

Finalised Assessment of the South African anchovy resource using data from 1984 – 2011: results at the posterior mode

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Abstract

The operating model (OM) for the South African anchovy resource has been updated from that used to develop OMP-08 given five more years of data, a revised time series of commercial catch and November survey proportion-at-age 1 estimates provided by a new approach. A Beverton Holt stock recruitment relationship is used, marginally supported by the AICc model selection criterion over a Ricker stock recruitment relationship. Time-invariant natural mortality is assumed at 1.2year^{-1} for both juvenile and adult natural mortality; an increase from that assumed for the OM from which OMP-08 was developed, with the change made because of a better fit to the data and avoidance of the questionable implication that the recruit survey detects a greater proportion of the recruits than the November survey detects of the adult biomass. There has been a decrease in recruitment residual standard deviation and in recruitment autocorrelation for this updated OM compared to the values used in previous OMs. The impact of this on the appropriate choices of a risk definition and threshold for the new OMP to be developed needs to be considered. The resource abundance has dropped below the historic (1984-2010) average, with a model-estimated spawner biomass of 1.2 million tons in November 2011, following 2 years of below average recruitment. Only four out of the past 13 years have produced below average recruitment. The harvest proportion over the past 11 years has not exceeded 0.13.

Introduction

Although the base case operating model for the South African anchovy resource was updated from the last assessment (Cunningham and Butterworth 2007, with further updates) to take account of new data collected between 2007 and 2010 (de Moor and Butterworth 2011a), the International Review Panel for the 2011 International Fisheries Stock Assessment Workshop suggested some revisions to this model (Anon. 2011) before it is used in the development of a new MP.

de Moor and Butterworth (2011a) proposed two base case operating models; one which estimated random effects about adult natural mortality over time while the other assumed constant (time-invariant) adult natural mortality. The inclusion of the random effects was in response to a perceived trend in the residuals from the model fit to May recruitment and November proportion-at-age 1 data (de Moor and Butterworth 2011b). Anon. (2011) suggested the November proportion-at-age 1 data may have been overfit, and suggested instead that a base case with constant natural mortality be used and a revision of the time series of proportion-at-age 1 data be attempted.

de Moor and Butterworth (2012b) provided updated assessment results using a base case with an average time-invariant effective sample size for the assumed binomially distributed proportion of 1-year-old anchovy estimated by de Moor and Butterworth (2012a). This document presents such results using a base case with annually varying effective sample sizes. This update to the operating model for the South African anchovy resource contains the following changes from the last full assessment in 2007.

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- i) The time series of commercial catch data has been revised since 2007; the monthly cut-off lengths for recruits now vary on an annual basis in accordance with the cut-off length estimated by the annual recruit survey (de Moor *et al.* 2012).
- ii) The inclusion of one more year's survey data from November 2010 to 2011 from those used by de Moor and Butterworth (2011a).
- iii) The time series of proportions-at-age 1 in the November survey has been revised (de Moor and Butterworth 2012a).
- iv) The method used to calculate weight-at-age corresponding to the November survey has been changed as an age-length key is no longer used. The new method involves assuming a time-invariant ratio of weight at ages 2, 3 and 4+ to age 1, and uses the time series of average weight-at-age in the November survey (de Moor *et al.* 2012).

This document presents the updated base case operating models assuming a Beverton Holt stock recruitment relationship to apply. A number of robustness tests are also considered. Results are given at the posterior mode only. A separate document will show the full posterior distributions.

Population Dynamics Model

The operating model used for the South African anchovy resource is detailed in Appendix A. A glossary of all parameters used in this document is given in Appendix B. The data used in this assessment are listed in de Moor *et al.* (2012). The majority of prior distributions for the estimated parameters were chosen to be relatively uninformative.

Stock recruitment relationship

The following alternative stock recruitment relationships have been considered (Table 1):

- A_{BH} – Beverton Holt stock-recruitment curve, with uniform priors on steepness and carrying capacity
- A_{2BH} – two Beverton Holt stock-recruitment curves, with uniform priors on steepness and carrying capacity, one estimated using data from 1984 to 1999 and the other from 2000 to 2010
- A_R – Ricker stock-recruitment curve, with uniform priors on steepness and carrying capacity
- A_{HS} – hockey stick stock-recruitment curve, with uniform priors on the log of the maximum recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity
- A_{2HS} – two hockey stick stock-recruitment curves, with uniform priors on the log of the maximum recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity, one estimated using data from 1984 to 1999 and the other from 2000 to 2010
- $A_{fixedHS}$ – hockey stick stock-recruitment curve with a uniform prior on the log of the maximum recruitment, with the spawning biomass at the inflection point set equal to 20% of K (to correspond to the assumption made for the 2007 assessment)

In cases where a second curve is estimated from 2000 to 2012, the variance about the stock recruitment curve over this time period, $(\sigma_{r,2000+}^A)^2 \sim U(0.04,10)$, is estimated separately from that for the earlier time period, $(\sigma_r^A)^2$.

Natural mortality

A number of combinations of time-invariant juvenile and median adult natural mortality values are tested, covering the range 0.6 to 1.8 year⁻¹, and for the case where a Beverton Holt stock recruitment relationship is assumed. For realism, only combinations with $\bar{M}_j^A \geq \bar{M}_{ad}^A$ are tested.

Variable natural mortality

Alternatives to the assumption of constant natural mortality over time will be considered through the following robustness tests (which may be further augmented later):

A_{Mad} – annually varying adult natural mortality, i.e. random effects model with $\sigma_{ad} \sim U(0.2,0.5)$ ¹, and

$\rho \sim U(0,1)$. Initial results showed there was no substantial improvement in the model fit to the data if juvenile natural mortality was allowed to vary annually.

A_{M2000+} – natural mortality is assumed to have increased at the turn of the century. In this case

$\bar{M}_j^A = \bar{M}_{ad}^A = 0.9 \text{ year}^{-1}$ prior to 2000 and $\bar{M}_j^A = \bar{M}_{ad}^A = 1.2 \text{ year}^{-1}$ from 2000 onwards.

A_{Mden} – density dependent natural mortality, i.e. $\bar{M}_{j,y}^A = \bar{M}_{ad,y}^A = \bar{M} + \omega(B^{avg} - B_y^A)$, where B^{avg} is a coarse estimate of the average model predicted biomass over time, $\bar{M} \sim U(0.9,1.5)$ and $\omega \sim U(-1,1)$.

Further robustness tests

The following robustness tests to A_{BH} are also considered:

A_{Neff} – average N^{eff} value, $N^{eff} = \frac{\sum_{y=1984}^{yn} \hat{p}_{y,1}^A (1 - \hat{p}_{y,1}^A)}{\sum_{y=1984}^{yn} (\hat{\sigma}_{y,p}^A)^2}$, rather than the annually

varying N_y^{eff} given in Appendix A

A_{prop} – alternative time series of proportion-at-age 1 data (and corresponding average weights at ages 1 and 2+), corresponding to the “Constant σ_1 ” model of de Moor and Butterworth (2012a)

A_{noprop} – no proportion-at-age 1 data in the likelihood

A_{kegg1} – negatively biased egg surveys, i.e., $k_g^A = 0.75$ (testing sensitivity to assumption 8 of Appendix A)

¹ The lower bound of 0.2 was chosen from initial results by which indicated that there was a change in the model fit to the data when σ_{ad} decreased from 0.20 to 0.19, with a poorer fit obtained for the fit to the proportion-at-age 1 data. In general, the negative log posterior distribution decreases with decreasing σ_{ad} , primarily due to the contributions from the prior on η_y^{ad} .

A_{kegg2} – positively biased egg surveys, i.e., $k_g^A = 1.25$ (testing sensitivity to assumption 8 of Appendix A)

A_{lamR} – fix the additional variance (over and above the survey sampling CV) associated with the recruit survey $(\lambda_r^A)^2 = 0$

A_{lamN} – fix the additional variance (over and above the survey sampling CV) associated with the November survey $(\lambda_N^A)^2 = 0.02$

Retrospective runs

A_{BH} is run using data from 1984 to 1999, to 2003 and to 2006 to compare the base case model estimates to those which would have resulted from data corresponding to the years used as input to the OMs used for testing OMP-02, OMP-04 and OMP-08. Note that the data used in A_{BH} and the retrospective runs do NOT compare directly with those used for the former OMs due to methodological updates over time, corrections to historic time series of data and the revision of the time series of proportion-at-age 1.

Results

Natural mortality

Table 2 lists the various contributions to the negative log posterior probability distribution function (pdf) at the posterior mode for the full range of combinations of juvenile and adult natural mortality tested. There is little change in the posterior distribution as \bar{M}_j^A is changed for a given \bar{M}_{ad}^A . Given \bar{M}_j^A , the posterior distribution indicated an improved fit to the data for increasing \bar{M}_{ad}^A . This latter feature may, however, be an artefact of the assessment methodology in that a higher natural mortality results in a higher loss of “memory” of cohorts, making the November survey data easier to fit.

The following criterion was used to distinguish “reasonable” from “unrealistic” combinations (“unrealistic” combinations are shaded in Table 2):

- the ratio $k_r^A / k_N^A \in [0.5, 1.0]$, as the November spawner biomass survey is expected to have a greater coverage of the full distribution of the resource than the May recruit survey so that the latter should reflect a smaller relative bias;
- the multiplicative bias for the proportion-at-age 1 in the November survey, k_p^A , should not be markedly different from 1; a value much lower than 1 would indicate the 1 year olds are not fully sampled by the survey, while a value much higher than 1 would indicate the 2+ year olds are not fully sampled by the survey; the latter of these seems less likely.

Considering these criteria, the following combinations were chosen for a set of robustness tests:

A_{BH} - $\bar{M}_j^A = 1.2$ and $\bar{M}_{ad}^A = 1.2$ (base case)

A_{M1} - $\bar{M}_j^A = 0.9$ and $\bar{M}_{ad}^A = 0.9$ (robustness test: for comparison with the base case assessment of 2007)

- A_{M2} - $\bar{M}_j^A = 1.5$ and $\bar{M}_{ad}^A = 1.2$ (robustness test: alternative \bar{M}_j^A , similar to A_{BH} in terms of value of the negative log joint posterior mode and k_p^A)
- A_{M3} - $\bar{M}_j^A = 1.2$ and $\bar{M}_{ad}^A = 0.9$ (robustness test: alternative \bar{M}_{ad}^A , with a worse negative log joint posterior mode value and higher k_r^A/k_N^A and k_p^A than A_{BH})
- A_{M4} - $\bar{M}_j^A = 1.8$ and $\bar{M}_{ad}^A = 1.2$ (robustness test: alternative \bar{M}_j^A , similar to A_{BH} in terms of value of negative log joint posterior mode and k_p^A)

Normally a change in the base case value of \bar{M}_j^A and \bar{M}_{ad}^A from that used previously would be avoided in the interests of consistency over time in assessments, but here this consideration was considered to be outweighed by an appreciably better fit to the data in likelihood terms together with avoidance of the questionable implication that the recruit survey detects a greater proportion of the recruits than the November survey detects of the adult biomass.

Stock recruitment relationship

Table 3 lists the various contributions to the negative log posterior pdf at the posterior mode for the alternative stock-recruitment relationships considered. AIC_c is used to approximately² compare amongst alternative stock-recruitment relationships, suggesting that the preferred stock-recruitment relationship is the Beverton Holt, with the Ricker being a close second choice. Thus A_{BH} is chosen as the base case operating model for OMP-13 development, with robustness being tested to A_R and A_{HS} (Figures 1 and 2). Models with different stock-recruitment relationships before and after the turn of the century were not favoured by AIC_c, primarily due to the additional number of estimable parameters required for these models. To enable comparison with the 2007 assessment, the hockey stick curve with a fixed inflection point, A_{fixedHS}, is also maintained as an alternative (Table 4).

Base case (A_{BH}) results at posterior mode

The estimated parameter values and key outputs for A_{BH} are listed in Table 4. The population model fits to the time series of abundance estimates of November 1+ biomass, DEPM estimates of spawner biomass, May recruitment and proportion-at-age 1 in November are shown in Figures 3 to 6. There is some trend in the residuals from the model fit to the May survey estimates of recruitment. The model projected posterior mode estimates of May recruitment in 2010 and November 2011 fall outside the 95% CIs for the survey results due to the model struggling to match a sharp decrease in the survey estimates of 1+ biomass from 2009 to 2011 after a relatively good recruitment estimate in May 2010. The historic annual harvest rates are plotted in Figure 7 and the annual losses of anchovy to predation are listed in Table 5.

Variable natural mortality

² Strictly AIC_c is for use in comparing between alternative frequentist models; the comparison here is made at the joint posterior mode.

The alternative robustness test which allows for adult natural mortality to vary with time through the use of random effects, A_{Mad} , results in a better fit to the data (Table 4, Figure 6), though there is little change in the residuals (results not shown). However, in this case the adult natural mortality is estimated to increase over time, ranging between 1.33 and 2.24, with strong autocorrelation ($\rho = 0.93$) (Table 4, Figure 8), which one could argue to be unrealistic given the consistent estimation of adult natural mortality which are above that of juvenile natural mortality. A slightly better fit to the May recruitment data is obtained if natural mortality is assumed to increase at the turn of the century (A_{M2000+}), and the perceived trend in residuals from the model fit to the May survey estimates of recruitment disappears. However, this alternative results in an unreliable estimate of K^A at the upper boundary of the prior distribution (Table 4). A_{Mden} similarly results in an unreliable estimate of K^A at the upper boundary of the prior distribution with an estimated range for juvenile and adult natural mortality above that assumed for A_{BH} (Figure 8). The fit to the data is, however, improved (Table 4).

Proportion-at-age 1

The fit to the November and May hydroacoustic data is poorer for A_{Neff} (with constant average rather than annually varying effective sample sizes for the proportion-at-age inputs) compared to A_{BH} , while the fit to the proportion-at-age 1 inputs is improved (Figure 6). A worse fit to the overall data is obtained for A_{prop} (with the alternative proportion-at-age 1 inputs) compared to A_{BH} , though the difference in fits is only noticeable in the proportion-at-age 1 data (Figure 6). Excluding the proportion-at-age 1 data from the assessment, A_{noprop} , results in an improved fit to the November survey estimates of abundance, without a substantial change to the remaining key model parameters (Table 4).

Further robustness tests

The model parameters, contributions to the negative log posterior pdf and key model outputs at the posterior mode for the robustness tests are given in Table 4. The remaining robustness tests, not discussed above, did not result in unanticipated changes from the parameter estimates for A_{BH} . Naturally, the magnitude of the resource biomass is dependent on the assumption made regarding the bias (if any) in the time series of abundance estimates resulting from the November egg surveys.

Retrospective analysis

There is little difference in the historic November 1+ biomass trajectory for the retrospective runs (Figure 9). The shape of the Beverton Holt stock recruitment curve changes between these runs, as do the estimates of carrying capacity and steepness, though the extent of the variability about the stock recruitment curve remains relatively constant across the retrospective runs (Table 6). The average model predicted 1984-1999 spawner biomass remains relatively stable over the retrospective runs.

Discussion

This document has detailed the updated assessment of the South African anchovy resource. The base case hypothesis assumes a Beverton Holt stock recruitment curve and time-invariant natural mortality. Results at

the posterior mode have also been presented for a number of robustness tests to the base case hypothesis, A_{BH} . The resource abundance in November 2011 is estimated to have dropped below the historic (1984-2010) average, and is now estimated at 1.2 million tons by A_{BH} . The two most recent years have seen below average recruitment, after a sustained period (9 out of 11) years of above average recruitment. The harvest proportion over the past 11 years has not exceeded 0.13 (Figure 7). Figure 10 demonstrates the change in the assumptions for anchovy recruitment to be used as a base case OM during OMP-13 development compared to that used in the development of OMP-08, while the effect of the change in assumed time-invariant natural mortality from that assumed during the development of OMP-08 can be seen on the time series of model predicted anchovy spawner biomasses in Figure 11.

References

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Table 1. The alternative stock-recruitment relationships considered. The parameter h^A denotes the “steepness” of the stock-recruitment relationship, which is the proportion of the virgin recruitment that is realised at a spawning biomass level of 20% of average pre-exploitation (virgin) spawning biomass K^A (shown in units of thousands of tons). For the hockey stick model, $X = \sum_{a=1}^3 \bar{w}_a^A e^{-M_j^A - (a-1)\bar{M}_{ad}^A} + \bar{w}_{4+} e^{-M_j^A - 3\bar{M}_{ad}^A} \frac{1}{1 - e^{-\bar{M}_{ad}^A}}$, where \bar{w}_a^A is the average of $w_{y,a}^A$ as defined in Appendix A. For the hockey stick model, a^A denotes the maximum recruitment (in billions) and b^A denotes the spawner biomass below which the expectation for recruitment is reduced below the maximum.

Test	Stock recruitment relationship	$f(SSB_{y,N}^A) =$	Parameters
A _{BH}	Beverton Holt	$\frac{\alpha^A SSB_y^A}{\beta^A + SSB_y^A}$	$h^A \sim U(0.2,1)$ $K^A \sim U(0,10\ 000)$ $\alpha^A = \frac{4h^A}{5h^A - 1} \frac{K^A}{X}$ $\beta^A = \frac{K^A(1-h^A)}{5h^A - 1}$
A _{2BH}	Beverton Holt (2 curves)	$\frac{\alpha_1^A SSB_y^A}{\beta_1^A + SSB_y^A}$ if $y < 2000$ $\frac{\alpha_2^A SSB_y^A}{\beta_2^A + SSB_y^A}$ if $y \geq 2000$	$h_{1/2}^A \sim U(0.2,1)$ $K_{1/2}^A \sim U(0,10\ 000)$ $\alpha_{1/2}^A = \frac{4h_{1/2}^A}{5h_{1/2}^A - 1} \frac{K_{1/2}^A}{X}$ $\beta_{1/2}^A = \frac{K_{1/2}^A(1-h_{1/2}^A)}{5h_{1/2}^A - 1}$
A _R	Ricker	$\alpha^A SSB_y^A e^{-\beta^A SSB_{y,N}^A}$	$h^A \sim U(0.2,1.5)$ $K^A \sim U(0,10\ 000)$ $\alpha^A = \frac{1}{X} \left(\frac{h^A}{0.2} \right)^{1/0.8}$ $\beta^A = \frac{\ln(h^A/0.2)}{0.8K^A}$
A _{ModR}	Modified Ricker	$\alpha^A SSB_y^A e^{-\beta^A (SSB_{y,N}^A)^c}$	$h^A \sim U(0.2,1.5)$ $K^A \sim U(0,10\ 000)$ $c \sim U(0,1)$ $\alpha^A = \frac{1}{X} \left(\frac{h^A}{0.2} \right)^{\frac{1}{1-0.2^c}}$ $\beta^A = \frac{\ln(h^A/0.2)}{(K^A)^c [1-0.2^c]}$

Table 1 (continued).

Test	Stock recruitment relationship	$f(SSB_{y,N}^A) =$	Parameters
A _{HS}	Hockey stick	$\begin{cases} a^A & , \text{if } SSB_y^A \geq b^A \\ \frac{a^A}{b^A} SSB_y^A & , \text{if } SSB_y^A < b^A \end{cases}$	$\ln(a^A) \sim U(0,7.2)^3$ $\frac{b^A}{K^A} \sim U(0,1)$ $K^A = a^A X^4$
A _{2HS}	Hockey stick (2 curves)	<p>if $y < 2000$:</p> $\begin{cases} a_1^A & , \text{if } SSB_y^A \geq b_1^A \\ \frac{a_1^A}{b_1^A} SSB_y^A & , \text{if } SSB_y^A < b_1^A \end{cases}$ <p>if $y \geq 2000$:</p> $\begin{cases} a_2^A & , \text{if } SSB_y^A \geq b_2^A \\ \frac{a_2^A}{b_2^A} SSB_y^A & , \text{if } SSB_y^A < b_2^A \end{cases}$	$\ln(a_{1/2}^A) \sim U(0,7.2)^1$ $\frac{b_{1/2}^A}{K_{1/2}^A} \sim U(0,1)$ $K_{1/2}^A = a_{1/2}^A X^2$
A _{fixedHS}	Hockey stick	$\begin{cases} a^A & , \text{if } SSB_y^A \geq b^A \\ \frac{a^A}{b^A} SSB_y^A & , \text{if } SSB_y^A < b^A \end{cases}$	$\ln(a^A) \sim U(0,7.2)$ $b^A = 0.2K^A$ $K^A = a^A X$

³ Given the lack of *a priori* information on the scale of a^A , a log-scale was used, with a maximum corresponding to about 10 million tons.

⁴ For consistency, K relates throughout to corresponding MLEs. These will be less than the corresponding average pre-exploitation levels because of the lognormal distributions assumed for recruitment.

Table 2. The contributions to the negative log posterior pdf at the posterior mode for a range of combinations of juvenile, \bar{M}_j^A , and adult, \bar{M}_{ad}^A , natural mortality for models assuming the Hockey Stick stock recruitment relationship. The ratio of the multiplicative bias in the recruit survey to that in the November survey, k_r^A/k_N^A , and the multiplicative bias in the proportion-at-age 1 in the November survey, k_p^A , are given for diagnostic purposes. Shaded cells represent unrealistic choices in terms of the criteria applied.

\bar{M}_j^A	\bar{M}_{ad}^A	-ln (Posterior)	-ln(Likelihood)				-ln(Prior)	k_r^A	k_N^A	k_r^A/k_N^A	k_p^A
			Nov	Egg	Rec	Prop	ϵ_y^A				
0.6	0.6	293.49	5.31	8.56	20.80	231.06	27.77	1.30	1.89	1.45	2.01
0.9	0.6	294.48	5.60	8.59	20.85	231.05	28.39	1.30	1.70	1.31	2.01
0.9	0.9	279.82	-4.74	6.35	19.18	230.36	28.67	1.21	1.26	1.04	1.26
1.2	0.6	295.47	5.88	8.61	20.95	231.05	28.98	1.29	1.53	1.18	2.01
1.2	0.9	280.56	-4.53	6.39	19.15	230.47	29.09	1.21	1.12	0.93	1.25
1.2	1.2	272.08	-9.37	5.54	16.55	231.34	28.02	1.16	0.90	0.77	0.73
1.5	0.6	296.47	6.16	8.62	21.08	231.06	29.54	1.29	1.37	1.06	2.01
1.5	0.9	281.30	-4.33	6.42	19.16	230.58	29.47	1.21	1.00	0.83	1.25
1.5	1.2	272.61	-9.23	5.59	16.52	231.44	28.30	1.16	0.80	0.69	0.73
1.5	1.5	267.20	-12.01	5.16	14.57	232.21	27.28	1.12	0.68	0.60	0.47
1.8	0.6	297.46	6.43	8.63	21.24	231.09	30.08	1.29	1.23	0.96	2.01
1.8	0.9	282.03	-4.14	6.46	19.20	230.70	29.82	1.20	0.89	0.74	1.24
1.8	1.2	273.14	-9.11	5.64	16.53	231.53	28.55	1.15	0.71	0.61	0.73
1.8	1.5	267.61	-11.93	5.21	14.59	232.26	27.48	1.12	0.60	0.53	0.47
1.8	1.8	264.23	-13.61	4.97	13.26	233.00	26.61	1.10	0.54	0.49	0.31

Table 3. The contributions to the negative log posterior pdf at the joint posterior mode, together with the values of various quantities at that mode, for alternative stock recruitment relationships.

	A_{BH}	A_{2BH}	A_R	A_{HS}	A_{2HS}	$A_{fixedHS}$
$-\ln(\text{Posterior})$	272.08	271.43	272.23	273.62	271.61	276.34
$-\ln(L_{Nov})$	-9.37	16.84	-9.31	-9.98	17.28	-10.75
$-\ln(L_{Egg})$	5.54	11.91	5.57	5.42	11.93	5.12
$-\ln(L_{Rec})$	16.55	-10.78	16.52	17.16	-11.10	17.65
$-\ln(L_{Prop})$	231.34	231.37	231.32	231.23	231.35	231.40
$-\ln(\text{Prior rec residuals})$	28.02	22.09	28.13	29.80	22.15	32.93
# parameters	36	39	36	36	39	35
Sample size (i.e. data points)	93	93	93	93	93	93
AIC	616.16	620.85	616.45	619.24	621.22	622.68
AIC_c	663.73	679.72	664.02	666.82	680.09	666.89
h^A	0.33	0.27	0.30			
K^A	2705	10000	2928	2291	4109	1848
a^A	1078	6149	0.33	445	798	359
b^A	2846	21658	0.00	1340	2979	370
h_2^A		0.41				
K_2^A		4959			4929	
a_2^A		1516			957	
b_2^A		2845			1376	

Table 4. Key parameter values estimated at the joint posterior mode together with key model outputs. All robustness tests are defined in the main text and all parameters are defined in Appendix B. Fixed values are given in **bold**. Numbers are reported in billions and biomass in thousands of tons.

	A _{BH}	A _R	A _{HS}	A _{fixedHS}	A _{M1}	A _{M2}	A _{M3}	A _{M4}	A _{Mad}	A _{M2000+}	A _{Mden}	A _{Neff}	A _{prop}	A _{noprop}	A _{kegg1}	A _{kegg2}	A _{lamR}	A _{lamN}	
-ln(Posterior)	272.1	272.2	273.6	276.3	279.8	272.6	280.6	273.1	240.4	269.6	260.6	161.1	442.5	35.6	273.1	271.3	288.3	272.9	
-ln(L _{Nov})	-9.4	-9.3	-10.0	-10.6	-4.7	-9.2	-4.5	-9.1	-14.2	-7.2	-13.6	-7.2	-6.3	-12.9	-9.0	-9.7	10.2	-1.59	
-ln(L _{Egg})	5.5	5.6	5.4	5.1	6.3	5.6	6.4	5.6	5.7	7.0	4.9	6.0	5.2	5.4	5.7	5.3	9.5	6.20	
-ln(L _{Rec})	16.6	16.5	17.2	17.6	19.2	16.5	19.2	16.5	9.11	12.5	10.8	13.6	19.1	16.3	16.4	16.7	10.0	13.10	
-ln(L _{Prop})	231.3	231.3	231.2	231.4	230.4	231.4	230.5	231.5	227.4	229.2	230.4	122.7	394.7		231.2	231.5	233.7	229.2	
-ln(Prior rec residuals)	28.0	28.1	29.8	32.9	28.7	28.3	29.1	28.6	25.6	28.1	28.0	25.8	29.8	26.8	28.8	27.4	25.0	26.0	
-ln(Prior Mad residuals)									-13.2										
Fixed/Estimated parameters																			
\bar{M}_j^A	1.2	1.2	1.2	1.2	0.9	1.5	1.2	1.8	1.2	0.9-1.2	1.6-2.4	1.2	1.2	1.2	1.2	1.2	1.2	1.2	
\bar{M}_{ad}^A	1.2	1.2	1.2	1.2	0.9	1.2	0.9	1.2	est	0.9-1.2	1.6-2.4	1.2	1.2	1.2	1.2	1.2	1.2	1.2	
$N_{1983,0}^A$	143.0	143.2	140.0	141.5	91.3	184.3	116.0	238.3	183.7	99.1	280.5	129.4	128.9	106.5	168.2	128.2	160.9	146.3	
$N_{1983,1}^A$	136.0	136.2	131.9	135.5	100.7	136.0	100.9	136.1	115.5	91.9	162.2	152.4	127.3	167.6	178.6	110.8	151.1	137.5	
$N_{1983,2}^A$	40.9	41.0	39.7	40.8	41.0	41.0	41.0	41.0	30.4	37.4	25.7	45.9	38.4	50.5	53.8	33.4	45.5	41.4	
$N_{1983,3}^A$	12.3	12.4	12.0	12.3	16.6	12.3	16.7	12.3	8.0	15.2	4.1	13.8	11.6	15.2	16.2	10.0	13.7	12.5	
k_N^A	1.16	1.16	1.18	1.17	1.21	1.16	1.21	1.15	1.13	1.21	1.14	1.13	1.22	1.19	0.87	1.45	0.96	1.10	
k_r^A	0.90	0.89	0.91	0.91	1.26	0.80	1.12	0.70	0.70	1.06	0.52	0.86	0.95	0.91	0.71	1.06	0.73	0.84	
k_r^A/k_N^A	0.77	0.77	0.77	0.78	1.04	0.69	0.93	0.61	0.62	0.88	0.45	0.76	0.78	0.76	0.82	0.73	0.76	0.77	
k_g^A	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.75	1.25	1.00	1.00
k_p^A	0.74	0.74	0.74	0.75	1.26	0.73	1.25	0.73	0.34	0.95	0.29	1.00	0.82		0.75	0.72	0.67	0.71	
$(\lambda_N^A)^2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02

Table 4 (continued).

	A _{BH}	A _R	A _{HS}	A _{fixedHS}	A _{M1}	A _{M2}	A _{M3}	A _{M4}	A _{Mad}	A _{M2000+}	A _{Mden}	A _{Neff}	A _{prop}	A _{noprop}	A _{kegg1}	A _{kegg2}	A _{lam1}	A _{lam2}
$(\lambda_r^A)^2$	0.16	0.16	0.17	0.18	0.20	0.16	0.20	0.16	0.08	0.11	0.09	0.12	0.18	0.16	0.16	0.16	0.00	0.12
σ_j																		
σ_{ad}									0.20 ⁵									
ρ									0.93									
a^A	1078	0.33	445	359	600	1465	812	1989	2476	2472	2541	1191	756	983	1458	863	1806	1342
b^A	2846	0.0002	1340	370	2301	2974	2424	3097	6797	11456	3080	3239	1626	2463	4295	2063	5620	3823
K^A	2705	2928	2291	1849	2911	2612	2798	2523	5951	10000 ⁶	10000 ⁶	2894	2265	2600	3213	2381	3678	3085
h^A	0.33	0.30			0.36	0.32	0.35	0.31	0.32	0.32	0.52	0.32	0.37	0.34	0.30	0.35	0.29	0.31
σ_r^A	0.68	0.69	0.73	0.82	0.70	0.69	0.71	0.70	0.62	0.68	0.68	0.63	0.73	0.65	0.70	0.67	0.61	0.63
	Model outputs																	
B_{2011}^A	1172	1176	1149	1126	1247	1180	1258	1187	917	1091	1033	1231	1106	1030	1594	926	1731	1411
\bar{B}_{Nov}^A ⁷	1157	1159	1143	1160	1104	1159	1107	1162	1178	1112	1194	1176	1116	1124	1543	930	1279	1187
η_{2009}^A	-1.19	-1.21	-1.29	-0.92	-1.07	-1.18	-1.06	-1.17	-1.35	-0.93	-1.21	-1.43	-0.91	-1.68	-1.17	-1.20	-1.36	-1.12
s_{cor}^A	0.10	0.69	0.73	0.82	0.12	0.10	0.13	0.11	0.09	0.16	0.27	0.15	0.03	0.15	0.12	0.08	0.10	0.12

⁵ Estimated on the lower bound of the prior distribution

⁶ Estimated on the upper bound of the prior distribution

⁷ This is the average over 1984 to 1999. OMP-04 and OMP-08 were developed using Risk defined as “the probability that adult anchovy biomass falls below 10% of the average adult anchovy biomass between November 1984 and November 1999 at least once during the projection period of 20 years”.

Table 5. The annual estimated anchovy loss to predation (in '000t), P_y^A in Appendix C, compared to the annual anchovy catch (in '000t), and the annual total proportion fished, F_y^A in Appendix C.

Year	Catch	A_{BH}			A_{Mad}		
		Loss to M	Catch / Loss to M	Annual proportion fished	Loss to M	Catch / Loss to M	Annual proportion fished
1984	265	54842	0.00	0.15	56446	0.00	0.18
1985	280	4079	0.07	0.19	4333	0.06	0.20
1986	300	7119	0.04	0.15	7424	0.04	0.15
1987	600	6173	0.10	0.22	6963	0.09	0.21
1988	570	5675	0.10	0.23	6142	0.09	0.23
1989	297	1868	0.16	0.25	2320	0.13	0.24
1990	152	2329	0.07	0.20	2626	0.06	0.19
1991	151	7208	0.02	0.07	7200	0.02	0.07
1992	349	5651	0.06	0.15	6623	0.05	0.15
1993	236	3017	0.08	0.16	3682	0.06	0.17
1994	156	1815	0.09	0.19	2126	0.07	0.20
1995	177	2081	0.08	0.25	2603	0.07	0.24
1996	42	850	0.05	0.10	1262	0.03	0.09
1997	60	2590	0.02	0.08	3018	0.02	0.07
1998	108	3786	0.03	0.10	4930	0.02	0.10
1999	179	6072	0.03	0.11	7525	0.02	0.11
2000	268	19225	0.01	0.07	21896	0.01	0.07
2001	285	24182	0.01	0.05	32799	0.01	0.05
2002	216	10820	0.02	0.04	16733	0.01	0.04
2003	256	10074	0.03	0.07	14470	0.02	0.07
2004	192	5846	0.03	0.08	8650	0.02	0.09
2005	282	7472	0.04	0.11	10163	0.03	0.11
2006	136	5626	0.02	0.07	8261	0.02	0.07
2007	251	8364	0.03	0.10	11849	0.02	0.10
2008	259	13710	0.02	0.08	18887	0.01	0.07
2009	181	10057	0.02	0.06	16021	0.01	0.06
2010	220	5670	0.04	0.09	9391	0.02	0.10
2011	120	3530	0.03	0.08	4920	0.02	0.10

Table 6. Key parameter values estimated at the joint posterior mode for A_{BH} and the retrospective runs assuming a Beverton Holt stock recruitment relationship (with parameters α^A, β^A). A_{1999}, A_{2003} and A_{2006} assume data available up to 1999, 2003 and 2006 only. Comparisons are also shown to the values at the joint posterior mode from former operating models used to develop OMP-02, OMP-04 and OMP-08, which were developed using operating models assuming a Hockey Stick stock recruitment relationship (with parameters a^A, b^A). Note that the carrying capacity, K^A , is not directly comparable between A_{BH} and the retrospective runs on the one hand, and those from previous assessments on the other, as a bias correction factor was used for the latter. Numbers are reported in billions and biomass in thousands of tons.

	A_{HS}	A_{2006}	A_{2003}	A_{1999}	Previous assessments		
					OMP-02	OMP-04	OMP-08
\bar{M}_j^A	1.2	1.2	1.2	1.2	0.9	0.9	0.9
\bar{M}_{ad}^A	1.2	1.2	1.2	1.2	0.9	0.9	0.9
k_N^A	1.16	1.14	1.13	1.15	0.99	1.22	1.23
k_r^A	0.90	0.79	0.75	0.71	0.84	0.93	1.03
α^A / a^A	1078	1242	3170	446	179	228	213
β^A / b^A	2846	3427	10187	681	360	461	368
K^A	2705	2967	6132	1613	1802	2492	2925
h^A	0.33	0.32	0.29	0.46	1.00	1.00	1.00
σ_r^A	0.68	0.66	0.67	0.62	0.69	0.88	0.86
\bar{B}_{Nov}^A ⁸	1157	1175	1185	1173		1169	1103
s_{cor}^A	0.10	0.07	0.03	-0.01	0.32	0.47	0.43

⁸ See footnote 7.

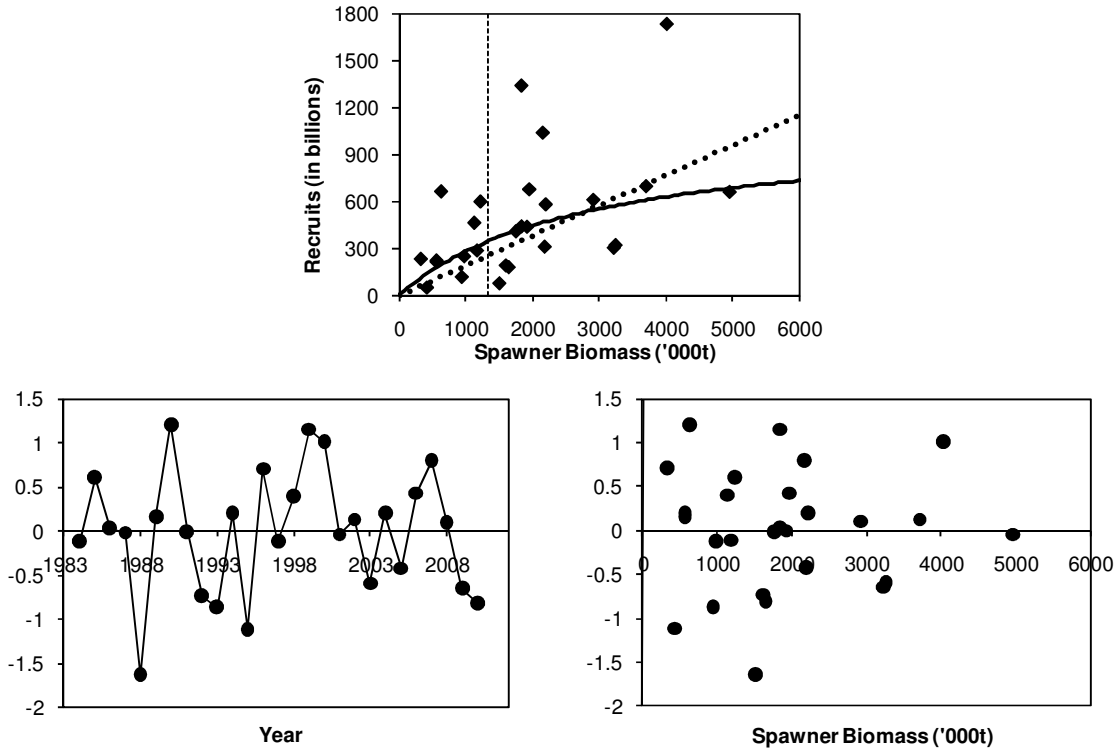


Figure 1. Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to November 2009 for A_{BH} with the Beverton Holt stock recruitment relationship. The vertical thin dashed line indicates the average 1984 to 1999 spawner biomass (used in the definition of risk in OMP-04 and OMP-08). The dotted line indicates the replacement line. The standardised residuals from the fit are given in the lower plots, against year and against spawner biomass.

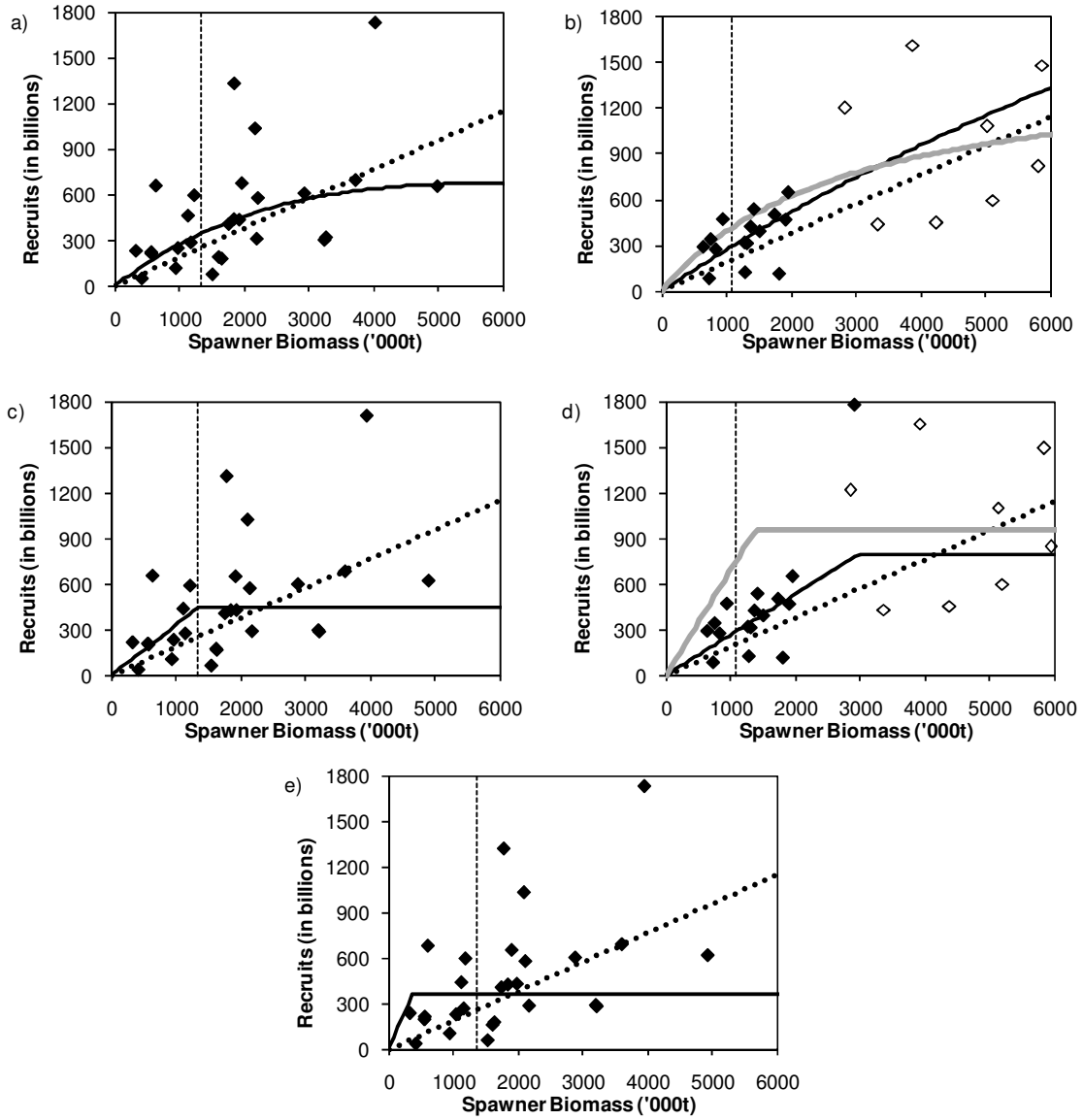


Figure 2. Stock-recruit relationships for a) A_R , b) A_{2BH} (grey being the 2000+ relationship), c) A_{HS} , d) A_{2HS} (grey line showing the 2000+ relationship), and e) $A_{fixedHS}$.

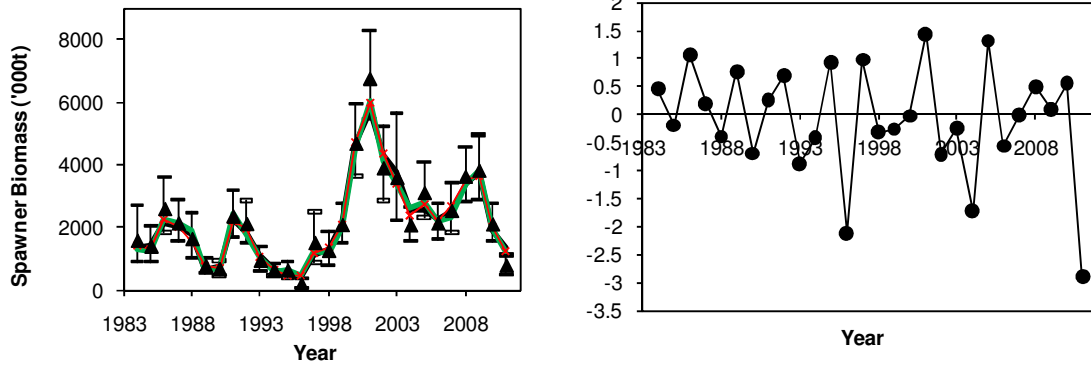


Figure 3. Acoustic survey results and model estimates for November anchovy spawner biomass from 1984 to 2011 for A_{BH} (black), A_{prop} (alternative time series of proportion-at-age 1 data; green) and A_{noprop} (no proportion-at-age 1 data; thin red line with crosses). The survey indices are shown with 95% confidence intervals. The standardised residuals (i.e. the residual divided by the corresponding standard deviation, including additional variance where appropriate, given in equation (A.9)) from the A_{BH} fit are given in the right hand plot.

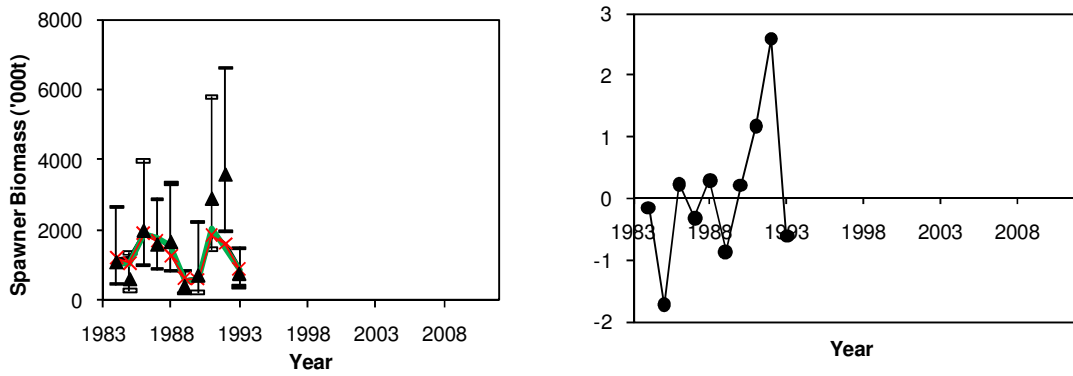


Figure 4. Egg survey results and model estimates for November anchovy spawner biomass from 1984 to 1993 for A_{BH} (black), A_{prop} (alternative time series of proportion-at-age 1 data; green) and A_{noprop} (no proportion-at-age 1 data; thin red line with crosses). The survey indices are shown with 95% confidence intervals. The standardised residuals from the A_{BH} fit are given in the right hand plot.

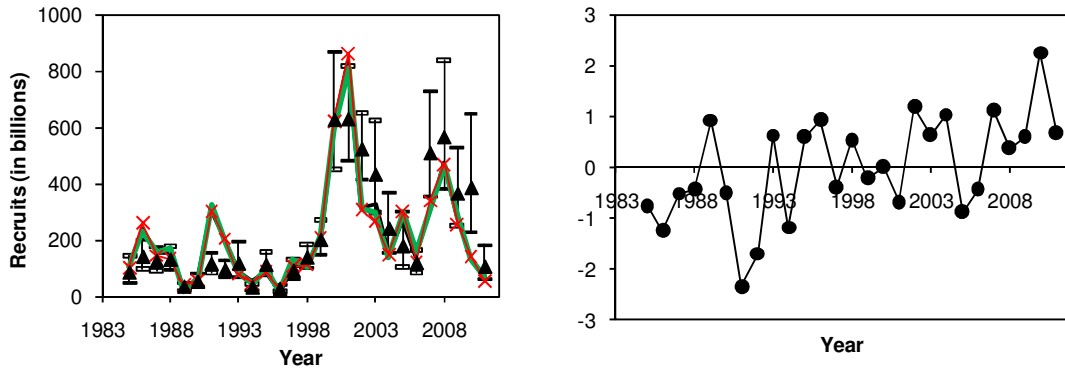


Figure 5. Acoustic survey results and model estimates for anchovy recruitment numbers from May 1985 to May 2010 for A_{BH} (black), A_{prop} (alternative time series of proportion-at-age 1 data; green) and A_{noprop} (no proportion-at-age 1 data; thin red line with crosses). The survey indices are shown with 95% confidence intervals. The standardised residuals from the A_{BH} fit are given in the right hand plot.

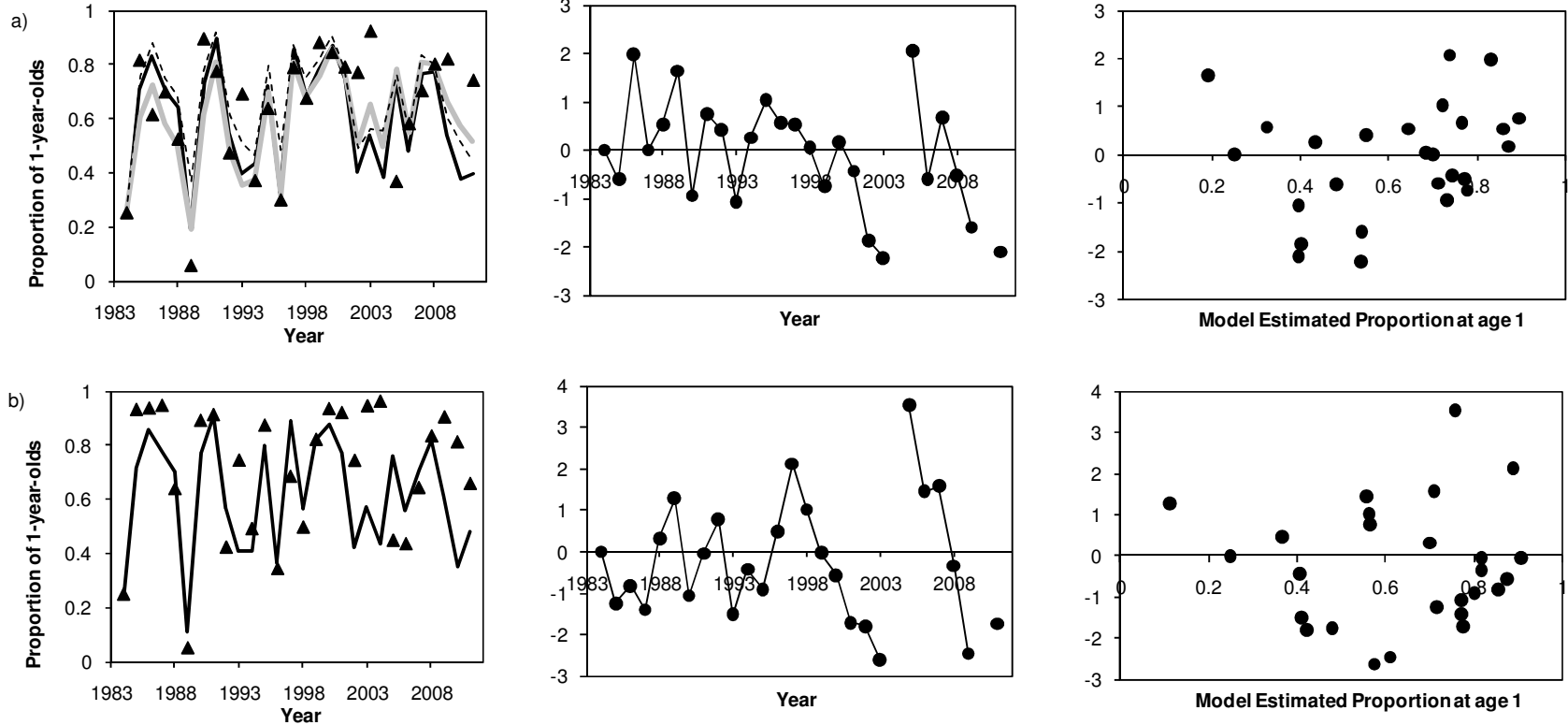


Figure 6. Acoustic survey results and model estimates for proportions of 1-year-olds in the November survey from 1984 to 2010 for a) A_{BH} (black), A_{Neff} (constant average rather than annually varying effective sample sizes for the proportion-at-age 1 inputs; dashed), and A_{Mad} (annually varying adult natural mortality; grey), and b) A_{prop} (alternative time series of proportion-at-age 1 data). The standardised residuals from the A_{BH} and A_{prop} fits are given in the middle and right hand plots, against year and against model estimates of proportions at age 1.

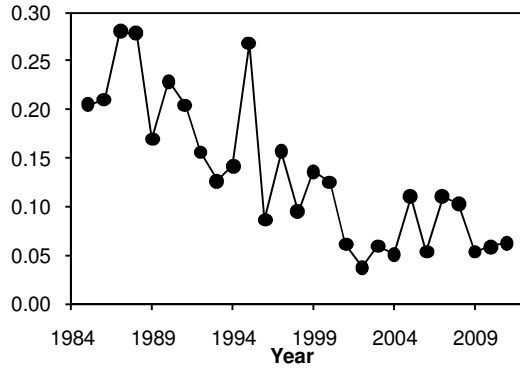


Figure 7. The historic harvest proportion (catch by mass as a proportion of 1+ biomass) for anchovy for A_{BH} .

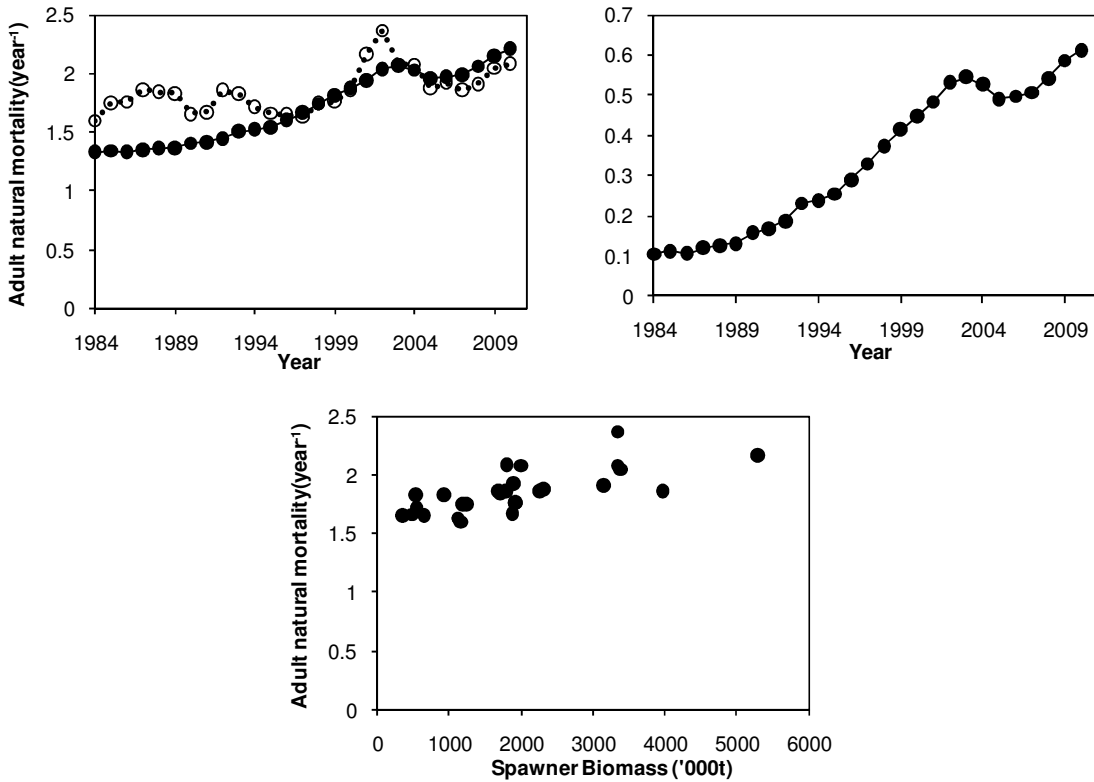


Figure 8. Model estimated annual adult natural mortality for A_{Mad} (annually varying adult natural mortality; solid line and circles) and A_{Mden} (density dependent natural mortality; dotted line with open circles). The random effects for A_{Mad} are plotted in the right hand panel and the natural mortality as a function of spawner biomass is plotted in the lower panel for A_{Mden} .

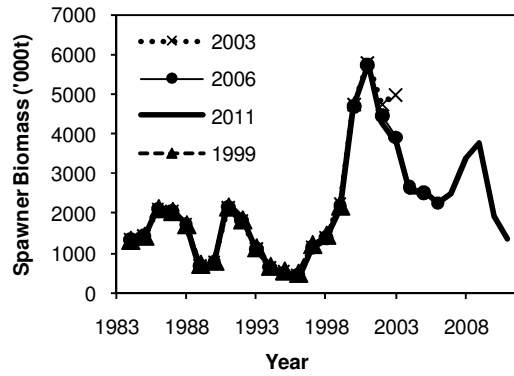


Figure 9. The model predicted November anchovy spawner biomass for A_{BH} and the retrospective runs A_{2006} using data up to 2006, A_{2003} using data up to 2003 and A_{1999} using data up to 1999.

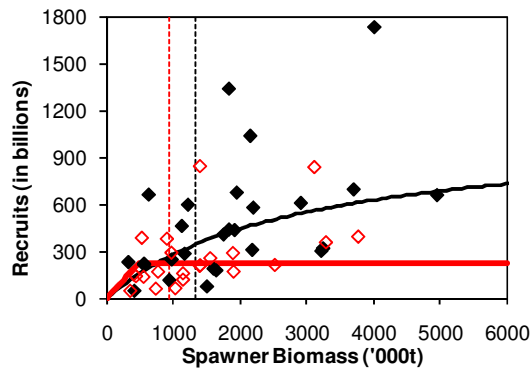


Figure 10. Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to November 2009 for A_{BH} with the Beverton Holt stock recruitment relationship (black with filled diamonds). The recruitment from November 1984 to November 2006 and corresponding Hockey Stick stock recruitment curve estimated by the 2007 assessment are also given (red with open diamonds). The vertical thin dashed line indicates the average 1984 to 1999 spawner biomass for A_{BH} (black) and the 2007 assessment (red).

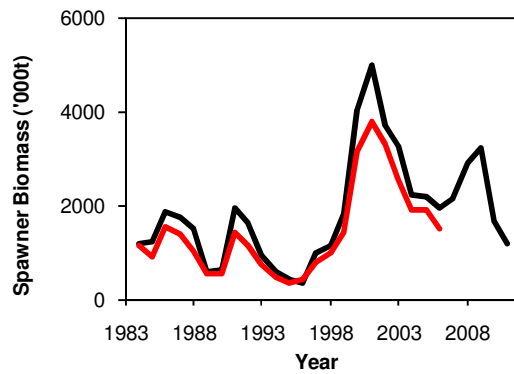


Figure 11. The model predicted spawner biomass (without bias) from A_{BH} (black) and the 2007 assessment (red).

Appendix A: Bayesian operating model for the South African anchovy resource

In the below equations a “ $\hat{}$ ” is used to represent an estimate of a quantity (e.g. biomass) from a source external to this model (e.g. a survey). Model predicted quantities are represented by terms without any additional super-/sub-scripts other than dependencies on, for example, year, length etc.

Model Assumptions

- 1) All fish have a birthdate of 1 November.
- 2) Anchovy spawn for the first time (and are called adult anchovy) when they turn one year old.
- 3) A plus group of age 4 is used, thus assuming that natural mortality is the same for age 4 and older ages.
- 4) Natural mortality is age-invariant for adult fish.
- 5) Two acoustic surveys are held each year: the first takes place in November and surveys the adult stock; the second is in May/June (known as the recruit survey) and surveys juvenile anchovy only.
- 6) The November acoustic survey provides a relative index of abundance of unknown bias.
- 7) The recruit survey provides a relative index of abundance of unknown bias.
- 8) The egg survey observations (derived from data collected during the earlier November surveys) provide estimates of abundance in absolute terms.
- 9) The survey designs have been such that they result in survey estimates of abundance whose bias is invariant over time.
- 10) Pulse fishing occurs five months after 1 November for 1-year-old anchovy; for 0-year-old anchovy this occurs 7½ months after 1 November prior to 1999, and 8½ months after 1 November from 1999 onwards; these two ages (0 and 1) are the only ages targeted by the fishery.
- 11) Catches are measured without error. (Selectivity of age 0 and age 1 anchovy varies from year to year. This would prove problematic were model predicted catch to be estimated and fitted to observed catch, but here the “observed” catches-at-age (inferred as detailed in de Moor *et al.* 2012) are directly incorporated into the dynamics.)

Population Dynamics

The basic dynamic equations for anchovy are as follows, where $y_n = 2011$.

Numbers-at-age at 1 November

$$\begin{aligned}
 N_{y,1}^A &= (N_{y-1,0}^A e^{-(7.5)M_{j,y}^A/12} - C_{y,0}^A) e^{-(4.5)M_{j,y}^A/12} & y = 1984, \dots, 1998 \\
 N_{y,1}^A &= (N_{y-1,0}^A e^{-(8.5)M_{j,y}^A/12} - C_{y,0}^A) e^{-(3.5)M_{j,y}^A/12} & y = 1999, \dots, y_n \\
 N_{y,2}^A &= (N_{y-1,1}^A e^{-5M_{ad,y}^A/12} - C_{y,1}^A) e^{-7M_{ad,y}^A/12} & y = 1984, \dots, y_n \\
 N_{y,3}^A &= N_{y-1,2}^A e^{-M_{ad,y}^A} & y = 1984, \dots, y_n \\
 N_{y,4+}^A &= N_{y-1,3}^A e^{-M_{ad,y}^A} + N_{y-1,4+}^A e^{-M_{ad,y}^A} & y = 1984, \dots, y_n
 \end{aligned} \tag{A.1}$$

where

$N_{y,a}^A$ is the model predicted number (in billions) of anchovy of age a at the beginning of November in year y ;

$C_{y,a}^A$ is the number (in billions) of anchovy of age a caught from 1 November in year $y - 1$ to 31 October in year y (de Moor *et al.* 2012);

$M_{j,y}^A$ is the annual natural mortality (in year⁻¹) of juvenile anchovy (i.e. fish of age 0) in year y ; and

$M_{ad,y}^A$ is the annual natural mortality (in year⁻¹) of adult anchovy (i.e. fish of age 1+) in year y .

Natural mortality

Natural mortality is modelled to vary annually using a random effects model:

$$M_{j,y}^A = \bar{M}_j^A e^{\varepsilon_{j,y}} \text{ with } \varepsilon_{1984}^j = \eta_{1984}^j \text{ and } \varepsilon_y^j = \rho \varepsilon_{y-1}^j + \sqrt{1 - \rho^2} \eta_y^j, \quad y > 1984 \quad (\text{A.2})$$

$$M_{ad,y}^A = \bar{M}_{ad}^A e^{\varepsilon_{ad,y}} \text{ with } \varepsilon_{1984}^{ad} = \eta_{1984}^{ad} \text{ and } \varepsilon_y^{ad} = \rho \varepsilon_{y-1}^{ad} + \sqrt{1 - \rho^2} \eta_y^{ad}, \quad y > 1984 \quad (\text{A.3})$$

where $\eta_y^j \sim N(0, \sigma_j^2)$, $\eta_y^{ad} \sim N(0, \sigma_{ad}^2)$ and

σ_j - is the standard deviation in the annual residuals about juvenile natural mortality;

σ_{ad} - is the standard deviation in the annual residuals about adult natural mortality; and

ρ - is the annual autocorrelation coefficient.

Biomass associated with the November survey

$$B_y^A = \sum_{a=1}^{4+} N_{y,a}^A w_{y,a}^A \quad y = 1984, \dots, y_n \quad (\text{A.4})$$

where:

B_y^A is the model predicted biomass (in thousand tons) of adult anchovy at the beginning of November in year y , which are taken to be associated with the November survey; and

$w_{y,a}^A$ is the mean mass (in grams) of anchovy of age a sampled during the November survey of year y .

Anchovy are assumed to mature at age 1 and thus the spawning stock biomass is:

$$SSB_y^A = \sum_{a=1}^{4+} N_{y,a}^A w_{y,a}^A \quad y = 1984, \dots, y_n \quad (\text{A.5})$$

As only $w_{y,1}^A$ and $w_{y,2+}^A$ are available (de Moor *et al.* 2012), $w_{y,a}^A$ for ages 2, 3 and 4+ are calculated as follows:

$$w_{y,2}^A = w_{y,2+}^A \frac{\sum_{a=2}^{4+} N_{y,a}^A}{N_{y,2}^A + \bar{w}_3 / \bar{w}_2 N_{y,3}^A + \bar{w}_4 / \bar{w}_2 N_{y,4+}^A},$$

$$w_{y,3}^A = w_{y,2}^A \times \bar{w}_3 / \bar{w}_2$$

$$w_{y,4+}^A = w_{y,2}^A \times \bar{w}_4 / \bar{w}_2$$

where

\bar{w}_a / \bar{w}_2 is the ratio of the mean mass (in grams) of anchovy of age a to age 2, sampled during the November survey of year y , i.e. these ratios are assumed to be year-independent, and are calculated as detailed in the Fixed Parameters section below.

Recruitment

Recruitment at the beginning of November is assumed to fluctuate lognormally about a stock-recruitment curve (see Table 1):

$$N_{y,0}^A = f(SSB_y^A) e^{\varepsilon_y^A} \quad y = 1984, \dots, y_{n-1} \quad (\text{A.6})$$

where

ε_y^A is the annual lognormal deviation of anchovy recruitment.

Number of recruits at the time of the recruit survey

The following equation projects $N_{y,0}^A$ to the start of the recruit survey, taking natural and fishing mortality into account, and assuming pulse fishing of juveniles at 1 May (based on historic data).

$$N_{y,r}^A = (N_{y-1,0}^A e^{-0.5M_j^A} - C_{y,obs}^A) e^{-t_y^A \times M_j^A / 12} \quad y = 1985, \dots, y_n \quad (\text{A.7})$$

where

$N_{y,r}^A$ is the model predicted number (in billions) of juvenile anchovy at the time of the recruit survey in year y ;

$C_{y,obs}^A$ is the number (in billions) of juvenile anchovy caught between 1 November and the day before the start of the recruit survey in year y (de Moor *et al.* 2012);

t_y^A is the time lapsed (in months) between 1 May and the start of the recruit survey that provided the estimate $N_{y,rec}^A$ in year y (de Moor *et al.* 2012).

Proportions of 1-year-olds associated with November survey

$$p_{y,1}^A = \frac{k_p^A N_{y,1}^A}{k_p^A N_{y,1}^A + \sum_{a=2}^{4+} N_{y,a}^A} \quad y = 1984, \dots, y_n \quad (\text{A.8})$$

where

$p_{y,1}^A$ is the model predicted proportion of 1-year-old anchovy at the beginning of November in year y , which pertains to the November survey; and

k_p^A is a multiplicative bias associated with the proportion of 1-year-olds in the November survey.

Fitting the Model to Observed Data (Likelihood)

The survey observations of abundance are assumed to be log-normally distributed, and sampling CVs (squared) of the untransformed survey observations are used to approximate the “sampling” component of the total variance of the corresponding log-distributions. The proportions of 1-year-olds are assumed to be multinomially distributed. Thus we have:

$$\begin{aligned}
 -\ln L = & \frac{1}{2} \sum_{y=1984}^{yn} \left\{ \frac{(\ln \hat{B}_y^A - \ln(k_N^A B_y^A))^2}{(\sigma_{y,N}^A)^2 + (\lambda_N^A)^2} + \ln[2\pi((\sigma_{y,N}^A)^2 + (\lambda_N^A)^2)] \right\} \\
 & + \frac{1}{2} \sum_{y=1984}^{1991} \left\{ \frac{(\ln \hat{B}_{y,egg}^A - \ln(k_g^A B_y^A))^2}{(\sigma_{y,egg}^A)^2} + \ln[2\pi(\sigma_{y,egg}^A)^2] \right\} \\
 & + \frac{1}{2} \sum_{y=1985}^{yn} \left\{ \frac{(\ln \hat{N}_{y,r}^A - \ln(k_r^A N_{y,r}^A))^2}{(\sigma_{y,r}^A)^2 + (\lambda_r^A)^2} + \ln[2\pi((\sigma_{y,r}^A)^2 + (\lambda_r^A)^2)] \right\} \\
 & + N_y^{eff} \sum_{y=1984}^{yn} \left\{ -\hat{p}_{y,1}^A \ln(p_{y,1}^A) - (1 - \hat{p}_{y,1}^A) \ln(1 - p_{y,1}^A) \right\}
 \end{aligned} \tag{A.9}$$

where

\hat{B}_y^A is the acoustic survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year y (de Moor *et al.* 2012), with associated CV $\sigma_{y,N}^A$ and constant of proportionality (multiplicative bias) k_N^A ;

$\hat{B}_{y,egg}^A$ is the egg survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year y (de Moor *et al.* 2012), with associated CV $\sigma_{y,egg}^A$ and constant of proportionality k_g^A ;

$\hat{N}_{y,r}^A$ is the acoustic survey estimate (in billions) of anchovy recruitment from the recruit survey in year y (de Moor *et al.* 2012), with associated CV $\sigma_{y,r}^A$ and constant of proportionality k_r^A ;

$\hat{p}_{y,1}^A$ is an estimate of the proportion (by number) of 1-year-old anchovy in the November survey of year y (de Moor *et al.* 2012, de Moor and Butterworth 2012a);

$(\lambda_{N/r}^A)^2$ is the additional variance (over and above the survey sampling CV $\sigma_{y,N/r}^A$ that reflects survey inter-transect variance) associated with the November/recruit surveys;

$N_y^{eff} = \frac{\hat{p}_{y,1}^A (1 - \hat{p}_{y,1}^A)}{(\hat{\sigma}_{y,p}^A)^2}$ is the effective sample size for the assumed binomially distributed proportion of 1-year-old anchovy estimated by de Moor and Butterworth (2012a), taken as input to this model⁹;

⁹ For ease of comparison between models, N_y^{eff} is taken to be fixed at that corresponding to the baseline data (i.e. that from the model which allowed for annual variation in the variance about mean length at age 1 (de Moor and Butterworth, 2012)

$\hat{\sigma}_{y,p}^A$ is the standard deviation about the proportion of 1-year-olds in the November survey of year y , as estimated by de Moor and Butterworth (2012a).

Fixed Parameters

Two parameters are fixed externally in this assessment (see main text for variations for robustness tests); $(\lambda_N^A)^2 = 0$, and $k_g^A = 1$, as the egg survey estimates of abundance are assumed to be absolute. In the base case assessment, natural mortality is assumed to be time-invariant, thus $\sigma_j = \sigma_{ad} = 0$, giving $\epsilon_y^j = \epsilon_y^{ad} = 0$.

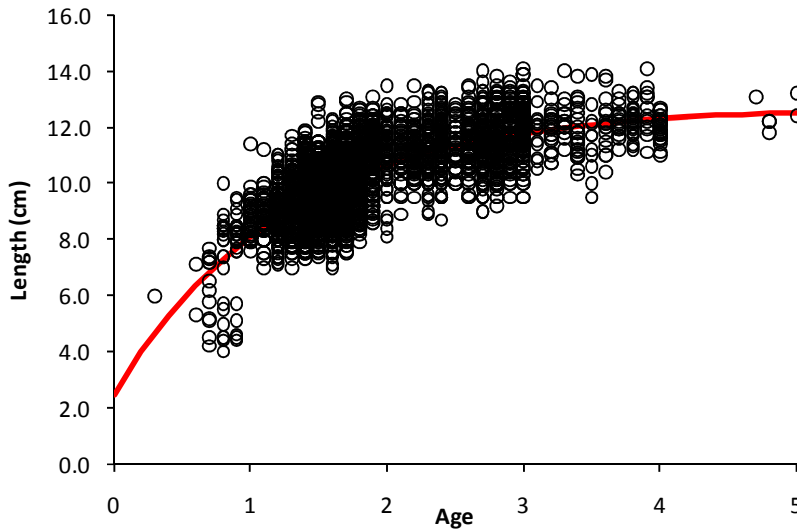


Figure A.1. A von Bertalanffy growth curve fitted to ageing data from Kerstan from November surveys in 1990, 1992 – 1995 (Deon Durholtz pers comm.)

The ratio of the weight-at-age 3 to weight-at-age 2 and weight-at-age 4 to weight-at-age 2 was calculated from the von Bertalanffy growth curve fitted to ageing data from Kerstan¹⁰ from the November surveys in 1990, 1992-1995 (Figure A.1), and using the anchovy length-weight relationship $mass = 0.00750 \times L_c^{3.110}$, where mass is in grams and length in centimetres (Lynne Shannon pers. comm. using 1990-1996 data):

$$w_3/w_2 = 1.37$$

$$w_4/w_2 = 1.57 .$$

The equilibrium assumptions:

$$N_{1983,a}^A \sim N_{1983,a-1}^A e^{-M_{ad,1983}^A}, \quad a = 2,3$$

$$N_{1983,4+}^A \sim N_{1983,3}^A \frac{e^{-M_{ad,1983}^A}}{1 - e^{-M_{ad,1983}^A}}$$

¹⁰ Although there are concerns that Kerstan’s ageing was biased (de Moor and Butterworth, 2012), the impact of such bias on these ratios should be rather less.

are used, given the absence of data that would allow the estimation of any variation from this. It is assumed

that $M_{ad,1983}^A = M_{ad,1984}^A$.

Estimable Parameters and Prior Distributions

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve:

$$\varepsilon_y^A \sim N\left(0, (\sigma_r^A)^2\right), \quad y = 1984, \dots, y_{n-1}$$

The remaining estimable parameters are defined as having the near non-informative prior distributions:

$$\ln(k_N^A) \sim U(-100, 0.7) \text{ (upper bound corresponding to } k_N^A \sim 2)$$

$$\ln(k_r^A) \sim U(-100, 0.7) \text{ (upper bound corresponding to } k_r^A \sim 2)$$

$$\ln(k_p^A) \sim U(-100, 0.7) \text{ (upper bound corresponding to } k_p^A \sim 2)$$

$$(\lambda_r^A)^2 \sim U(0, 100)$$

$$(\sigma_r^A)^2 \sim U(0.04, 10)$$

$$N_{1983,a}^A \sim U(0, 500), \quad a = 0, 1$$

Further Outputs

Recruitment serial correlation:

$$s_{cor}^A = \frac{\sum_{y=1984}^{yn-2} \varepsilon_y \varepsilon_{y+1}}{\sqrt{\left(\sum_{y=1984}^{yn-2} \varepsilon_y^2\right) \left(\sum_{y=1984}^{yn-2} \varepsilon_{y+1}^2\right)}} \tag{A.10}$$

and the standardised recruitment residual value for 2010:

$$\eta_{yn-1}^A = \frac{\varepsilon_{yn-1}^A}{\sigma_{r,2000+}^A} \tag{A.11}$$

where $\sigma_{r,2000+}^A$ denotes the standard deviation in the residuals about the stock recruitment curve corresponding to the years 2000-2011 (which for some sensitivity tests is different to that estimated pre-2000).

Appendix B: Glossary of parameters used in this document

Annual numbers and biomass:

- $N_{y,a}^A$ - model predicted number (in billions) of anchovy of age a at the beginning of November in year y
- $C_{y,a}^A$ - model predicted number (in billions) of anchovy of age a caught from 1 November in year $y-1$ to 31 October in year y
- $B_{y,N}^A$ - model predicted biomass (in thousand tons) of adult anchovy at the beginning of November in year y , which are taken to be associated with the November survey
- SSB_y^A - model predicted spawning stock biomass (in thousand tonnes) at the beginning of November in year y
- $w_{y,a}^A$ - mean mass (in grams) of anchovy of age a sampled during the November survey of year y
- $N_{y,r}^A$ - model predicted number (in billions) of juvenile anchovy at the time of the recruit survey in year y
- $C_{y,Obs}^A$ - number (in billions) of juvenile anchovy caught between 1 November and the day before the start of the recruit survey in year y
- t_y^A - time lapsed (in months) between 1 May and the start of the recruit survey in year y .

Natural mortality:

- $M_{j,y}^A$ - annual natural mortality (in year⁻¹) of juvenile anchovy (i.e. fish of age 0) in year y
- $M_{ad,y}^A$ - annual natural mortality (in year⁻¹) of adult anchovy (i.e. fish of age 1+) in year y
- \overline{M}_{ad}^A - median adult rate of natural mortality (in year⁻¹)
- ε_y^{ad} - annual residuals about adult natural mortality
- η_y^{ad} - normally distributed error used in calculating ε_y^{ad}
- σ_{ad} - standard deviation in the annual residuals about adult natural mortality
- ρ - annual autocorrelation coefficient in annual residuals about adult natural mortality

Recruitment:

- h^A - steepness associated with the stock-recruitment curve
- K^A - carrying capacity
- a^A - maximum median recruitment in the Hockey Stick stock-recruitment curve
- b^A - biomass above which median recruitment is not impaired in the Hockey Stick stock-recruitment curve
- α^A - stock-recruitment curve parameter, linked to K^A and h^A (for Beverton Holt and Ricker curves)
- β^A - stock-recruitment curve parameter, linked to K^A and h^A (for Beverton Holt and Ricker curves)
- ε_y^A - annual lognormal deviation of anchovy recruitment
- σ_r^A - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve

Proportions of 1-year-olds:

$p_{y,1}^A$ - model predicted proportion of 1-year-old anchovy at the beginning of November in year y

k_p^A - multiplicative bias associated with the proportion of 1-year-olds in the November survey

Likelihoods:

\hat{B}_y^A - acoustic survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year y

$\sigma_{y,N}^A$ - survey sampling CV associated with \hat{B}_y^A that reflects survey inter-transect variance

k_N^A - constant of proportionality (multiplicative bias) associated with \hat{B}_y^A

$\hat{B}_{y,egg}^A$ - egg survey estimate (in thousand tons) of adult anchovy biomass from the November survey in year y

$\sigma_{y,egg}^A$ - survey sampling CV associated with $\hat{B}_{y,egg}^A$ that reflects survey inter-transect variance

k_g^A - constant of proportionality (multiplicative bias) associated with $\hat{B}_{y,egg}^A$

$\hat{N}_{y,r}^A$ - acoustic survey estimate (in billions) of anchovy recruitment from the recruit survey in year y

$\sigma_{y,r}^A$ - survey sampling CV associated with $\hat{N}_{y,r}^A$ that reflects survey inter-transect variance

k_r^A - constant of proportionality (multiplicative bias) associated with $\hat{N}_{y,r}^A$

$\hat{p}_{y,1}^A$ - estimate of the proportion (by number) of 1-year-old anchovy in the November survey of year y

$(\lambda_{N/r}^A)^2$ - additional variance (over and above $\sigma_{y,N/r}^A$) associated with the November/recruit surveys

N^{eff} is the effective sample size for the assumed binomially distributed proportion of 1-year-old anchovy, taken as input to this model;

Other:

s_{cor}^A - recruitment serial correlation

η_{2009}^A - standardised recruitment residual value for 2009

\bar{w}_a^A - mean mass (in grams) of anchovy of age a during each November survey

Appendix C: Calculation of Annual Total Proportion Fished and Loss to Predation of Anchovy

The assessment model assumes catch is taken in a single pulse during the year. The loss in numbers of age a in year y is calculated by:

$$P_{y,1}^A = N_{y-1,0}^A \left(1 - e^{-7.5M_{j,y}^A/12}\right) + \left(N_{y-1,0}^A e^{-7.5M_{j,y}^A/12} - C_{y,0}^A\right) \left(1 - e^{-4.5M_{j,y}^A/12}\right) \quad y = 1984, \dots, 1998$$

$$P_{y,1}^A = N_{y-1,0}^A \left(1 - e^{-8.5M_{j,y}^A/12}\right) + \left(N_{y-1,0}^A e^{-8.5M_{j,y}^A/12} - C_{y,0}^A\right) \left(1 - e^{-3.5M_{j,y}^A/12}\right) \quad y = 1999, \dots, y_n$$

$$P_{y,2}^A = N_{y-1,1}^A \left(1 - e^{-5M_{ad,y}^A/12}\right) + \left(N_{y-1,1}^A e^{-5M_{ad,y}^A/12} - C_{y,1}^A\right) \left(1 - e^{-7M_{ad,y}^A/12}\right) \quad y = 1984, \dots, y_n$$

$$P_{y,3}^A = N_{y-1,2}^A \left(1 - e^{-M_{ad,y}^A}\right), \quad y = 1984, \dots, y_n$$

$$P_{y,4+}^A = N_{y-1,3}^A \left(1 - e^{-M_{ad,y}^A}\right) + N_{y-1,4+}^A \left(1 - e^{-M_{ad,y}^A}\right) \quad y = 1985, \dots, y_n$$

The loss in biomass of fish of age a to predation in year y is therefore given by:

$$P_{y,1}^A = \left[N_{y-1,0}^A \left(1 - e^{-7.5M_{j,y}^A/12}\right) + \left(N_{y-1,0}^A e^{-7.5M_{j,y}^A/12} - C_{y,0}^A\right) \left(1 - e^{-4.5M_{j,y}^A/12}\right) \right] \frac{1}{2} (w_{y-1,0} + w_{y,1}) \quad y = 1984, \dots, 1998$$

$$P_{y,1}^A = \left[N_{y-1,0}^A \left(1 - e^{-8.5M_{j,y}^A/12}\right) + \left(N_{y-1,0}^A e^{-8.5M_{j,y}^A/12} - C_{y,0}^A\right) \left(1 - e^{-3.5M_{j,y}^A/12}\right) \right] \frac{1}{2} (w_{y-1,0} + w_{y,1})$$

$$P_{y,2}^A = \left[N_{y-1,1}^A \left(1 - e^{-5M_{ad,y}^A/12}\right) + \left(N_{y-1,1}^A e^{-5M_{ad,y}^A/12} - C_{y,1}^A\right) \left(1 - e^{-7M_{ad,y}^A/12}\right) \right] \frac{1}{2} (w_{y-1,1} + w_{y,2}) \quad y = 1984, \dots, y_n$$

$$P_{y,3}^A = N_{y-1,2}^A \left(1 - e^{-M_{ad,y}^A}\right) \frac{1}{2} (w_{y-1,2} + w_{y,3}), \quad y = 1984, \dots, y_n$$

$$P_{y,4+}^A = N_{y-1,3}^A \left(1 - e^{-M_{ad,y}^A}\right) \frac{1}{2} (w_{y-1,3} + w_{y,4}) + N_{y-1,4+}^A \left(1 - e^{-M_{ad,y}^A}\right) \frac{1}{2} (w_{y-1,4} + w_{y,4}) \quad y = 1985, \dots, y_n$$

The assumption is made that $w_{1983,a} = w_{1984,a}$, $a = 1, \dots, 4 +$.

The total loss in anchovy biomass to predation in year y is then given by:

$$P_y^A = \sum_{a=1}^{4+} P_{y,a}^A$$

The anchovy biomass at the time of pulse fishing is given by:

$$B_0^{Available} = N_{y-1,0}^A e^{-7.5M_{j,y}^A/12} w_{y,0}^{catch} \quad y = 1984, \dots, 1998$$

$$B_0^{Available} = N_{y-1,0}^A e^{-8.5M_{j,y}^A/12} w_{y,0}^{catch} \quad y = 1999, \dots, y_n$$

$$B_1^{Available} = N_{y-1,1}^A e^{-5M_{ad,y}^A/12} w_{y,1}^{catch}$$

$$B_2^{Available} = N_{y-1,2}^A e^{-M_{ad,y}^A/2} \frac{w_{y-1,2}^A + w_{y,3}^A}{2}$$

$$B_3^{Available} = N_{y-1,3}^A e^{-M_{ad,y}^A / 2} \frac{w_{y-1,3}^A + w_{y,4}^A}{2}$$

$$B_{4+}^{Available} = N_{y-1,4+}^A e^{-M_{ad,y}^A / 2} \frac{w_{y-1,4}^A + w_{y,4}^A}{2}$$

The annual total proportion fished (catch/biomass) mortality is thus given by:

$$F_y^A = \frac{C_{y,0}^A w_{y,0}^{catch} + C_{y,1}^A w_{y,1}^{catch}}{\sum_{a=0}^{4+} B_a^{Available}}.$$

where

$w_{y,a}^{catch}$ - mean mass (in grams) in the catch of anchovy of age a in year y (from de Moor *et al.* 2012).