# Preliminary assessment of the western and south-western Cape Carpenter resource using an age-structured production model 

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#### Abstract

An age-structured production model (ASPM) is used to assess the Carpenter resource off the south western coast of South Africa. The model is fitted to standardized CPUE and length frequency linefish data. Problems encountered when attempting to fit the model to these data are explored. Reasons may include conflicting data, or complexities in the resource dynamics that are not incorporated in the simple density-dependent ASPM dynamics. Plausible parameter estimates are only achieved when fixing or imposing penalty functions for key parameters. However, this leads to deterioration in the fit to the data with systematic trends in the residuals which render results suspect and management advice based on such assessments dubious. Indeed, a management procedure approach may be better suited to circumstances when a "best assessment" is problematic, as seems to be the case here.


## 1 Introduction

An age-structured production model (ASPM - see Appendix 1) is used here to assess the Carpenter (silverfish) stock off the south western coast (including the Agulhas Bank) of South Africa. No comprehensive assessment has been conducted for this resource since 1999 (Linefish Scientific Working Group Report, 2011). That Report notes that the resource is estimated to be "collapsed" with an approximate $88 \%$ decrease since the fishery commenced in the late 1800s (Griffiths 2000). It has thus become a pressing priority to undertake a thorough assessment of this resource, with the eventual aim of developing a management procedure for the Carpenter fishery to ensure future recovery and sustainable long-term use. This assessment intends to be first step in this direction.

## 2 Data

Total annual Carpenter catches (in tons) for the western South African zone as well as standardised CPUE data and associated CVs for the commercial linefish fleet are given in Table 1 of the Appendix 2. The catch-at-length data can be found in Table 2 of Appendix 2. All data were kindly provided by H. Winkler.

## 3 Methods

An age-structured production model (ASPM) is used for this assessment. The technical specifications can be found in Appendix 1 of this document. The Pope approximation for the catch equation was employed to facilitate comparison with results of Booth et al. (2011). CPUE data are incorporated in the likelihood function using the "additional variance" approach of Geromont and Butterworth (2001). The catch-at-length data were fitted using a proportionally weighted method set out in Brandao and Butterworth (2009) where model predicted catches-at-age are converted to catches-at-length using an age-length transformation matrix.

Parameters estimated include $K$ (carrying capacity), $h$ (steepness of the stock-recruitment curve), the raio of the initial spawning biomass in the first year of the assessment to its pre-exploitation level, $B_{1} / K$, the selectivity-at-age vector $S_{a}$, natural mortality rate, $M_{a}$, which is assumed to be ageindependent, and the stock-recruit residuals. Prior distributions adopted for these parameters are given in Section 1.3 of Appendix 1.

## 4 Results

Both standardized linefish CPUE and length data are incorporated in the likelihood for the model fit for all runs. Results are grouped into three categories according to values input for the initial spawning biomass compared to the pre-exploitation level ( $B_{1} / K$ ), ranging from $10 \%$ to $50 \%$ of pristine (Tables 1,2 and 3 ). This was necessary as preliminary runs showed that this parameter was not well-estimated (it approached an upper bound of 1, implying that the 1985 biomass was at its preexploitation level), so that a prior/penalty needed to be included for this parameter, or else $B_{1} / K$ had to be fixed. For the base case runs $M_{a}$ was fixed to a value of $0.2 y r^{-1}$ for all ages $a$, the central value of the prior distribution proposed by Kerwath and Winkler (pers. commn). There are four options shown for each case: estimate the steepness $(h)$ of the stock-recruitment curve, or fix $h$ to 0.6 ; and estimate the recruitment residuals, or set them all to zero.

- $\quad B_{1} / K=0.3$ : Results are shown in Table 1 and Figures $1 \mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$ and e.
- $\quad B_{1} / K=0.1$ : Results are shown in Table 2 and Figures $2 \mathrm{a}, \mathrm{b}, \mathrm{c}$ and d.
- $\quad B_{1} / K=0.5$ : Results are shown in Table 3 and Figures 3a, b, c and d.

Table 4 shows results when estimating an age-independent $M_{a}$ rather than fixing it to $0.2 y r^{-1}$ while forcing the selectivity-at-age to be flat at older ages, as well as the introduction of informative (tighter) priors for $B_{1} / K, M_{a}$ and $h$.

## 5 Discussion

Initial runs demonstrated that the model has difficulty fitting the data within the permissible parameter space. Key model parameters, such as $h, M_{a}$ and $B_{I} / K$ were estimated at the boundaries of their
allowed ranges, rendering these fits dubious. It was therefore decided that best practice would be to fix these parameters at plausible values and examine the resultant fits to the data in order to understand why the model is unable to reconcile the data with the prior distributions of parameters.

The goodness of fit for each of the assessments is indicated by the total negative log-likelihood value $\left(-\ln L_{\text {TOTAL }}\right)$. In addition, the Akaike Information Criterion (AIC) statistic is given to aid model selection, where $A I C=-2 \ln L_{\text {TOTAL }}+2 p$ where $p$ is the number of estimable parameters. The first term is a measure of how well the model fits the data, while the second term is a penalty for the addition of further estimable parameters (Burnham and Anderson, 1998). Both these statistics are shown at the bottom of the Tables. Note that although this assessment is set up in a Bayesian framework, results are given only as posterior modes, corresponding to penalised MLEs in a frequent context; thus strictly AIC can be used to select amongst models only in instances when the penalty functions are unchanged.

Table 1 shows fits to the data when fixing the initial spawning biomass in 1985 to $30 \%$ of the preexploitation level while at the same time fixing $M_{a}$ to $0.2 y r^{-1}$. The model has difficulty estimating the steepness parameter, $h$, both with and without recruitment fluctuations (columns 1 and 3 ), and prefers an unrealistically low value of $h=0.21$ (the lower bound for specified for this prior) corresponding to a resource with effectively no productivity. However, when fixing $h$ to a more realistic value of 0.6 , the fit to the CPUE data deteriorates markedly: the AIC increasing from -44.78 to -3.34 for one estimable parameter less for the case when not allowing for recruitment fluctuations; and an increase in AIC from -42.84 to 9.53 when allowing for recruitment fluctuations. According to the Akaike criterion the preferred model is clearly $\mathrm{SC1a}$ (column1) with an AIC score of -44.8 . The addition of the 21 additional stock-recruitment residual parameters are therefore not justified in terms of the extent of improvement of fit to the data, though this is not a completely reliable comparison for the reasons given above, essentially here as these residuals are not completely free parameters but instead constrained by their prior.

Figure 1a shows the model estimated and observed CPUE corresponding to the runs in Table 1. To illustrate the effect of an increase in the stock-recruit steepness parameter, $h$, twenty year projections for a zero future catch are plotted. For the $h=0.6$ scenarios, the model estimated CPUE values clearly do not fit the observed CPUE data. For the two scenarios when $h$ is estimated, the fit to the CPUE data is much improved, with an historic decrease in CPUE that mimics the trend in the data. However, due to the low estimate of $h=0.21$ (prior lower bound), there is very little recovery of the resource even for a zero future catch. The corresponding spawning biomass plots are given in Figure 1b. The estimated stock recruitment residuals are plotted in Figure 1c for fixed and estimated $h$. Systematic effects amongst the residuals are evident for SC1c (estimate $h$ ): positive for the 1980s and then negative thereafter. This is the only way the model is able to fit the length data at low $h$. For $h=0.6$, allowing for fluctuations about the stock-recruitment curve had no effect whatsoever on the complete lack of fit to the CPUE and length data (purple plots). The estimated selectivity vectors are shown in Figure 1d. As in the previous graphs, the two plots when $h$ are estimated lie on top of the other, both with a negative slope at older ages. However for $h=0.6$, both selectivity vectors have zero slope (i.e. standard logistic curve). Lastly, Figure le gives the log residuals of the fit to the catch-at-length data for SC1a. The bubble plot shows that the model systematically over-estimates the proportion of older/larger fish while under-estimating the proportion of fish caught in length groups 250 mm to 350 mm , which constitute the highest proportion of the catch (see Appendix 2, Figures A-3 and A-4 for length frequency plots).

According to preliminary baseline estimates by Griffiths (2000), the resource in 1985 could well have been far more depleted than the $30 \%$ assumed in Table 1. A lower initial spawning biomass depletion of $B_{l} / K=0.1$ is assumed for the runs depicted in Table 2. In the case of a severely depleted resource, the steepness, $h$, is estimated at 0.32 and 0.26 (columns 1 and 3 respectively), with associated AICs of -25 and -38 corresponding to the deterministic and stochastic stock-recruitment relationships. However, when fixing $h$ at a more realistic value of 0.6 , there is a complete lack of fit to both the CPUE and length data, with a substantial increase in additional variance to $\sigma_{A D D}=0.89$. This is illustrated in Figure 2a, where both $h=0.6$ scenarios (the one lying on top of the other) show an increase in estimated CPUE in contrast to the observed downward CPUE trend. Scenario 2a mimics the decline in historic CPUE, as well as a subsequent recovery when no catch is taken, although perhaps not quite as fast as expected under zero catch. The estimated biomass plots are given in Figures 2b, with the estimated recruitment residuals shown in Figure 2c. As before, due to the inability to fit the data at higher $h$, the recruitment residuals cannot be estimated (purple plot). The residuals when estimating $h$ show the same systematic patterns as for scenario 1 c . Of these runs, the preferred model in terms of the AIC is scenario 2c which allows for recruitment fluctuations (despite the restrictions the prior places on these fluctuations). Not surprisingly, the low $10 \%$ initial depletion necessitates a high estimate of $K$ (here estimated at its upper prior bound of 60000 tons).

When assuming that the 1985 spawning biomass was at $50 \%$ of the pre-exploitation level, the same problem persists: the best model fit is achieved when estimating $h$ (AIC=-44.74), however this is only achieved for an unrealistically low $h$ (no productivity) - see Figure 3a for the fit to the CPUE data. Fixing $h$ to 0.6 leads to severe deterioration in fit to the CPUE data when not allowing for recruitment fluctuation (column 2). This is less apparent when estimating the stock recruitment residuals (last two columns of Table 3), with the AIC increasing from -42.80 to -30.34 with increasing $h$. The log recruitment residuals plots in Figure 3c show the same systematic trends as before, the patterns being even more pronounced for the $h=0.6$ case.

The assessments thus far have all assumed an age-independent natural mortality, $M_{a}$, of $0.2 \mathrm{yr}^{-1}$ while estimating the fishing selectivity-at-age vector, which decreases with age at older ages. However, the decline in the proportion of older fish caught may equally be due to the lesser numbers of these fish because of higher natural mortality. Table 4 column 1 shows results for estimating the natural mortality rate while fixing the fishing selectivity to that provided by H. Winkler (see Appendix 1). This has the effect of pushing the estimate for age-independent natural mortality up to $0.36 \mathrm{yr}^{-1}$. However, the fit to the length data deteriorates markedly. Estimating a logistic selectivity vector (fully selected at older ages) improves the fit to both the length and CPUE data, with an associated estimate for natural mortality rate, $M_{a}$, of $0.21 \mathrm{yr}^{-1}$. Figure 4 a shows the input and estimated selectivity vectors and associated mortality rates. In both cases the steepness is estimated at an unrealistically low 0.21 . Columns 3 and 4 of Table 4 show results when imposing penalty functions for the steepness, $h$, initial biomass ratio, $B_{l} / K$, as well as age-independent natural mortality rate, $M_{a}$ (see Appendix 1, Section 1.3 for penalty functions). The model has difficulty to fit the trend in CPUE data when not allowing for recruitment fluctuations for an $h$ estimated at 0.44 . Allowing for fluctuations about the stock recruitment relationship improves the fit to both CPUE and length data, however this is achieved by a series of positive residuals from 1985 to 1991 followed by a series of negative residuals (Figure 4c). Model estimated and observed CPUE are shown in Figure 4b for these runs.

The exploitable biomass trajectories for all the assessments that fit the data reasonable well are plotted in Figure 5 for comparison. There are clearly many different interpretations, which renders management advice based on these assessments impossible, with current depletion ranging from $6 \%$ to over $80 \%$. In terms of model fit to both CPUE and length data, SC1c, SC2c and SC3c fair best, i.e. those assessments for which recruitment fluctuations are allowed. However, for these runs $h$ is estimated at 0.21 (the lower bound of the prior), which is unrealistic, or slightly above at 0.26 for SC3c (column 3, Table 3, which corresponds to an initial depletion of $50 \%$ ). When $h$ is fixed at a higher more believable level of 0.6 , the only model that fits the data reasonably well is SC3d (column 4, Table 3).

A reasonable fit to the data is also obtained when incorporating a tighter (more informative) prior for $h$ (SC4d), with a corresponding estimate of $h=0.46$. For this case tighter priors were also imposed for $B_{l} / K$ and $M_{a}$, subsequently estimated at 0.43 and 0.17 respectively, with current spawning biomass estimated to be $38 \%$ of $K$. However, these assessments cannot be used to base management decisions on as the recruitment residuals are anything but randomly distributed, which renders these results suspect.

## 6 Conclusions

At this stage the only conclusions that one can draw from these analyses is that this assessment is inconclusive and possibly flawed. There is no way to determine the key model parameters with any confidence and the resultant estimates of statistics pertinent to management decisions are therefore dubious at best. There are two possible reasons for this outcome: either that the data are inconsistent (note that the recent decreasing trend CPUE data is not synchronous with the recent decrease in annual catches), and/or there are complex effects in the resource dynamics that are not incorporated in a standard ASPM assessment with its underlying simple density dependent population regulation mechanism.

## 7 Further development

An obvious extension of the assessment process is the management procedure (MP) approach. This is particularly useful when there is substantial uncertainty and a "best" assessment cannot be selected due to conflicting or noisy data, or other complexities. The MP approach does not rely on any one assessment, but integrates over a range of possibilities, called operating models (OMs), testing management rules on each such OM to determine which harvesting control rule is the "best" (in the sense of the most robust) across all plausible scenarios.

The present ASPM analysis is particularly useful to highlight some of the shortcomings of a "best assessment" approach when there is little or conflicting information in the data: any number of interpretations are possible depending on what is included in/excluded from the likelihood function and what parameter ranges/prior distributions are allowed. This uncertainty will need to be taken into account in a quantitatively defensible and consistent manner in order to give scientifically rigorous management advice.

## Acknowledgements

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Fit to CPUE and length data: $M=0.2, B_{I} / K=0.3$, est $S_{a}$
(project 20 years under zero catch)


Table 1: Model estimates (posterior modes) shown for pertinent parameters and management quantities, along with the associated negative (penalized) log likelihood values when fitting to CPUE and length data. Ageindependent natural mortality rate is fixed to $0.2 y r^{-1}$, while the initial (1985) spawning biomass ration $B_{1} / K$, is set to 0.3 for these runs. Note: parameters that are estimated to be at the edge of their prior distribution are marked with an exclamation as these values are suspect.

| Fit to CPUE and length data: $M=0.2, B_{1} / K=0.1$, est $S_{a}$ (project 20 years under zero catch) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | SC2a: <br> Est $h$ | SC2b: <br> Fix $h=0.6$ | Est SR residuals 1985-2005 |  |
|  |  |  | $\begin{aligned} & \hline \text { SC2c: } \\ & \text { Est: } h \end{aligned}$ | SC2d: <br> Fix: $h=0.6$ |
| Parameters: | 5 | 4 | 26 | 25 |
| $K^{s p}$ | 37797 | 48031 | 60000 ! | 48031 |
| $h$ | 0.32 | 0.6 | 0.26 | 0.6 |
| $M_{\alpha}$ | 0.2 | 0.2 | 0.2 | 0.2 |
| $r^{s p}$ | 0.1 | 0.1 | 0.1 | 0.1 |
| agec | 4.74 | 5.10 | 4.39 | 5.10 |
| $\delta$ | 0.16 | 0.73 | 0.83 | 0.73 |
| slope | -0.06 | -0.05 | 0.0 | -0.05 |
| Stats: |  |  |  |  |
| $B_{n}^{s p} / K^{s p}$ | 0.06 | 0.74 | 0.07 | 0.74 |
| $B_{\text {final }}^{s p} / K^{s p}$ | 0.20 | 0.97 | 0.11 | 0.97 |
| Max likelihood: |  |  |  |  |
| $\sigma_{\text {ADD }}$ | 0.26 | 0.89 | 0.18 | 0.89 |
| $\sigma_{l e n}$ | 0.10 | 0.15 | 0.08 | 0.15 |
| $-\ln L_{\text {CPUE }}$ | -16.41 | 8.09 | -23.46 | 8.42 |
| $-\ln L_{\text {len }}$ | -1.19 | 32.11 | -10.73 | 32.30 |
| $-\ln L_{S R}$ |  |  | -11.29 | -14.56 |
|  |  |  |  |  |
| $-\ln L_{\text {total }}$ | -17.61 | 40.20 | -45.48 | 17.17 |
| $A I C=2 p-2 \ln L$ | -25.22 | 88.4 | -38.96 | 84.34 |

Table 2: Same as Table 1, but here assuming that the initial spawning biomass is $10 \%$ of the pre-exploitable level. Note: parameters that are estimated to be at the edge of their prior distribution are marked with an exclamation as these values are suspect.

| Fit to CPUE and length data: $M=0.2, B_{1} / K=0.5$, est $S_{a}$ (project 20 years under zero catch) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | SC3a: <br> Est $h$ | SC3b: <br> Fix $h=0.6$ | Est SR residuals 1985-2005 |  |
|  |  |  | SC3c: <br> Est: $h$ | SC3d: <br> Fix: $h=0.6$ |
| Parameters: | 5 | 4 | 26 | 25 |
| $K^{s p}$ | 29008 | 16790 | 25735 | 11632 |
| $h$ | 0.21 ! | 0.6 | 0.21 ! | 0.6 |
| $M_{\alpha}$ | 0.2 | 0.2 | 0.2 | 0.2 |
| $r^{s p}$ | 0.1 | 0.1 | 0.1 | 0.1 |
| agec | 4.07 | 4.54 | 4.06 | 4.49 |
| $\delta$ | 0.76 | 0.88 | 0.74 | 0.85 |
| slope | -0.03 | 0.0 | -0.03 | 0.0 |
| Stats: |  |  |  |  |
| $B_{n}^{s p} / K^{s p}$ | 0.36 | 0.81 | 0.32 | 0.51 |
| $B_{\text {final }}^{s p} / K^{s p}$ | 0.31 | 0.98 | 0.34 | 0.92 |
| Max likelihood: |  |  |  |  |
| $\sigma_{\text {ADD }}$ | 0.20 | 0.34 | 0.18 | 0.20 |
| $\sigma_{l e n}$ | 0.09 | 0.09 | 0.08 | 0.08 |
| $-\ln L_{\text {CPUE }}$ | -21.56 | -10.44 | -24.25 | -20.89 |
| $-\ln L_{l e n}$ | -5.80 | -5.92 | -11.24 | -10.08 |
| $-\ln L_{S R}$ |  |  | -11.90 | -9.20 |
|  |  |  |  |  |
| $-\ln L_{\text {TOTAL }}$ | -27.37 | -16.26 | -47.40 | -40.17 |
| $A I C=2 p-2 \ln L$ | -44.74 | -24.52 | -42.80 | -30.34 |

Table 3: Same as Table 1, but here assuming that the initial spawning biomass is $50 \%$ of the pre-exploitable level. Note: parameters that are estimated to be at the edge of their prior distribution are marked with an exclamation as these values are suspect.

| Fit to CPUE and length data: est $\boldsymbol{M}_{a}$ (project 20 years under zero catch) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Uninformative priors |  | Tighter priors: $B_{l} / K, h, M_{a}$ |  |
|  | SC4a: <br> Est $h$ <br> Input $S_{a}$ <br> Fix: $B_{l} / K=0.3$ | SC4b: <br> Est $h$ <br> Est $S_{a(\text { slope }=0)}$ <br> Fix: $B_{l} / K=0.3$ | SC4c: <br> Est h, <br> Est $S_{a(\text { slope }=0)}$ <br> Est: $B_{1} / K$ | SC4d: <br> Est h, <br> Est $S_{a(\text { slope }=0)}$ <br> Est: $B_{l} / K$ <br> Est: SR resid |
| Parameters: | 3 | 5 | 6 | 25 |
| $K^{s p}$ | 49067 | 45353 | 13030 | 12310 |
| $h$ | 0.21 ! | 0.21 ! | 0.44 | 0.46 |
| $M_{\alpha}$ | 0.355 | 0.211 | 0.171 | 0.173 |
| $r^{s p}$ | 0.3 | 0.3 | 0.43 | 0.43 |
| agec |  | 4.13 | 4.40 | 4.21 |
| $\delta$ |  | 0.77 | 0.90 | 0.79 |
| slope |  | 0 | 0 | 0 |
| Stats: |  |  |  |  |
| $B_{n}^{s p} / K^{s p}$ | 0.26 | 0.21 | 0.48 | 0.38 |
| $B_{\text {final }}^{s p} / K^{s p}$ | 0.29 | 0.23 | 0.82 | 0.78 |
| Max likelihood: |  |  |  |  |
| $\sigma_{\text {ADD }}$ | 0.22 | 0.20 | 0.26 | 0.19 |
| $\sigma_{l e n}$ | 0.13 | 0.09 | 0.09 | 0.08 |
| $-\ln L_{\text {CPUE }}$ | -19.44 | -21.41 | -14.87 | -21.64 |
| $-\ln L_{l e n}$ | 14.82 | -5.82 | -5.63 | -11.84 |
| $-\ln L_{S R}$ |  |  |  | -10.05 |
|  |  |  |  |  |
| $-\ln L_{\text {TOTAL }}$ | -4.62 | -27.23 | -19.04 | -42.82 |
| $A I C=2 p-2 \ln L$ | -3.24 | -44.46 | -26.08 | -35.66 |

Table 4: Parameter estimates (posterior modes) shown for pertinent parameters when estimating natural mortality and keeping the selectivity-at-age vector flat at older ages. Note: parameters that are estimated to be at the edge of their prior distribution are marked with an exclamation as these values are suspect.



Figures 1a and b: Past and future model estimated CPUE and spawning biomass for each of the scenarios in Table 1. The current year (2010) is marked by the vertical line. A 20 year projection period with a zero future catch was chosen to illustrate possible resource recovery, or lack thereof.


Figure 1c: Model estimated stock recruitment residuals when for fixed $\boldsymbol{h - 0 . 6}$ and estimable $\boldsymbol{h}$.


Figure 1d: Original input and model estimated selectivity-at-age for the four scenarios.




Figure 1e: Model estimated residuals of the fit to the length data for estimated $\boldsymbol{h}$ (SC1a) for the years 1985 to 2007. The size of the bubbles are proportional to the $\log$ residuals: solid circles for positive residuals and empty circles for negative residuals.



Figures 2a and b: Top: Same as Figures 1a, b and c, but here for an initial biomass ratio of 0.1.


Figure 2c: Model estimated stock recruitment residuals when for fixed $\boldsymbol{h} \mathbf{- 0 . 6}$ and estimable $\boldsymbol{h}$.


Figure 2d: Original input and model estimated selectivity-at-age vectors when the initial biomass is taken to be $10 \%$ of the pristine level.



Figures 3a and b: Same as Figures 1a and b, but here for an initial biomass ratio of 0.5.


Figure 3c: Model estimated stock recruitment residuals when for fixed $\boldsymbol{h - 0 . 6}$ and estimable $\boldsymbol{h}$.


Figure 3d: Original input and model estimated selectivity-at-age for an initial biomass ratio of $\mathbf{5 0 \%}$.


Figure 4a: Model estimated mortality rate and selectivities-at-age for SC4a and b for scenarios where this selectivity is constrained to be flat at large ages.


Figure 4b: Model estimated and observed CPUE for SC4a, b, c and d.


Figure 4c: Recruitment residuals when imposing alternative tighter priors for $h, M_{a}$ and $B_{l} / K$ for SC4d.


Figure 5: Plots of estimated exploitable biomass series for those scenarios that fit the data reasonably well. Twenty year projections shown for a zero future catch.

## References

Booth, T, Attwood, C, Kerwath, S and H Winkler. 2011. An age-structured production model for assessing exploitation rate and population size for exploited linefish species. Linefish Working Group Document 2011. Draft.

Brandão, A. and Butterworth, D.S. 2009. A proposed Management Procedure for the toothfish (Dissostichus eleginoides) resource in the Prince Edward Islands vicinity. CCAMLR Science: 16: 3369.

Brouwer, S.L. and M.H. Griffiths. 2005a. Stock separation and life history of the Argyrozona argyrozona (Pisces: Sparidae) on the South African east coast. African Journal of Marine Science 2005, 27(3):585-595

Brouwer, S.L. and M.H. Griffiths. 2005b. Reproductive biology of carpenter seabream (Argyrozona argyrozona) (Pisces: Sparidae) in a marine protected area. Fish. Bull. 103:258-269 (2005).

Butterworth, D.S. and H.F. Geromont. 2001. Evaluation of a class of possible simple interim management procedures for the Namibian Hake fishery. S. Afr. J. mar. Aci. 23: 357-374

Burnham, K.P. and D.R. Anderson. 1998. Model selection and inference - A practical informationtheoretic approach. Springer-Verlag, New York, 355pp.

Geromont, H.F. and D.S. Butterworth. 2001. Possible extensions to the ADAPT VPA model applied to Western North Atlantic bluefin tuna, addressing in particular the need to account for "additional variance". Col. Vol. Sci. Pap. ICCAT, 52(5): 1663-1705 (2001)

Griffiths, M.H. 2000. Long-term trends in catch and effort of commercial linefish off South Africa's Cape Province: snapshots of the $20^{\text {th }}$ century. S.Afr.J.mar. Sci. 22: 81-110

Recommendation of the Scientific Working Group for the management of sustainable linefish resources for the 2011 season. Linefish Working Group Document. August 2011.

## Appendix 1: <br> The age-structured production model (ASPM)

The resource dynamics are modeled by the following equations, depending whether a continuous or pulse fishery is assumed. For the Baranov approximation (continuous fishing throughout year), the resource dynamics are modeled by the equations:
$N_{y+1, a \min }=R_{y+1}$
$N_{y+1, a+1}=N_{y, a} e^{-\left(M_{a}+S_{y, a} F_{y}\right)}=N_{y, a} e^{-Z_{y, a}}$ for $a_{\min } \leq a<m-2$

$$
\begin{equation*}
N_{y+1, m}=N_{y, m-1} e^{-\left(M_{m-1}+S_{y, m-1} F_{y}\right)}+N_{y, m} e^{-\left(M_{m}+S_{y, m} F_{y}\right)} \tag{1.3}
\end{equation*}
$$

When employing Pope's approximation (assuming a mid-year pulse fishery), the resource dynamics are modeled by the equations:

$$
\begin{array}{r}
N_{y+1, a+1}=N_{y, a} \exp ^{-M_{a}}-C_{y, a} e^{-M_{a} / 2} \quad \text { for } a_{\min } \leq a<m-2 \\
N_{y+1, m}=N_{y, m-1} e^{-M_{m-1}}-C_{y, m-1} e^{-M_{m-1} / 2}+N_{y, m} e^{-M_{m}}-C_{y, m} e^{-M_{m} / 2} \tag{1.5}
\end{array}
$$

where
$N_{y, a}$ is the number of fish of age $a$ at the start of year $y$,
$M_{a}$ denotes the natural mortality rate on fish of age $a$,
$S_{y, a}$ is the age-specific selectivity for year $y$,
$F_{y}$ is the fishing mortality for year $y$,
$m$ is the maximum age considered (taken to be a plus-group), and
$a_{\text {min }}$ is the minimum age considered ( 0 in this case).

The number of recruits at the start of year $y$ (for $y>1$ ) is related to the spawning stock size by a stockrecruitment relationship:
$R_{y}=\frac{\alpha B_{y-a \min }^{s p}}{\beta+\left(B_{y-a \text { min }}^{s p}\right)^{\gamma}} e^{\varsigma_{y}}$
where
$\alpha, \beta$ and $\gamma$ are spawning biomass-recruitment parameters ( $\gamma=1$ for a Beverton-Holt and $\gamma>1$ for a Ricker-like relationship, and can either be input or treated as an estimable parameter),
$\varsigma_{y}$ reflects fluctuation about the expected recruitment for year $y$, and
$B_{y-a \text { min }}^{s p}$ is the spawning biomass at the start of year $y-a_{\text {min }}$, given that:
$B_{y}^{s p}=\sum_{a=0}^{m} f_{a} w_{a} N_{y, a}$
where $w_{a}$ is the begin-year mass of fish of age $a$ and $f_{a}$ is the proportion of fish of age a that are mature.

In order to work with estimable parameters that are more meaningful biologically, the stockrecruitment relationship is re-parameterised in terms of the pre-exploitation equilibrium spawning biomass, $K^{s p}$, and the "steepness" of the stock-recruitment relationship (recruitment at $B^{s p}=0.2 K^{s p}$ as a fraction of recruitment at $\left.B^{s p}=K^{s p}\right)$ :
$\alpha=\frac{\left(5-0.2^{\gamma-1}\right) h R_{1}\left(K^{s p}\right)^{\gamma-1}}{5 h-1}$
and
$\beta=\frac{\left(K^{s p}\right)^{\gamma}\left(1-h 0.2^{\gamma-1}\right)}{5 h-1}$
where

$$
\begin{equation*}
R_{1}=K^{s p} /\left[f_{0} w_{0}+\sum_{a=a \min +1}^{m-1} f_{a} w_{a} e^{-\left(\sum_{a=a \min }^{a-1} M_{a}\right)}+f_{m} w_{m} \frac{e^{-\left(\sum_{a}^{m-1}=a \min M_{a}\right)}}{1-e^{-M_{m}}}\right] \tag{1.10}
\end{equation*}
$$

Note: A Beverton-Holt stock-recruitment relationship is assumed for these analyses, i.e. $\gamma=1$.

For the Baranov approximation, the total number of fish caught of age $a$ in year $y$ is given by

$$
\begin{equation*}
C_{y, a}=N_{y, a} \frac{S_{y, a} F_{y}}{Z_{y, a}}\left(1-e^{-Z_{y, a}}\right) \tag{1.11}
\end{equation*}
$$

where the fishing mortality $F_{y}$ cannot be calculated directly, but is computed using the bisection method.

When assuming Pope's approximation, the number of fish caught of age $a$ in year $y$ is given by

$$
\begin{equation*}
C_{y, a}=N_{y, a} S_{y, a} F_{y} e^{-M_{a} / 2} \tag{1.12}
\end{equation*}
$$

where the estimate fishing mortality is simply $F_{y}=C_{y} / B_{y}^{\exp }$

The corresponding catch by mass for each year is given by

$$
\begin{equation*}
C_{y}=\sum_{a=a_{\min }}^{m} w_{a+1 / 2} C_{y, a} \tag{1.14}
\end{equation*}
$$

where $w_{a+1 / 2}$ denotes the mid-year mass of fish of age.

The model estimate of the exploitable ("available") component of biomass is given by:
$B_{y}^{\exp }=\sum_{a=a \min }^{m} w_{a} S_{y, a} N_{y, a} \quad$ for begin-year biomass, and
for the mid-year biomass:
$B_{y}^{\exp }=\sum_{a=a \min }^{m} w_{a+1 / 2} S_{y, a} N_{y, a} e^{-Z_{y, a} / 2}$ for the Baranov approximation, or
$B_{y}^{\exp }=\sum_{a=a \min }^{m} w_{a+1 / 2} S_{y, a} N_{y, a} e^{-M_{a} / 2}$ for Pope's approximation.

It is usually assumed that the resource is at the deterministic equilibrium that corresponds to an absence of harvesting at the start of the initial year ( $B_{1}^{s p}=K^{s p}$ ). However, if the initial year does not correspond to the start of the fishery then the initial spawning biomass ratio to the pristine level can be estimated such that $B_{1}^{s p}=r^{s p} K^{s p}$ where $r^{s p} \neq 1$. In this case the age-structure of $B_{1}^{s p}$ cannot be assumed to be that corresponding to the equilibrium with zero fishing mortality. An initial fishing mortality, $F_{0}$, corresponding to the initial year needs to be computed such that $B_{1}^{s p}=r^{s p} K$, where the number of recruits in the first year, adjusted to account for previous catches, is given by

$$
\begin{equation*}
R_{1}^{*}=\frac{\alpha r^{s p} K}{\left(\beta+\left(r^{s p} K\right)^{\gamma}\right)} \tag{1.18}
\end{equation*}
$$

where $\alpha$ and $\beta$ are given by equations (1.8) and (1.9), while the associated initial spawning biomass is given by

$$
\begin{equation*}
B_{1}^{s p}=R_{1}^{*}\left[\sum_{a=a \min +1}^{m-1} f_{a} w_{a} e^{-\left(\sum_{a=a \min }^{a-1} M_{a^{\prime}}+S_{a} F 0\right)}+f_{m} w_{m} \frac{e^{-\left(\sum_{a^{\prime}=a \min }^{m-1} M_{a^{\prime}}+S_{a} F 0\right)}}{1-e^{-\left(M_{m}+S_{m} F 0\right)}}\right] \tag{1.19}
\end{equation*}
$$

In order to generate the initial population numbers using $F_{0}$ defined above, we assume that the catches prior to the first year considered in the model are of the same magnitude. A more defensible approach would be to include estimates of historic catches in the model data, even if these are not well recorded, to get improved estimates of $R_{1}^{*}$ and $F_{0}$.

Note: Pope's approximation, which assumes a pulse fishery, was adopted for these analyses to facilitate comparison with analyses done by Booth et al. (2011).

### 1.1 The likelihood function

The model is fitted to generated abundance and length data, as well as catch-at-age data to estimate model parameters. Contributions by each of these to the negative of the $\log$-likelihood $(-\ln L)$ are as follows.

### 1.1.1 Abundance data:

The likelihood is calculated assuming that the observed abundance index is log-normally distributed about its expected value:
$I_{y}^{i}=\hat{I}_{y}^{i} \exp \left(\varepsilon_{y}^{i}\right) \quad$ or $\quad \varepsilon_{y}^{i}=\ln \left(I_{y}^{i}\right)-\ln \left(\hat{I}_{y}^{i}\right)$
where
$I_{y}^{i}$ is the abundance index for year $y$ and series i ,
$\hat{I}_{y}^{i}=\hat{q}^{i} \hat{B}_{y}$ is the corresponding model estimate, where $B_{y}$ is the model estimate of exploitable resource biomass, given by equations (1.16) and (1.17),
$\hat{q}^{i}$ is the constant of proportionality for abundance series $i$ (effectively the multiplicative bias if the series reflects abundance in absolute terms), and $\varepsilon_{y}^{i}$ from $N\left(0,\left(\sigma_{y}^{i}\right)^{2}\right)$.

Note therefore that in any year, the selectivity ( $S_{y, a}$ ) is taken to be the same for all abundance indices i. The only factor that distinguishes such indices is potentially differing values of the catchability coefficients $\hat{q}^{i}$.
The contribution of the abundance data to the negative of the log-likelihood function (after removal of constants) is given by:
$-\ln L=\sum_{f} \sum_{i}\left[\sum_{y} \ln \sigma_{y}^{i}+\left(\varepsilon_{y}^{i}\right)^{2} / 2\left(\sigma_{y}^{i}\right)^{2}\right]$

## Estimate variance:

In this case, homoscedasticity of residuals is assumed, so that $\sigma_{y}^{i}=\sigma^{i}$, the standard deviation of the residuals for the logarithms of abundance index $i$ is estimated in the fitting procedure by its maximum likelihood value:

$$
\begin{equation*}
\sigma^{i}=\sqrt{1 / n^{i} \sum_{y}\left(\ln I_{y}^{i}-\ln \hat{I}_{y}^{i}\right)^{2}} \tag{1.22}
\end{equation*}
$$

where $n^{i}$ is the number of data points for abundance series $i$.

The catchability coefficient $q^{i}$ for abundance index $i$ is estimated by its maximum likelihood value:

$$
\begin{equation*}
\ln \hat{q}^{i}=1 / n^{i} \sum_{y}\left(\ln I_{y}^{i}-\ln \hat{B}_{y}\right) \tag{1.23}
\end{equation*}
$$

## Input variance:

In this case, $\sigma_{y}^{i}$ is taken to be the estimate of the coefficient of variation (CV) of the resource abundance estimate for year $y$, which is input.

The constant of proportionality for this abundance index is estimated by its maximum likelihood value which, for the case of a log-normal error distribution, is given by:

$$
\begin{equation*}
\ln \hat{q}^{i}=\frac{\sum_{y} 1 /\left(\sigma_{y}^{i}\right)^{2}\left(\ln I_{y}^{i}-\ln \hat{B}_{y}\right)}{\sum_{y} 1 /\left(\sigma_{y}^{i}\right)^{2}} \tag{1.24}
\end{equation*}
$$

## Additional variance:

For this approach, index variances $\left(\left(\sigma_{y}^{i}\right)^{2}\right)$ corresponding to abundance index $i$ in year $y$ are incorporated in the model fitting procedure in the same manner as for the "input variance" scenario described above. Furthermore, the additional variance for each index $\left(\left(\sigma_{A}^{i}\right)^{2}\right)$ is then estimated using an extension of the maximum likelihood approach as proposed in Geromont and Butterworth (2001). In this extension, the catchability coefficient for each index is estimated in the fitting procedure by its maximum likelihood value using equation (1.28) given below, where the total variance for each data point incorporates both input variance $\left(\left(\sigma_{y}^{i}\right)^{2}\right)$ and (estimated) additional variance $\left(\left(\sigma_{A}^{i}\right)^{2}\right)$.

The objective function minimised is thus given by the negative of the log-likelihood, ignoring constants:

$$
\begin{equation*}
-\ln L=\sum_{i} \sum_{y}\left[\left(\varepsilon_{y}^{i}\right)^{2} / 2\left(\left(\sigma_{y}^{i}\right)^{2}+\left(\sigma_{A}^{i}\right)^{2}\right)+\ln \sqrt{\left(\sigma_{y}^{i}\right)^{2}+\left(\sigma_{A}^{i}\right)^{2}}\right] \tag{1.25}
\end{equation*}
$$

where
$\sigma_{y}^{i}$ is the (minimum) standard error of the value for abundance series $i$ in year $y$, which is input, and
$\sigma_{A}^{i}$ is the square root of the additional variance for abundance series $i$, estimated by its maximum likelihood value from the following relationship which follows from differentiating equation 17 :

$$
\begin{equation*}
\sum_{y} \frac{1}{\left(\sigma_{y}^{i}\right)^{2}+\left(\sigma_{A}^{i}\right)^{2}}=\sum_{y} \frac{\left(\varepsilon_{y}^{i}\right)^{2}}{\left(\left(\sigma_{y}^{i}\right)^{2}+\left(\sigma_{A}^{i}\right)^{2}\right)^{2}} \tag{1.26}
\end{equation*}
$$

where
$\varepsilon_{y}^{i}=\ln \left(I_{y}^{i}\right)-\ln \left(q^{i} \hat{B}_{y}^{i}\right)$
for log-normally distributed errors, where
$I_{y}^{i}$ is the abundance index for year $y$ and series $i$ from fleet $f$,
$\hat{B}_{y}^{i}$ is the corresponding resource population model estimate, and
$q^{i}$ is the catchability coefficient for abundance series $i$, estimated by its maximum likelihood value:
$\ln \hat{q}^{i}=\frac{\sum_{y}\left[1 /\left(\sigma_{y}^{i}+\sigma_{A}^{i}\right)^{2}\right]\left(\ln I_{y}^{i}-\ln \hat{B}_{y}\right)}{\sum_{y} 1 /\left(\sigma_{y}^{i}+\sigma_{A}^{i}\right)^{2}}$
for log-normally distributed errors.

Note that for the special case where $\sigma_{y}^{i}=\sigma^{i}$ (a constant), equation (1.26) above can be simplified so that the "additional variance" is estimated as follows (similar to equation (1.22)):
$\sigma_{A}^{i}=\sqrt{\left[1 / n^{i} \sum_{y}\left(\varepsilon_{y}^{i}\right)^{2}\right]-\left(\sigma^{i}\right)^{2}}$

This procedure was carried out enforcing the constraint that $\left(\sigma_{A}^{i}\right)^{2} \geq 0$, i.e. the overall variance cannot be less that its externally input component. Thus this method avoids one of the potential problems of the "maximum likelihood" approach described above using equation (1.22): that in certain circumstances unrealistically high precision (and so high weight) can be ascribed to certain indices.

Note: This option is used for the reference case assessment model. This approach is useful when fitting to the Carpenter linefish estimates of abundance thus allowing the input (fixed) variance associated with these indices to be interpreted as the minimum overall variance and letting the model estimate any possible additional variance.

### 1.1.2 Catches-at-length:

The contribution of the catch-at-length data to the negative of the log-likelihood function when assuming a log-normal error distribution is given by:
$-\ln L=w \sum_{y} \sum_{l}\left[\ln \sigma_{l e n}+\left(\ln p_{y, l}-\ln \hat{p}_{y, l}\right)^{2} / 2\left(\sigma_{l e n}\right)^{2}\right]$
or, when making an adjustment to effectively weight in proportion to sample size:
$-\ln L=w \sum_{y} \sum_{l}\left[\ln \left(\sigma_{l e n} / \sqrt{p_{y, l}}\right)+p_{y, a}\left(\ln p_{y, l}-\ln \hat{p}_{y, l}\right)^{2} / 2\left(\sigma_{l e n}\right)^{2}\right]$
where
$w=0.1$ to down-weight the contribution of the length data to the likelihood function to allow for their non-independence,
$p_{y, l}=C_{y, l} / \sum_{l^{\prime}} C_{y, l^{\prime}}$ is the observed proportion of fish caught in year $y$ that are of length $l$,
$\hat{p}_{y, l}=\hat{C}_{y, l} / \sum_{l^{\prime}} \hat{C}_{y, l^{\prime}}$ is the estimated proportion of fish caught in year $y$ that are of length $l$, which is derived from the corresponding model-predicted catches-at-age using the transformation suggested in Brandao and Butterworth (2009) such that :

$$
\begin{equation*}
\hat{C}_{y, l}=\sum_{a} \hat{C}_{y, a} T_{a, l} \tag{1.32}
\end{equation*}
$$

where
$\hat{C}_{y, a}=N_{y, a} \frac{S_{y, a} F_{y}}{Z_{y, a}}\left(1-\exp \left(-Z_{y, a}\right)\right)$
and $T_{a, l}$ is the transformation matrix which contains the proportion of fish of age $a$ that fall into length group $l$. The expected proportion of fish in any length group is sampled from a normal distribution with mean given by the von Bertalanffy equation (1.39), such that:

$$
\begin{equation*}
l_{a} \sim N\left[l_{\infty}\left(1-e^{-\kappa\left(t-t_{0}\right)}\right),\left(\sigma_{a}^{T}\right)^{2}\right] \tag{1.34}
\end{equation*}
$$

with the associated standard deviation, $\sigma_{a}^{T}$, which is assumed to be proportional to the expected length for age $a$, such that

$$
\begin{equation*}
\sigma_{a}^{T}=\sigma^{T}\left(l_{\infty}\left(1-e^{-\kappa\left(t-t_{0}\right)}\right)\right) \tag{1.35}
\end{equation*}
$$

where $\sigma^{T}$ is an estimable parameter.

The standard deviation associated with the catch-at-length data, $\sigma_{l e n}$, is estimated in the fitting procedure by:

$$
\sigma_{l e n}=\sqrt{\sum_{y} \sum_{l}\left(\ln p_{y, l}-\ln \hat{p}_{y, l}\right)^{2} / \sum_{y} \sum_{l} 1}
$$

if equation (1.30) applies, or:

$$
\sigma_{l e n}=\sqrt{\sum_{y} \sum_{l} \hat{p}_{y, l}\left(\ln p_{y, l}-\ln \hat{p}_{y, l}\right)^{2} / \sum_{y} \sum_{l} 1}
$$

if equation (1.31) has been used.

The log-normal error distribution underlying equation (1.30) is chosen on the grounds that (assuming no aging error) variability is likely dominated by a combination of inter-annual variation in the distribution of fishing effort, and fluctuations (partly as a consequence of such variations) in selectivity-at-age, which suggests that the assumption of a constant coefficient of variation is appropriate. However, for ages poorly represented in the sample, sampling variability considerations must at some stage start to dominate the variance. To take this into account weighting by the expected proportions (equation (1.31)) is effected so that undue importance is not attached to data based upon a few samples only.

Note: For the present application, the minimum and maximum length were chosen as 220 and 480 $m m$ respectively with a length interval of 10 mm .These minus and plus groups were chosen after inspection of the data and to avoid undue systematic trends in the residuals corresponding to the shorter and longer length ranges were the proportion of fish caught is relatively low.

### 1.1.3 Stock-recruitment function residuals:

These residuals are assumed to be log-normally distributed and serially correlated. Thus, the contribution of the recruitment residuals to the negative of the log-likelihood function is given by:

$$
\begin{equation*}
-\ln L=\sum_{y=y 1+1}^{y 2}\left[\ln \sigma_{R}+\left[\frac{\varsigma_{y}-\rho \varsigma_{y-1}}{\sqrt{1-\rho^{2}}}\right]^{2} / 2 \sigma_{R}^{2}\right] \tag{1.36}
\end{equation*}
$$

where
$\varsigma_{y}=\rho \varsigma_{y-1}+\sqrt{1-\rho^{2}} \varepsilon_{y}$ is the recruitment residual for year $y$, which is estimated for years $y l$
to $y 2$,
$\varepsilon_{y} \sim N\left(0, \sigma_{R}^{2}\right)$,
$\sigma_{R}=0.5$ is the standard deviation of the log-residuals, which is input, and
$\rho$ is the serial correlation coefficient, which is input ( 0 for these analyses).

In the interest of simplicity, equation (1.36) omits a term in $\zeta_{y 1}$ for the case when serial correlation is assumed ( $\rho \neq 0$ ), which is generally of little quantitative consequence to values estimated.

### 1.2 Model parameters:

Natural mortality: An age-independent mortality rate, $M_{a}=0.2 y r^{-1}$ is assumed for the base case runs.

Fishing selectivity: Commercial linefish fishing selectivity can either be input or estimated. In the former case a time-invariant age-dependent fishing selectivity of

| $S_{a}=[0.00$ |  | 0.01 | 0.02 | 0.05 | 0.13 | 0.29 | 0.52 | 0.75 | 0.89 | 0.96 | 0.98 | 0.99 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | $]$ |  |  |  |  |  |

is assumed (provided by H. Winkler).
Alternatively, the selectivities-at-age can be approximated in terms of the following logistic curve:

$$
\begin{equation*}
S_{a}=\frac{1}{1+\exp \left(-\left(a-a_{c}\right) / \delta\right)} \tag{1.37}
\end{equation*}
$$

and, for $a>10$ :
$S_{a}=S_{a} \exp (s(a-10))$
where
$a_{c} \mathrm{yrs}$ is the age-at- $50 \%$ selectivity,
$\delta \mathrm{yr}^{-1}$ defines the steepness of the ascending limb of the selectivity curve, and $s$ measures the rate of decrease ("slope") in selectivity with age for older fish ( $a>10$ ).

Note: A value of 10 was assumed for these analyses after inspection of Figure A-1 and Figures A-4 and $A-5$ : The largest proportion of catches fall in the 250 mm (minimum allowed length) and 350 mm age group (ages 4 to 10 years).

Initial spawning biomass ratio: $r^{s p}$ is estimated for these analyses.

Minimum and maximum age: $a_{\text {min }}$ is taken to be $1 ; m$ is taken as a plus-group and set to 20 (Henning Winkler pers. commn).

Age-at-maturity: The proportion of fish of age $a$ that are mature is input. For the reference case this is approximated by a logistic form with $a_{50}=4 y r$ (Brouwer and Griffiths (2005b)):
$f_{a}=\left[\begin{array}{lllllllllllllllllllllllll}0.05 & 0.12 & 0.27 & 0.50 & 0.73 & 0.88 & 0.95 & 0.98 & 0.99 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0\end{array}\right.$ 1.0]

Mass-at-age: The mass ( $w$ ) of a fish at age $a$ is given by:
$w_{a}=\alpha\left(l_{a}\right)^{\beta}$
where $l_{a}$ is the length of a fish at age $a$, assumed to be given by the von Bertalanffy growth equation:
$l_{a}=l_{\infty}\left(1-\exp \left(-\kappa\left(a-t_{0}\right)\right)\right)$

The following values, taken from Brouwer and Griffiths (2005a), are assumed here:
$\alpha=0.0002 \mathrm{~g}$,
$\beta=2.924$,
$l_{\infty}=619 \mathrm{~mm}$,
$\kappa=0.06 \mathrm{yr}^{-1}$, and
$t_{0}=-4.5 \mathrm{yr}$.


Figure A-1: The von Bertalanffy growth curve assumed for these analyses (Brouwer and Griffiths (2005a)).

### 1.3 Bayesian approach:

A Bayesian approach is followed where prior distributions are specified for key model parameters such as initial spawning biomass depletion $r^{s p}=B_{1}^{s p} / K^{s p}$ (when assuming that the resource is not at its pre-exploitation equilibrium spawning biomass, $\mathrm{K}^{\text {sp }}$, in the first year), "steepness" of the stockrecruit relationship, $h$, natural mortality rate, $M_{a}$, as well as stock-recruit. These prior distributions reflect some qualitative information available about the resource. When quantitative data are available for the fishery, such as indices of abundance (CPUE) and length frequency data, the prior distributions are updated with respect to the respective likelihoods of the associated population model fits to these data, to provide posterior distributions of model parameters and other management quantities.

Most of the analyses presented assume uniform prior distributions for some key model parameters with the intention of being relatively uninformative:

- Initial biomass ration $r^{s p} \sim U[0.1,1.0]$
- Steepness: $h \sim U[0.21,0.95]$
- Mortality: $M_{\alpha} \sim U[0.17,0.23]$
- Stock-recruit residuals: $\varepsilon_{y} \sim N\left(0,0.5^{2}\right)$

Alternative tighter prior distributions imposed for selected runs are:

- Initial biomass ration $r^{s p} \sim N^{6}\left(0.3,0.15^{6}\right)$
- Mortality: $M_{\alpha} \sim N^{6}\left[0.2,0.03^{6}\right]$
- Steepness: $h \sim N^{6}\left[0.6,0.15^{6}\right]$
where $\mathrm{N}^{6}$ refers to a "flattened" normal distribution of the form
$-\ln L=-\ln L+(x-\mu)^{6} / 2(\sigma)^{6}$
where $x$ is the estimated parameter and $\mu$ is the mean with an associated measure of spread of $\sigma$. A "flattened" normal was chosen instead of a uniform distribution to ensure continuity of the respective contributions to the negative log-likelihood function.

No priors distributions were assumed for the von Bertalanffy growth parameters. The total annual catches, $C_{y}$, were taken to be exact for the base case runs.

## Appendix 2:

## Input data

Total annual catches, CPUE and length frequency data were provided by Henning Winkler and Sven Kerwath (pers. commn).

| Year | Catch (tons) | CPUE | CV |
| :--- | :--- | :--- | :--- |
| 1985 | 313.105 | 164.00474 | 0.08249524 |
| 1986 | 443.748 | 125.25751 | 0.07490713 |
| 1987 | 348.301 | 101.9071 | 0.07451718 |
| 1988 | 488.808 | 126.78956 | 0.06701664 |
| 1989 | 331.956 | 110.12566 | 0.06864165 |
| 1990 | 650.638 | 169.11762 | 0.06615197 |
| 1991 | 796.713 | 127.23586 | 0.07035701 |
| 1992 | 461.191 | 121.48753 | 0.07231863 |
| 1993 | 376.016 | 117.06365 | 0.08219 |
| 1994 | 361.268 | 144.16709 | 0.08246802 |
| 1995 | 471.676 | 122.02317 | 0.0826848 |
| 1996 | 646.383 | 131.02412 | 0.08700869 |
| 1997 | 582.861 | 123.62224 | 0.09073707 |
| 1998 | 315.828 | 93.70692 | 0.07991473 |
| 1999 | 391.69 | 110.80606 | 0.08758011 |
| 2000 | 293.019 | 124.46037 | 0.08287509 |
| 2001 | 127.216 | 99.99206 | 0.09843403 |
| 2002 | 160.201 | 115.39171 | 0.10527154 |
| 2003 | 107.332 | 177.14267 | 0.11454538 |
| 2004 | 115.359 | 120.0586 | 0.10489975 |
| 2005 | 86.953 | 46.88033 | 0.17632287 |
| 2006 | 92.043 | 54.24737 | 0.21597996 |
| 2007 | 128.4 | 89.00879 | 0.15706431 |
| 2008 | 111.198 | 116.22998 | 0.22298034 |
| 2009 | 152.986 | 70.73533 | 0.25386441 |
| 2010 | 50.572 | 39.76746 | 0.52617524 |

Table A-1. Annual catches in tons and standardized CPUE used for input for these analyses.


Figure A-2: Annual catch and standardised linefish CPUE data

| Length (FL) mm | 1985 | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 120 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 130 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 140 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 150 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 160 | 6 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 170 | 7 | 0 | 4 | 1 | 2 | 0 | 2 | 1 | 0 | 2 |
| 180 | 11 | 0 | 2 | 1 | 5 | 1 | 4 | 1 | 1 | 4 |
| 190 | 25 | 1 | 10 | 1 | 7 | 14 | 8 | 4 | 1 | 11 |
| 200 | 32 | 15 | 16 | 4 | 10 | 6 | 29 | 6 | 10 | 12 |
| 210 | 49 | 28 | 40 | 11 | 14 | 12 | 40 | 25 | 14 | 29 |
| 220 | 93 | 30 | 100 | 37 | 41 | 34 | 85 | 103 | 25 | 42 |
| 230 | 140 | 36 | 253 | 138 | 84 | 88 | 212 | 317 | 146 | 118 |
| 240 | 115 | 53 | 239 | 339 | 124 | 146 | 606 | 703 | 404 | 388 |
| 250 | 110 | 87 | 223 | 568 | 178 | 200 | 1102 | 1219 | 535 | 650 |
| 260 | 104 | 112 | 260 | 806 | 197 | 198 | 1399 | 1821 | 690 | 767 |
| 270 | 120 | 154 | 276 | 953 | 240 | 198 | 1545 | 2213 | 805 | 831 |
| 280 | 140 | 155 | 337 | 990 | 306 | 196 | 1770 | 2187 | 873 | 834 |
| 290 | 175 | 182 | 381 | 791 | 346 | 255 | 1885 | 2005 | 835 | 524 |
| 300 | 197 | 183 | 377 | 932 | 426 | 306 | 1933 | 1927 | 879 | 585 |
| 310 | 222 | 181 | 323 | 886 | 386 | 285 | 1419 | 1368 | 585 | 401 |
| 320 | 242 | 170 | 292 | 768 | 417 | 305 | 1127 | 991 | 513 | 404 |
| 330 | 259 | 181 | 258 | 731 | 367 | 272 | 902 | 661 | 427 | 284 |
| 340 | 278 | 202 | 244 | 544 | 369 | 260 | 771 | 689 | 411 | 263 |
| 350 | 307 | 193 | 203 | 434 | 255 | 237 | 610 | 430 | 264 | 164 |
| 360 | 244 | 167 | 160 | 312 | 216 | 188 | 490 | 351 | 234 | 113 |
| 370 | 227 | 173 | 129 | 262 | 171 | 186 | 379 | 342 | 276 | 125 |
| 380 | 183 | 128 | 113 | 209 | 146 | 164 | 280 | 272 | 204 | 79 |
| 390 | 150 | 111 | 88 | 141 | 99 | 118 | 247 | 286 | 179 | 99 |
| 400 | 117 | 103 | 68 | 120 | 77 | 97 | 286 | 261 | 178 | 75 |
| 410 | 86 | 60 | 47 | 79 | 42 | 104 | 191 | 184 | 137 | 63 |
| 420 | 81 | 53 | 38 | 62 | 36 | 93 | 157 | 172 | 153 | 65 |
| 430 | 74 | 57 | 34 | 53 | 19 | 62 | 151 | 122 | 102 | 50 |
| 440 | 73 | 60 | 30 | 51 | 17 | 51 | 133 | 163 | 111 | 44 |
| 450 | 72 | 26 | 16 | 33 | 21 | 38 | 92 | 62 | 75 | 32 |
| 460 | 64 | 22 | 15 | 32 | 21 | 24 | 77 | 45 | 59 | 24 |
| 470 | 46 | 20 | 15 | 16 | 6 | 22 | 80 | 66 | 97 | 35 |
| 480 | 33 | 13 | 15 | 7 | 6 | 19 | 46 | 33 | 69 | 32 |
| 490 | 40 | 13 | 9 | 10 | 6 | 12 | 40 | 23 | 51 | 16 |
| 500 | 25 | 6 | 8 | 7 | 3 | 11 | 29 | 13 | 30 | 10 |


| 510 | 18 | 2 | 4 | 3 | 2 | 10 | 13 | 9 | 28 | 3 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 520 | 21 | 3 | 9 | 4 |  | 8 | 12 | 12 | 27 | 8 |
| 530 | 6 | 2 | 2 | 3 | 3 | 4 | 10 | 3 | 17 | 3 |
| 540 | 7 | 4 | 4 | 0 | 0 | 3 | 4 | 2 | 19 | 8 |
| 550 | 6 | 1 | 1 | 0 | 0 | 2 | 3 | 1 | 10 | 2 |
| 560 | 2 | 1 | 3 | 0 | 1 | 2 | 1 | 4 | 5 | 3 |
| 570 | 3 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 12 | 0 |
| 580 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 16 | 2 |
| 590 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 7 | 0 |
| 600 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 10 | 0 |
| 610 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 620 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 |
| 630 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 640 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 650 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 660 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 670 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 680 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 3a: Length frequency data for the years 1985 to 1994.

| Length <br> (FL) <br> mm | 1995 | 1996 | 1997 | 1998 | 1999 | $\mathbf{2 0 0 0}$ | $\mathbf{2 0 0 1}$ | $\mathbf{2 0 0 2}$ | $\mathbf{2 0 0 3}$ | $\mathbf{2 0 0 4}$ | $\mathbf{2 0 0 5}$ | $\mathbf{2 0 0 6}$ | $\mathbf{2 0 0 7}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 100 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 120 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 130 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 140 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 |
| 150 | 0 | 1 | 0 | 0 | 3 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 160 | 0 | 2 | 0 | 0 | 1 | 4 | 0 | 0 | 0 | 0 | 8 | 0 | 0 |
| 170 | 0 | 11 | 0 | 3 | 10 | 3 | 0 | 0 | 0 | 0 | 15 | 2 | 0 |
| 180 |  | 17 | 0 | 4 | 28 | 2 | 0 | 0 | 0 | 0 | 16 | 2 | 0 |
| 190 | 2 | 14 | 0 | 5 | 14 | 5 | 0 | 0 | 0 | 0 | 15 | 13 | 0 |
| 200 | 2 | 21 | 2 | 6 | 12 | 11 | 0 | 0 | 0 | 0 | 47 | 22 | 0 |
| 210 | 19 | 26 | 17 | 43 | 14 | 10 | 0 | 0 | 0 | 0 | 25 | 33 | 0 |
| 220 | 43 | 30 | 29 | 67 | 33 | 18 | 0 | 0 | 0 | 0 | 78 | 33 | 3 |
| 230 | 92 | 77 | 80 | 175 | 89 | 23 | 0 | 0 | 0 | 0 | 104 | 23 | 0 |
| 240 | 218 | 236 | 169 | 337 | 188 | 5 | 0 | 0 | 0 | 0 | 99 | 4 | 1 |
| 250 | 230 | 309 | 296 | 481 | 351 | 7 | 0 | 0 | 0 | 0 | 102 | 6 | 1 |
| 260 | 394 | 369 | 443 | 642 | 555 | 10 | 0 | 0 | 0 | 0 | 110 | 8 | 0 |
| 270 | 556 | 447 | 580 | 803 | 725 | 19 | 0 | 0 | 0 | 0 | 123 | 4 | 0 |
| 280 | 708 | 545 | 661 | 826 | 837 | 25 | 0 | 0 | 0 | 0 | 111 | 5 | 0 |
| 290 | 712 | 836 | 682 | 859 | 895 | 39 | 0 | 0 | 0 | 0 | 103 | 38 | 8 |


| 300 | 775 | 927 | 911 | 1087 | 1054 | 57 | 0 | 0 | 0 | 0 | 116 | 73 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 310 | 565 | 833 | 817 | 1023 | 1006 | 39 | 0 | 0 | 0 | 0 | 153 | 95 | 26 |
| 320 | 556 | 817 | 971 | 986 | 976 | 25 | 0 | 0 | 0 | 0 | 142 | 98 | 47 |
| 330 | 434 | 638 | 888 | 910 | 947 | 28 | 0 | 0 | 0 | 0 | 213 | 116 | 57 |
| 340 | 365 | 618 | 918 | 853 | 918 | 40 | 0 | 0 | 0 | 0 | 183 | 77 | 56 |
| 350 | 273 | 447 | 791 | 696 | 812 | 35 | 0 | 0 | 0 | 0 | 197 | 100 | 70 |
| 360 | 220 | 358 | 621 | 605 | 703 | 50 | 0 | 0 | 0 | 0 | 182 | 74 | 55 |
| 370 | 204 | 405 | 608 | 512 | 656 | 30 | 0 | 0 | 0 | 0 | 130 | 78 | 51 |
| 380 | 201 | 335 | 534 | 407 | 566 | 35 | 0 | 0 | 0 | 0 | 124 | 70 | 73 |
| 390 | 209 | 314 | 414 | 353 | 482 | 38 | 0 | 0 | 0 | 0 | 76 | 37 | 72 |
| 400 | 208 | 273 | 392 | 309 | 446 | 39 | 0 | 0 | 0 | 0 | 65 | 51 | 66 |
| 410 | 160 | 162 | 315 | 252 | 396 | 35 | 0 | 0 | 0 | 0 | 30 | 36 | 100 |
| 420 | 144 | 152 | 308 | 240 | 351 | 39 | 0 | 0 | 0 | 0 | 30 | 16 | 86 |
| 430 | 105 | 120 | 272 | 214 | 237 | 34 | 0 | 0 | 0 | 0 | 22 | 22 | 67 |
| 440 | 90 | 116 | 208 | 190 | 207 | 28 | 0 | 0 | 0 | 0 | 16 | 20 | 37 |
| 450 | 73 | 75 | 253 | 179 | 146 | 28 | 0 | 0 | 0 | 0 | 11 | 14 | 28 |
| 460 | 60 | 68 | 201 | 141 | 150 | 18 | 0 | 0 | 0 | 0 | 12 | 8 | 28 |
| 470 | 70 | 75 | 172 | 149 | 126 | 15 | 0 | 0 | 0 | 0 | 1 | 7 | 22 |
| 480 | 46 | 34 | 178 | 89 | 96 | 13 | 0 | 0 | 0 | 0 | 4 | 12 | 17 |
| 490 | 46 | 41 | 125 | 93 | 61 | 4 | 0 | 0 | 0 | 0 | 4 | 8 | 14 |
| 500 | 26 | 26 | 109 | 129 | 35 | 5 | 0 | 0 | 0 | 0 | 3 | 14 | 10 |
| 510 | 27 | 21 | 73 | 85 | 36 | 5 | 0 | 0 | 0 | 0 | 1 | 1 | 8 |
| 520 | 21 | 19 | 63 | 89 | 36 | 1 | 0 | 0 | 0 | 0 | 0 | 5 | 20 |
| 530 | 21 | 16 | 45 | 73 | 26 | 4 | 0 | 0 | 0 | 0 | 0 | 3 | 6 |
| 540 | 5 | 6 | 27 | 66 | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 2 |
| 550 | 6 | 2 | 15 | 43 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 4 |
| 560 | 2 | 7 | 12 | 32 | 10 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 4 |
| 570 | 2 | 8 | 9 | 28 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 3 |
| 580 | 1 | 1 | 2 | 21 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 590 | 0 | 2 | 0 | 10 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 600 | 0 | 3 | 2 | 10 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 610 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 620 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 630 | 0 | 0 | 1 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 640 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 650 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 660 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 670 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 680 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 3b: Length frequency data for the years 1995 to 2007.


Figure A-3: Length frequencies from line fish catches for years 1985 to 2000 and 2005 to 2007. No data are available from 2001 to 2005.


Figure A-4: Proportions caught per length group (mm) for years 1985 to 2000 and 2005 to 2007.

