\max }\right]$ to obtain the annual reproductive success rates $H_{y}$ as follows:

$$
\begin{equation*}
H_{y}=H_{\max } H_{y}^{*} \tag{A17}
\end{equation*}
$$

We assume that the $H_{y}^{*}$ parameters are beta-distributed about the predicted values $\bar{H}_{y}^{*}$. Setting $\kappa_{H, y}=\bar{H}_{y}^{*}\left(1-\bar{H}_{y}^{*}\right) / \sigma_{H}^{2}-1$, the parameters of this beta distribution $\alpha_{H, y}$ and $\beta_{H, y}$ are:

$$
\begin{equation*}
\alpha_{H, y}=\bar{H}_{y}^{*} \kappa_{H, y} \quad \beta_{H, y}=\kappa_{H, y}\left(1-\bar{H}_{y}^{*}\right) \tag{A18}
\end{equation*}
$$

The prior added to the negative log likelihood for each year, which reflects the assumption that $H_{y}^{*}$ is beta-distributed, is:

$$
\begin{equation*}
P_{H, y}=-\ln \left(\frac{\Gamma\left(\alpha_{H, y}+\beta_{H, y}\right)}{\Gamma\left(\alpha_{H, y}\right) \Gamma\left(\beta_{H, y}\right)}\right)-\left(\alpha_{H, y}-1\right) \ln H_{y}^{*}-\left(\beta_{H, y}-1\right) \ln \left(1-H_{y}^{*}\right) \tag{A19}
\end{equation*}
$$

An additional penalty term ensures that the annual reproduction success rates are evenly distributed about the assumed relationship curve, i.e. the sum of the residuals is forced to zero (this was found to aid estimation stability):

$$
\begin{equation*}
P_{\mathrm{B}-\mathrm{R}}=10^{5}\left\{\sum_{y}\left[H_{y}-\bar{H}_{y}\right]\right\}^{2} \tag{A20}
\end{equation*}
$$

## Moult counts

The population model is fitted to annual moult count data for both adult and juvenile birds by taking them into account through the following negative log-likelihood functions:

$$
\begin{align*}
-\ln L_{\mathrm{M}} & =\sum_{y}\left[\ln \sigma_{\mathrm{M}, y}+\frac{1}{2 \sigma_{\mathrm{M}, y}^{2}}\left(\ln N_{y}^{\mathrm{obs}}-\ln N_{y}^{\mathrm{model}}\right)^{2}\right]  \tag{A21}\\
-\ln L_{\mathrm{J}} & =\sum_{y}\left[\ln \sigma_{\mathrm{J}}+\frac{1}{2 \sigma_{\mathrm{J}}^{2}}\left(\ln J_{y}^{\mathrm{obs}}-\ln J_{y}^{\mathrm{model}}\right)^{2}\right]
\end{align*}
$$

$\sigma_{\mathrm{M}, y}$ and $\sigma_{\mathrm{J}}$ are respectively the standard errors of the logarithms of the adult moult counts and juvenile proportions of these counts about their true values (i.e. these reflect observation errors). The terms in the likelihood for the adult moult counts are weighted according to the CVs of the observations (see Table 1) with

$$
\begin{equation*}
\sigma_{\mathrm{M}, y}=\sqrt{\mathrm{CV}\left(N_{y}^{\mathrm{obs}}\right)^{2}+0.05^{2}+\sigma_{\mathrm{add}}^{2}} \tag{A22}
\end{equation*}
$$

where the additional variance $\sigma_{\text {add }}^{2}$ is an estimable parameter, with the true additional variance forced to be at least $0.05^{2}$. The value of $\sigma_{\mathrm{J}}$ is fixed at $\sigma_{\mathrm{J}}=0.1$.
$N_{y}^{\text {model }}=q_{\mathrm{M}} \sum_{a=2}^{A} N_{y, a} S_{y}^{11 / 12}$ is the number of female birds in adult plumage (aged 2 and over) counted in year $y$, where the peak of the counting season is the end of November, and $q_{\mathrm{M}}$ is the proportion of these birds susceptible to observation (assumed here to be $q_{\mathrm{M}}=0.9$ ).
$J_{y}=\frac{p_{\mathrm{J}} N_{y, 1}}{\sum_{a=1}^{A} N_{y, a}}$ is the proportion of juvenile birds in the model at the time of the moult count (note that $S_{y}^{11 / 12}$ factors in numerator and denominator cancel), with $p_{\mathrm{J}}$ being the detectability of juvenile moulters relative to adults in the counting process, which is assumed here to be $p_{\mathrm{J}}=1$.
$N_{y}^{\text {obs }} \quad$ is the number of female adult moulters observed in year $y$ (taken to be one half of the total adult moulters counted).
$J_{y}^{\text {obs }} \quad$ is the observed proportion of moulters in immature plumage counted in year $y$.

## Tag data

In addition to the moult counts, tag data for the period 1989-2010 has been incorporated into the analysis. This provides an independent estimate of annual survival rates and allows for the estimation of immigration. The multinomial likelihood of the encounter histories is calculated through the estimation of re-sighting probabilities and survival rates for each year. This is the method used in program MARK. A variation on the standard MARK procedure which has been included in the analysis is the estimation of a "transient" factor relating to birds tagged which are never seen again, modelled as additional mortality in the year following tagging to reflect emigration. This factor is assumed to be equal for all years, except for the years of the major oil spills (Apollo Sea in 1994 and Treasure in 2000) for which separate values are estimated, as these birds are more likely to be linked to Robben Island than would be the case for normal "transients".

The log-likelihood function for the multinomial distribution, ignoring the constant multinomial coefficient, is:

$$
\begin{equation*}
\ln L_{\mathrm{T}-\mathrm{R}}=\frac{1}{\hat{c}} \sum_{j=1}^{m} n_{j} \ln \hat{p}_{j} \tag{A23}
\end{equation*}
$$

where $\hat{p}_{j}$ is the estimated probability of the $j$ th encounter history occurring and $n_{j}$ is the number of times which that history has been observed. The number of unique encounter histories is $m$. In order to account for over-dispersion, the likelihood is scaled by a factor of $1 / \hat{c}$, which was obtained from analysing the data using MARK.

## Objective function

The overall log posterior is thus:

$$
\begin{equation*}
-\ln P_{\mathrm{post}}=-\ln L_{\mathrm{M}}-\ln L_{\mathrm{J}}-\ln L_{\mathrm{T}-\mathrm{R}}+P_{S}+P_{\mathrm{B}-\mathrm{M}}+P_{H}+P_{\mathrm{B}-\mathrm{R}} \tag{A24}
\end{equation*}
$$

## Tables

Table 1: Counts of birds moulting at Robben Island with CVs (MCM/2010/SWG-PEL/Island Closure Task Team/05) and the proportion of juveniles.

| Year | Number of adult <br> female moulters | CV | Proportion of <br> juveniles |
| :---: | :---: | :---: | :---: |
| 1988 | 1734 | 0.197 | 0.188 |
| 1989 | 1698 | 0.184 | 0.202 |
| 1990 | 2362 | 0.176 | 0.160 |
| 1991 | 2470 | 0.101 | 0.240 |
| 1992 | 3272 | 0.097 | 0.200 |
| 1993 | 3958 | 0.100 | 0.166 |
| 1994 | 3967 | 0.115 | 0.147 |
| 1995 | 3337 | 0.112 | 0.174 |
| 1996 | 3650 | 0.069 | 0.238 |
| 1997 | 4278 | 0.066 | 0.218 |
| 1998 | 4750 | 0.058 | 0.224 |
| 1999 | 5847 | 0.056 | 0.194 |
| 2000 | 6441 | 0.065 | 0.158 |
| 2001 | 6181 | 0.058 | 0.180 |
| 2002 | 7234 | 0.044 | 0.198 |
| 2003 | 8488 | 0.047 | 0.164 |
| 2004 | 6221 | 0.063 | 0.168 |
| 2005 | 3830 | 0.059 | 0.259 |
| 2006 | 3227 | 0.083 | 0.229 |
| 2007 | 2579 | 0.114 | 0.212 |
| 2008 | 1872 | 0.087 | 0.326 |
| 2009 | 2215 | 0.090 | 0.284 |

Table 2: Model inputs relating to oiling

| Constant | Description | Value |
| :--- | :--- | :--- |
| $N_{1994}^{\text {oil }}$ | Number of adult females which died as a result of the <br> 1994 oil spill <br> $N_{2000}^{\text {oil }}$ | Number of adult females which died as a result of the <br> 2000 oil spill <br> Proportion of chicks which died due to the 1994 oil spill |
| $p_{1994}^{\text {oil }}$ | 750 |  |
| $p_{2000}^{\text {oil }}$ | Proportion of chicks which died due to the 2000 oil spill | 0.39 |
| $m_{1994}$ | Month of the 1994 oil spill <br> $m_{2000}$ | 5.7 |

Table 3: Parameter values fixed on input

| Parameter | Description | Value |
| :--- | :--- | :--- |
| $A$ | Plus group age | 5 |
| $a^{*}$ | Age of first breeding attempt | 4 |
| $\sigma_{\mathrm{J}}$ | Standard errors of the logarithms of the juvenile proportions | 0.1 |
| $\tilde{\sigma}$ | Parameter related to variability about adult mortality vs fish abundance (A9) | 0.1 |
| $\sigma_{H}$ | Standard deviation of reproductive success | 0.1 |
| $q_{\mathrm{M}}$ | Proportion of moulters susceptible to observation | 0.9 |
| $M_{\min }$ | Minimum allowed mortality rate (A7) | 0.04 |
| $M_{\min }^{*}$ | Additional term in biomass-mortality relationship (A7) | 0.02 |
| $H_{\max }$ | Maximum allowed reproductive success (A15) | 1.8 |
| $p_{\mathrm{J}}$ | Detectability of juveniles relative to adults in the moult count | 1.0 |

Table 4: List of estimated parameters, prior distributions and Bayesian posterior probability intervals

| Parameter | Description | Prior | Mode | 5\% | Median | 95\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p_{\text {TR }, ~ y}$ | Re-sighting probabilities | $\mathrm{U}[0,1]$ |  |  |  |  |
| $M_{\text {trans }}$ | "transient" mortality of tagged birds | $\mathrm{U}[0,1]$ | 0.379 | 0.311 | 0.402 | 0.497 |
| $M_{\text {trans,1994 }}$ |  | $\mathrm{U}[0,1]$ | 0.000 | 0.001 | 0.006 | 0.010 |
| $M_{\text {trans,2000 }}$ |  | $\mathrm{U}[0,1]$ | 0.000 | 0.000 | 0.000 | 0.000 |
| $\ln N_{0}$ | Log of initial population size | $\mathrm{U}[1,10]$ | 6.794 | 6.422 | 6.783 | 7.161 |
| $\lambda$ | Initial population profile parameter | $\mathrm{U}[0,3]$ | 0.211 | 0.179 | 0.214 | 0.256 |
| $\sigma_{\text {add }}$ | Additional variance in moult counts | $\mathrm{U}[0,1]$ | 0.000 | 0.000 | 0.000 | 0.000 |
| $I_{1989-1991}$ | Immigration of three year old birds | U [0, 3000] | 427.7 | 127.4 | 404.5 | 688.8 |
| $I_{1992-1994}$ |  | U [0, 3000] | 973.6 | 530.8 | 858.7 | 1202.1 |
| $I_{1995-1999}$ |  | $\mathrm{U}[0,3000]$ | 0.0 | 0.1 | 3.1 | 4.9 |
| $U$ | $M\left(B_{S}=1\right)$ | $\mathrm{U}[0.02,0.2]$ | 0.025 | 0.022 | 0.024 | 0.030 |
| V | $M\left(B_{S}=0.1\right)-M\left(B_{S}=1\right)$ | U[0, 0.75] | 0.437 | 0.376 | 0.429 | 0.476 |
| $n$ | Biomass-mortality relationship | $\mathrm{U}[1,8]$ | 8.000 | 6.718 | 7.331 | 7.938 |
| $X_{y}$ | Adult mortality random effects | U[-4.5, 4.5] |  |  |  |  |
| $h$ | Reproductive success relationship | $\mathrm{U}[0,1]$ | 0.509 | 0.486 | 0.517 | 0.547 |
| $H_{y}$ | Reproductive success | $\begin{aligned} & \mathrm{U}[0.0001, \\ & 0.9999] \end{aligned}$ |  |  |  |  |
| $-\ln L_{\mathrm{M}}$ | Moult count likelihood |  | -42.9 |  |  |  |
| $-\ln L_{\text {J }}$ | Juvenile proportion likelihood |  | -48.1 |  |  |  |
| $-\ln L_{\text {T-R }}$ | Tag re-sighting likelihood |  | 6112.7 |  |  |  |
| $P_{S}$ | Prior on $X_{y}$ parameters |  | 5.5 |  |  |  |
| $P_{H}$ | Prior on $H_{y}^{*}$ parameters |  | -24.3 |  |  |  |
| $-\ln P_{\text {post }}$ | Total negative log posterior |  | 6002.8 |  |  |  |

Table 5: Time series of the medians and $90 \%$ probability intervals of the Bayesian posterior distributions for the number of adult female moulting penguins $N$ (see Figure 6), the annual survival rate $S$ (see Figure 7) and the random effects on mortality $X$.

|  |  | $N$ |  | $S$ |  |  |  |  <br>  $\mathrm{5} \mathrm{\%}$ |  |  | $50 \%$ | $95 \%$ | $5 \%$ | $50 \%$ | $95 \%$ | $5 \%$ | $50 \%$ | $95 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1988 | 1421 | 1800 | 2270 | 0.56 | 0.65 | 0.73 | -0.48 | -0.17 | 0.12 |  |  |  |  |  |  |  |  |  |
| 1989 | 1693 | 1977 | 2326 | 0.69 | 0.77 | 0.83 | -0.80 | -0.39 | -0.03 |  |  |  |  |  |  |  |  |  |
| 1990 | 1962 | 2228 | 2540 | 0.69 | 0.78 | 0.85 | -0.76 | -0.24 | 0.19 |  |  |  |  |  |  |  |  |  |
| 1991 | 2391 | 2745 | 3148 | 0.85 | 0.93 | 0.96 | -2.90 | -0.95 | 0.25 |  |  |  |  |  |  |  |  |  |
| 1992 | 2449 | 2743 | 3078 | 0.53 | 0.62 | 0.70 | 0.20 | 0.53 | 0.81 |  |  |  |  |  |  |  |  |  |
| 1993 | 3143 | 3407 | 3699 | 0.74 | 0.84 | 0.93 | -0.99 | 0.21 | 0.99 |  |  |  |  |  |  |  |  |  |
| 1994 | 3483 | 3812 | 4173 | 0.82 | 0.87 | 0.92 | -1.20 | -0.44 | 0.15 |  |  |  |  |  |  |  |  |  |
| 1995 | 3593 | 3906 | 4234 | 0.82 | 0.87 | 0.92 | -1.29 | -0.60 | -0.05 |  |  |  |  |  |  |  |  |  |
| 1996 | 3529 | 3798 | 4097 | 0.75 | 0.80 | 0.85 | -0.73 | -0.33 | 0.03 |  |  |  |  |  |  |  |  |  |
| 1997 | 3935 | 4208 | 4504 | 0.79 | 0.85 | 0.91 | 0.46 | 1.26 | 1.82 |  |  |  |  |  |  |  |  |  |
| 1998 | 4635 | 4964 | 5306 | 0.87 | 0.93 | 0.96 | -3.41 | 0.03 | 1.22 |  |  |  |  |  |  |  |  |  |
| 1999 | 5194 | 5488 | 5797 | 0.80 | 0.86 | 0.92 | -0.20 | 0.71 | 1.33 |  |  |  |  |  |  |  |  |  |
| 2000 | 5553 | 5890 | 6251 | 0.96 | 0.96 | 0.96 | -4.48 | -4.37 | -3.91 |  |  |  |  |  |  |  |  |  |
| 2001 | 6220 | 6641 | 7065 | 0.90 | 0.95 | 0.96 | -4.00 | -1.64 | 0.06 |  |  |  |  |  |  |  |  |  |
| 2002 | 6562 | 7052 | 7612 | 0.80 | 0.88 | 0.96 | -3.22 | 1.19 | 1.91 |  |  |  |  |  |  |  |  |  |
| 2003 | 7295 | 7925 | 8588 | 0.80 | 0.91 | 0.96 | -3.07 | 0.84 | 2.00 |  |  |  |  |  |  |  |  |  |
| 2004 | 5784 | 6361 | 7001 | 0.57 | 0.65 | 0.74 | 0.21 | 0.60 | 0.92 |  |  |  |  |  |  |  |  |  |
| 2005 | 3790 | 4117 | 4482 | 0.47 | 0.53 | 0.59 | -0.23 | -0.03 | 0.18 |  |  |  |  |  |  |  |  |  |
| 2006 | 3205 | 3520 | 3866 | 0.58 | 0.65 | 0.72 | -0.26 | 0.06 | 0.33 |  |  |  |  |  |  |  |  |  |
| 2007 | 2222 | 2478 | 2758 | 0.47 | 0.53 | 0.60 | -0.33 | -0.15 | 0.04 |  |  |  |  |  |  |  |  |  |
| 2008 | 1796 | 1996 | 2221 | 0.57 | 0.65 | 0.73 | -0.13 | 0.24 | 0.52 |  |  |  |  |  |  |  |  |  |
| 2009 | 1941 | 2192 | 2477 | 0.72 | 0.80 | 0.87 | -0.88 | -0.32 | 0.15 |  |  |  |  |  |  |  |  |  |
| 2010 | 2028 | 2420 | 2843 | 0.73 | 0.83 | 0.90 | -1.08 | -0.33 | 0.29 |  |  |  |  |  |  |  |  |  |

Table 6: Results of fitting to penguin tagging data.

|  | Probability of re-sighting |  |  | Number of re-sightings |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mode | $5 \%$ | $50 \%$ | $95 \%$ | Observed | Expected |
| 1990 | 0.56 | 0.40 | 0.56 | 0.73 | 80 | 71.2 |
| 1991 | 0.28 | 0.18 | 0.29 | 0.45 | 57 | 68.6 |
| 1992 | 0.19 | 0.13 | 0.20 | 0.29 | 80 | 57.5 |
| 1993 | 0.26 | 0.19 | 0.27 | 0.37 | 107 | 156.3 |
| 1994 | 0.45 | 0.35 | 0.45 | 0.55 | 182 | 264.6 |
| 1995 | 0.53 | 0.46 | 0.53 | 0.59 | 469 | 550.0 |
| 1996 | 0.55 | 0.48 | 0.55 | 0.61 | 514 | 516.7 |
| 1997 | 0.53 | 0.46 | 0.53 | 0.60 | 443 | 460.5 |
| 1998 | 0.72 | 0.65 | 0.71 | 0.78 | 580 | 648.6 |
| 1999 | 0.67 | 0.60 | 0.67 | 0.73 | 616 | 618.3 |
| 2000 | 0.47 | 0.40 | 0.47 | 0.54 | 409 | 458.4 |
| 2001 | 0.55 | 0.53 | 0.55 | 0.57 | 4302 | 4250.2 |
| 2002 | 0.27 | 0.25 | 0.27 | 0.30 | 2081 | 1884.2 |
| 2003 | 0.22 | 0.20 | 0.22 | 0.25 | 1512 | 1420.1 |
| 2004 | 0.18 | 0.16 | 0.18 | 0.21 | 1181 | 818.9 |
| 2005 | 0.10 | 0.08 | 0.10 | 0.13 | 440 | 247.3 |
| 2006 | 0.31 | 0.26 | 0.31 | 0.36 | 712 | 484.9 |
| 2007 | 0.33 | 0.27 | 0.33 | 0.38 | 505 | 288.7 |
| 2008 | 0.52 | 0.44 | 0.53 | 0.61 | 469 | 320.2 |
| 2009 | 0.54 | 0.44 | 0.55 | 0.67 | 330 | 285.6 |
| 2010 | 0.14 | 0.10 | 0.15 | 0.21 | 100 | 71.6 |

## Figures

Sardine spawners


Figure 1: Index of sardine November survey biomass west of Cape Agulhas. Dashed horizontal lines indicate the three biomass levels used for the projections illustrated in Figure 11.

## Probability of re-sighting



Number of re-sightings


Figure 2: Results of fitting to tag-recapture data. Top: annual probability of re-sighting banded birds. Bottom: comparison of observed and expected numbers of banded penguins re-sighted each year for the joint posterior mode.


Figure 3: Fits to data/relationships and associated residuals at the joint posterior mode. Projections beyond 2010 assume $B_{S}=0.2$. The residuals in (b) and (d) are the differences between the logarithms of the observations and the model predicted values which appear in equation (A21). The residuals in (f) are the differences between the estimated reproductive success rates and the assumed relationship (A16).


Figure 4: Further fits to data/relationships and associated residuals for the joint posterior mode. Projections beyond 2010 assume $B_{S}=0.2$. The residuals in (b) are the differences between the estimated annual mortality rates and those predicted by the relationship with fish abundance (A8). The random effects in (c) are the $X_{y}$ in (A7), and are standardised in (d) by dividing by $\sigma_{y}$. Plot (e) shows the time series of the estimates for the adult survival rates, and (f) shows the corresponding mortality rates.

## Adult mortality/Fish abundance relationship



Figure 5: Estimated relationship between penguin adult mortality and the sardine spawner biomass west of Cape Agulhas showing the value at the joint posterior mode and the posterior median and 90\% probability interval.


Figure 6: Time series of observed counts of female moulting penguins and the median and $90 \%$ probability interval of the Bayesian posterior distribution of the model predicted moult counts.


Figure 7: Time series of the adult annual survival rates at the joint posterior mode and the median and $90 \%$ probability interval of the Bayesian posterior distribution.

## Posterior distributions



Figure 8: Prior and posterior distributions of the parameters in the mortality-biomass relationship.


Figure 9: "Transient" mortality comparison of priors and posteriors. The "transient M" applies for all years except those with major oil spills (1994 and 2000) from which many penguins were tagged for which separate estimates are made. The solid black bars indicate the posterior to be entirely at $\mathrm{M}=0$.


Figure 10: Immigration, taken to be constant over the periods shown: comparison of priors and posteriors.


Figure 11: Projections of penguin moult counts from the joint posterior mode from 2009 for fixed future sardine spawner biomass levels west of Cape Agulhas for $B_{S}=0.1, B_{S}=0.2$ and $B_{S}=0.3$.


[^0]:    ${ }^{1}$ william.robinson@uct.ac.za

