Assessment of the South African sardine resource using data from 1984-2011, with some results for a single stock hypothesis

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Introduction

Although the base case operating model for the South African sardine resource was updated from the last assessment (Cunningham and Butterworth 2007) to take account of new data collected between 2007 and 2010 (de Moor and Butterworth 2011a), the International Review Panel for the 2011 International Fisheries Stock Assessment Workshop suggested some revisions to this model (Smith *et al*. 2011) before it is used in the development of a new MP.

de Moor and Butterworth (2011a) proposed two base case operating models; one which estimated random effects about adult natural mortality over time while the other assumed constant (time-invariant) adult natural mortality. The inclusion of the random effects was in response to similar changes made to the base case operating models for anchovy (de Moor and Butterworth 2011b). Smith *et al*. (2011) suggested rather that a base case with constant natural mortality be used.

A number of key changes to the model and data used have been made.

- The inclusion of one more year's survey data from November 2011 to that used by de Moor and Butterworth (2011a).
- The time series of commercial catch data has been revised and updated form that used in 2007. Commercial catches from the primarily juvenile anchovy bycatch fishery have been considered separately to those from the primarily adult directed and redeye bycatch fisheries. In the former fishery catch is split between age 0 and 1 using cut-off lengths which vary by month and year (de Moor *et al*. 2012a). The model is fit directly to the raised length frequency of the quarterly catch from the directed and redeye bycatch fisheries, with the exclusion of the minus length class (see Appendix A for details).
- Both commercial and survey selectivity is estimated by length and no longer by age.
- The manner in which bias on the November 1+ biomass and May recruit surveys is modelled has been updated. Previously a single parameter was estimated for each survey and an informative prior distribution was given for both parameters. Bias is now estimated separately first for the hydroacoustic survey, using the same prior as had formally been developed for the November survey. A second bias parameter is estimated for the proportion of the stock abundance covered during the recruit survey relative to the November survey. Finally, for the two-stock hypothesis, the proportion of the recruit abundance covered for the "south" stock in comparison to the "west" stock is also estimated. It is assumed that full coverage of the sardine abundance is obtained during the November survey.

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- The method used to calculate weight-at-age corresponding to the November survey has been changed as age-length keys are no longer used. The new method involves assuming a time-invariant ratio of weight at ages 2, 3, 4 and 5+ to age 1, and uses the time series of average weights-at-age in the November survey (de Moor *et al*. 2012).

Both a single stock and two mixing-stock ("west" and "south") hypotheses have been proposed for consideration as operating models in simulation testing OMP-13. This document presents the updated base case operating models for a single sardine stock hypothesis, assuming a Hockey Stick stock recruitment relationship to apply. The model detailed in the Appendices is generalised for both the single and two mixing-stock hypotheses. Results for the single stock base case as well as some robustness tests are given in this document at the posterior mode only. A separate document will show the full posterior distributions. Initial results for the two mixing-stock hypothesis are given in de Moor and Butterworth (2012).

Population Dynamics Model

The generalised operating model for the South African sardine resource, which can apply to either the single or two mixing-stock hypotheses, is detailed in Appendix A. Informative prior distributions were constructed for the multiplicative bias in the hydroacoustic survey as well as the additional inter-annual variance associated with these surveys (see Appendix B). The fixed parameters and prior distributions for the growth curve parameters were informed by the von Bertalanffy growth curve estimated by available ageing data (Appendix D; Durholtz and Mtengwane pers. commn). The priors for the remaining estimable parameters were chosen to be relatively uninformative (see Appendices A and B for details), although the bounds on the survey selectivities and upper bound on commercial selectivities were chosen to constrain some parameter estimates. The data used in this assessment are listed in de Moor *et al*. (2012a). A glossary of terms used in this model is provided in Appendix C.

Multiplicative bias associated with the November survey is taken to be that associated with the hydroacoustic survey. The assumption is made that full coverage of the sardine abundance is obtained during the November survey. Multiplicative bias associated with the May recruit survey is taken to be that associated with the hydroacoustic survey multiplied by that associated with the proportion of the recruit abundance covered by the recruit survey compared to the November survey. Given that not all of the recruitment is assumed to be available to the survey by mid-May, this latter ratio is constrained by a maximum of 1. In the two-stock hypothesis, the multiplicative bias associated with the May recruit survey of the "south" stock is taken to be that associated with the May recruit survey of the "west" stock (as described in the preceding sentences) multiplied by the ratio of the proportion of the "south" stock recruit abundance covered by the recruit survey to the proportion of the "west" stock recruit abundance covered in the same survey. Further details are provided in Appendices A and B.

Stock recruitment relationship

The following alternative stock recruitment relationships have been considered (Table 1):

S_{HS} – hockey stick stock-recruitment curve, with uniform priors on the log of the maximum

recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity

S_{2HS} – hockey stick stock-recruitment curve over all years except 2000-2004, with uniform priors on the log of the maximum recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity; constant recruitment over the peak years of 2000-2004, with a uniform prior on the log of this constant.

S_{BH} – Beverton Holt stock-recruitment curve, with uniform priors on steepness and carrying capacity

S_R – Ricker stock-recruitment curve, with uniform priors on steepness and carrying capacity

In all of the alternatives above the standard deviations about the curve are estimated assuming a difference between peak (2000-2004) and non-peak years.

Natural mortality

A number of combinations of juvenile and median adult natural mortality values are tested, covering the range 0.6 to 1.2 year^{-1}, for the case where a Hockey Stick stock recruitment relationship is assumed. For realism, only combinations with $\overline{M}_j^S \geq \overline{M}_{ad}^S$ are tested.

Results

Natural mortality

Table 2 lists the various contributions to the objective function at the posterior mode for the full range of combinations of juvenile and adult natural mortality tested. Given the choice of prior distributions, the ratio k_r^S / k_N^S is by definition less than 1. Combinations of natural mortality which result in $k_r^S / k_N^S < 0.5$ are considered less plausible.

There is little change in the posterior distribution as $\overline{M}_j^S$ is changed for a given $\overline{M}_{ad}^S$ (<1.3 likelihood points, improving as $\overline{M}_j^S$ increases). Given $\overline{M}_j^S$, the posterior distribution indicated a worse fit to the data for both increasing and decreasing $\overline{M}_{ad}^S$ away from 0.8. The lowest values for the negative log posterior mode were obtained for $\overline{M}_{ad}^S = 0.8$, with $\overline{M}_j^S = 0.8$ and $\overline{M}_j^S = 1.0$. To maintain consistency with previous assessments, the base case hypothesis currently assumes $\overline{M}_j^S = 1.0$ and $\overline{M}_{ad}^S = 0.8$.

Stock recruitment relationship

Table 3 lists the various contributions to the negative log posterior pdf at the posterior mode for the alternative stock-recruitment relationships considered. AIC$_c$ is used to approximately compare amongst alternative stock-recruitment relationships, and suggests that the preferred stock-recruitment relationship is Ricker, which is marginally preferred over the Hockey Stick and Beverton Holt stock-recruitment relationships. The alternative assuming a different, constant recruitment during peak years, A$_{2HS}$, was not preferred as a result of the additional estimable parameter required. As there is little difference in the AIC$_c$ values between the alternatives, the base case hypothesis assumes a hockey stick stock-recruitment relationship to maintain consistency with previous assessments. The alternative stock

recruitment relationships are plotted in Figures 1 and 2. A much higher standard deviation about the curve is estimated for "peak" (2000-2004) years compared to non-peak years (Table 4).

Base case (S_{HS}) results at posterior mode

The estimated parameter values and other key outputs are listed in Table 4 together with the individual contributions to the negative log posterior probability density function (pdf) at the posterior mode.

The population model fits to the time series of abundance estimates of November 1+ biomass and May recruitment are shown in Figures 3 and 4, respectively. In both cases the fits to the survey data are reasonably good. The model does not predict as high a peak in 1+ biomass as is shown by the point estimates from the survey results, though the predicted 1+ biomass is within the 95% CI for the biomass estimated by the survey. The model also under-predicts recruitment in May 2010 as it is unable to reconcile the conflicting data of an above average recruitment estimate in May 2010, with almost no increase in the November 1+ biomass estimate from 2009 to 2010. One change in this assessment compared to those of former years is the lower value estimated for the bias associated with the May recruitment, $k_{r,1}^{S} = 0.39$, implying that the May recruit survey estimates correspond, on average, to only 39% of the true biomass. This results from the model estimating that the May recruit survey covers only 54% of the recruits compared to 100% coverage of the 1+ biomass in the November survey.

Figure 5 shows the model estimated survey selectivity at length, which is restricted to vary from 1 only for lengths contributing to the minus and plus length classes (see Appendix A for details). The residuals from the model fit to the survey proportions-at-length are given in Figure 6. The average (over all years) model predicted November survey proportion-at-length is relatively good, especially considering the restriction of survey selectivity to be 1 for all length classes other than the minus and plus length class (Figure 7).

Figure 8 shows the model estimated commercial selectivity at length which is constrained by two curves to mimic the pattern evident in the data (see Appendix A for details). Some trends in the residuals from the model fit to the commercial proportion-at-length are evident (Figure 9), but given the assumption of constant selectivity over time, these are considered to be acceptable. The average (over all years and quarters) model predicted commercial proportion-at-length matches the general pattern of that observed, although the peak about the lower lengths is over-predicted while the peak about the higher lengths is under-predicted (Figure 10).

A key factor in the model fits to the proportion-at-length data is the model estimated growth curve (Figure 11) and variability about this curve (Figure 12).

Summary

This document has detailed the updated operating model for the South African sardine resource. As a base case single stock hypothesis it is proposed that a hockey stick stock recruitment relationship be assumed and that $\overline{M}_{j}^{S} = 1.0$ and

$\overline{M}_{ad}^{S} = 0.8$. Consistency between the single and two stock hypotheses in terms of natural mortality assumptions is desirable, and thus these proposed values of $\overline{M}_{j}^{S} = 1.0$ and $\overline{M}_{ad}^{S} = 0.8$ are still to be tested for the two stock hypothesis (de Moor and Butterworth 2012). Under the current base case hypotheses, the resource abundance is predicted to be 1.4 million tons in November 2011. This is just above the long-term average of 1.2 million tons, and more than double the 1991-1994 average of 0.63 million tons. However, despite the recent increase in abundance, the resource has suffered below average recruitment in seven of the last eight years.

Further work will include further robustness tests will need to be considered as well as retrospective runs for the base case hypothesis.

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Table 1. The alternative stock-recruitment relationships considered. The parameter h_j^S denotes the "steepness" of the stock-recruitment relationship for stock j, which is the proportion of the virgin recruitment that is realised at a spawning biomass level of 20% of average pre-exploitation (virgin) spawning biomass K_j^S (shown in units of thousands of tons). For the hockey stick model, $X_j = \sum_{a=1}^{4} \overline{w}_{j,a}^S e^{-M_j^S - (a-1)\overline{M}_{ad}^S} + \overline{w}_{j,5+}^S e^{-M_j^S - 3\overline{M}_{ad}^S} \dfrac{1}{1 - e^{-\overline{M}_{ad}^S}}$, where $\overline{w}_{j,a}^S$ is the average of $w_{j,y,a}^S$ as defined in Appendix A. For this same model, a_j^S denotes the maximum recruitment (in billions) and b_j^S denotes the spawner biomass below which the expectation for recruitment is reduced below the maximum.

Test	Stock recruitment relationship	$f\left(SSB_{y,N}^S\right)=$	Parameters
S_{BH}	Beverton Holt	$\dfrac{\alpha_j^S SSB_{j,y}^S}{\beta_j^S + SSB_{j,y}^S}$	$h_j^S \sim U(0.2,1)$ $K_j^S \sim U(0,10\,000)$ $\alpha_j^S = \dfrac{4h_j^S}{5h_j^S - 1}\dfrac{K_j^S}{X_j}$ $\beta_j^S = \dfrac{K_j^S\left(1-h_j^S\right)}{5h_j^S - 1}$
S_R	Ricker	$\alpha_j^S SSB_{j,y}^S e^{-\beta_j^S SSB_{j,y,N}^S}$	$h_j^S \sim U(0.2,1.5)$ $K_j^S \sim U(0,10\,000)$ $\alpha_j^S = \dfrac{1}{X_j}\left(\dfrac{h_j^S}{0.2}\right)^{1/0.8}$ $\beta_j^S = \dfrac{\ln\left(h_j^S / 0.2\right)}{0.8K_j^S}$
S_{HS}	Hockey stick	$\begin{cases} a_j^S & \text{,if } SSB_{j,y}^S \geq b_j^S \\ \dfrac{a_j^S}{b_j^S} SSB_{j,y}^S & \text{,if } SSB_{j,y}^S < b_j^S \end{cases}$	$\ln(a_j^S) \sim U(0,5.4)$ [1] $b_j^S / K_j^S \sim U(0,1)$ $K_j^S = a_j^S X_j$ [2]

S_{2HS}	Hockey stick (2 curves) [3]	if $2000 \leq y \leq 2004$:
		 a'^S
		 else
		 $\begin{cases} a_j^S & \text{,if } SSB_{j,y}^S \geq b_j^S \\ \dfrac{a_j^S}{b_j^S} SSB_{j,y}^S & \text{,if } SSB_{j,y}^S < b_j^S \end{cases}$
			$\ln(a_j^S) \sim U(0,5.4)$ [1]
			 $\ln(a'^S) \sim U(0,5.4)$
			 $b_j^S / K_j^S \sim U(0,1)$
			 $K_j^S = a_j^S X_j$ [2]

[1] Given the lack of *a priori* information on the scale of a_j^S, a log-scale was used, with a maximum corresponding to about 10 million tons.

[2] For consistency, K relates throughout to corresponding MLEs, i.e. the approach works with median rather than mean estimates of K and thus a bias correction factor for the log-normal distribution is not used. These values for K will be less than the corresponding average pre-exploitation levels because of the lognormal distributions assumed for recruitment.

[3] This is attempted for the single stock hypothesis only.

Table 2. The contributions to the objective function at the posterior mode for a range of combinations of juvenile, $\overline{M}_j^S$, and adult, $\overline{M}_{ad}^S$, natural mortality for models assuming the Hockey Stick stock recruitment relationship. The ratio of the multiplicative bias in the recruit survey to that in the November survey, k_r^S / k_N^S, is given for diagnostic purposes. Shaded rows represent what are considered unrealistic values for this ratio.

$\overline{M}_j^S$	$\overline{M}_{ad}^S$	-ln(Post-erior)	-ln(Likelihood)					-ln(Prior)		k_N^S	k_r^S	k_r^S / k_N^S
			Nov	Rec	Com Prop-at-length	Survey Prop-at-minus length	Survey Prop-at-length	k_{ac}^S	ε_y^S			
0.6	0.6	265.62*	19.01	25.28	194.87	2.25	14.60	-1.61	11.23	0.74	0.60	0.81
0.8	0.6	265.90*	19.00	25.32	195.04	2.23	14.68	-1.61	11.24	0.74	0.54	0.73
0.8	0.8	264.83	18.32	24.89	195.99	2.07	14.13	-1.65	11.08	0.72	0.44	0.61
1	0.6	266.20*	19.00	25.37	195.22	2.20	14.76	-1.61	11.26	0.73	0.48	0.65
1	0.8	265.12	18.33	24.95	196.18	2.06	14.16	-1.65	11.09	0.72	0.39	0.54
1	1	266.11	18.08	24.85	197.60	1.89	14.58	-1.65	10.76	0.71	0.34	0.48
1.2	0.6	266.51*	18.99	25.42	195.41	2.18	14.85	-1.61	11.27	0.73	0.43	0.58
1.2	0.8	265.43	18.34	25.02	196.37	2.05	14.19	-1.65	11.11	0.72	0.35	0.49
1.2	1	266.45	18.09	24.93	197.82	1.88	14.60	-1.65	10.78	0.71	0.31	0.43
1.2	1.2	268.12	18.00	24.91	199.69	1.73	15.02	-1.65	10.41	0.71	0.28	0.39
1.4	0.6	266.83*	18.99	25.47	195.60	2.15	14.94	-1.61	11.29	0.73	0.38	0.52
1.4	0.8	265.75	18.35	25.09	196.58	2.03	14.23	-1.65	11.12	0.72	0.31	0.43
1.4	1	266.80	18.11	25.01	198.04	1.88	14.63	-1.65	10.79	0.71	0.27	0.38
1.4	1.2	268.53	18.02	25.00	199.96	1.73	15.05	-1.65	10.42	0.71	0.25	0.35

* Non positive definite Hessian

Table 3. The contributions to the negative log posterior pdf at the joint posterior mode, together with the values of various quantities at that mode, for alternative stock recruitment relationships.

	S_{HS}	S_{2HS}	S_{BH}	S_R
-ln(Posterior)	265.12	265.10	265.23	264.77
-ln(L_{Nov})	18.33	18.32	17.86	17.89
-ln(L_{Rec})	24.95	25.00	25.40	25.24
-ln($L_{ComPropL}$)	196.18	196.19	195.97	196.03
-ln($L_{SurPropLmin}$)	2.06	2.06	2.05	2.06
-ln($L_{SurPropL}$)	14.16	14.13	13.47	13.74
-ln(Prior k_acoustic)	-1.65	-1.65	-1.65	-1.65
-ln(Prior rec residuals)	11.09	11.04	12.13	11.46
# parameters	51	52	51	51
AIC	4619	4619	4619	4619
AIC$_c$	632.25	634.19	632.46	631.55
h_j^S			0.56	0.67
K_j^S	2040	1983	2633	1843
a_j^S or α_j^S	54	52	87	0.119
$a_j'^S$		59		
b_j^S or β_j^S	595	579	648	0.0008

Table 4. Key model parameter values and model outputs estimated at the joint posterior mode for S_{HS}. Values fixed on input are given in **bold**. Numbers are reported in billions and biomasses in thousands of tonnes.

Parameter	S_{HS}	Parameter	S_{HS}
-ln(posterior)	265.12	$N^S_{1983,0}$	6.87
$-\ln L^{Nov}$	18.3	$N^S_{1983,1}$	4.37
$-\ln L^{rec}$	25.0	$N^S_{1983,2}$	<0.001
$-\ln L^{coml\ propl}$	196.2	$Finit$	0.62
$-\ln L^{sur\ propl\ \min}$	2.1	a^S	54.0
$-\ln L^{surl\ propl}$	14.2	b^S	595
$-\ln(priors)$	9.4	K^S_{normal}	2040
$\overline{M}^S_j$	**1.0**	σ^S_r	0.40
$\overline{M}^S_{ad}$	**0.8**	$\sigma^S_{r,peak}$	0.88
$k^S_{j,N} = k^S_{ac}$	0.72	$\overline{B}^S_{Nov}$ [4]	626
k^S_{cov}	0.54	η^S_{2009}	-0.45
$k^S_{j,r}$	0.39	s^S_{cor}	0.41
$k^S_{j,r}/k^S_{j,N}$	0.54	L_∞	**19.9**
$\left(\lambda^S_N\right)^2$	0.0	κ	1.06
$\left(\lambda^S_r\right)^2$	0.0	t_0	0.07
		ϑ_0	3.0
		ϑ_1	2.3
		ϑ_2	1.6
		ϑ_3	1.6
		ϑ_4	1.6
		ϑ_{5+}	1.6

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[4] This is the average over the 1991-1994 perior. OMP-04 and OMP-08 were developed using Risk defined as "the probability that 1+ sardine biomass falls below the average 1+ sardine biomass between November 1991 and November 1994 at least once during the projection period of 20 years".
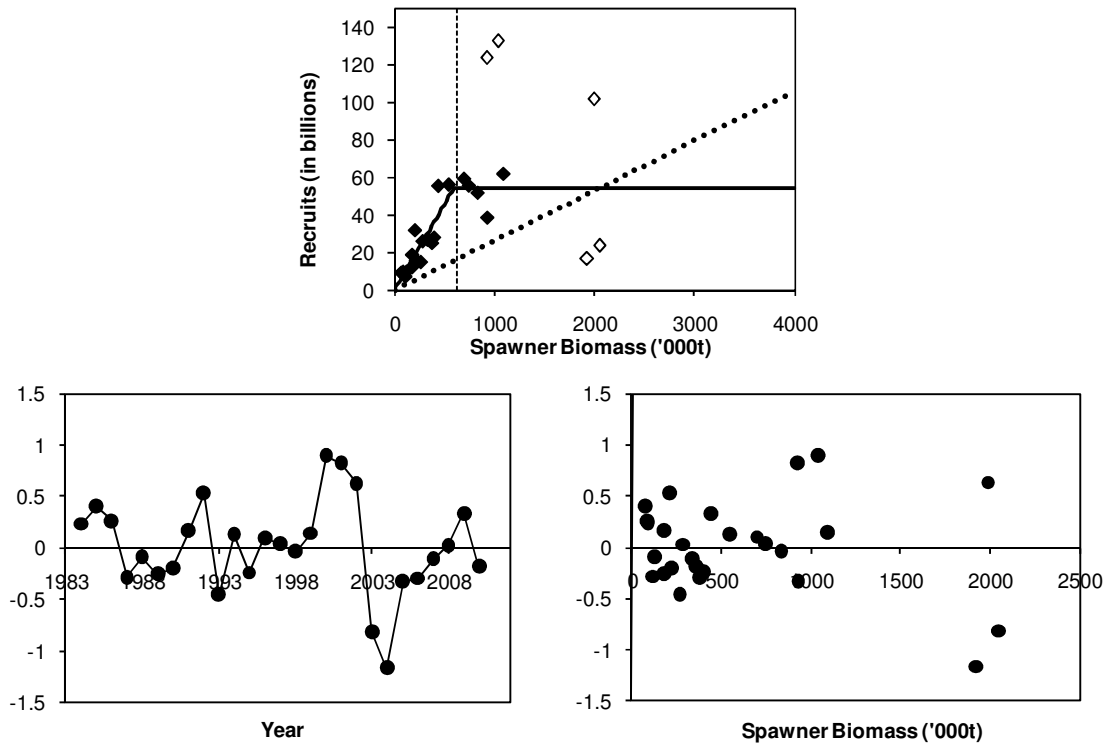
Figure 1. Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to November 2009 for S_{HS} with the Hockey stick stock recruitment relationship. The vertical thin dashed line indicates the average 1991 to 1994 spawner biomass (used in the definition of risk in OMP-04 and OMP-08). The dotted line indicates the replacement line. The standardised residuals from the fit are given in the lower plots, plotted against year and against spawner biomass.
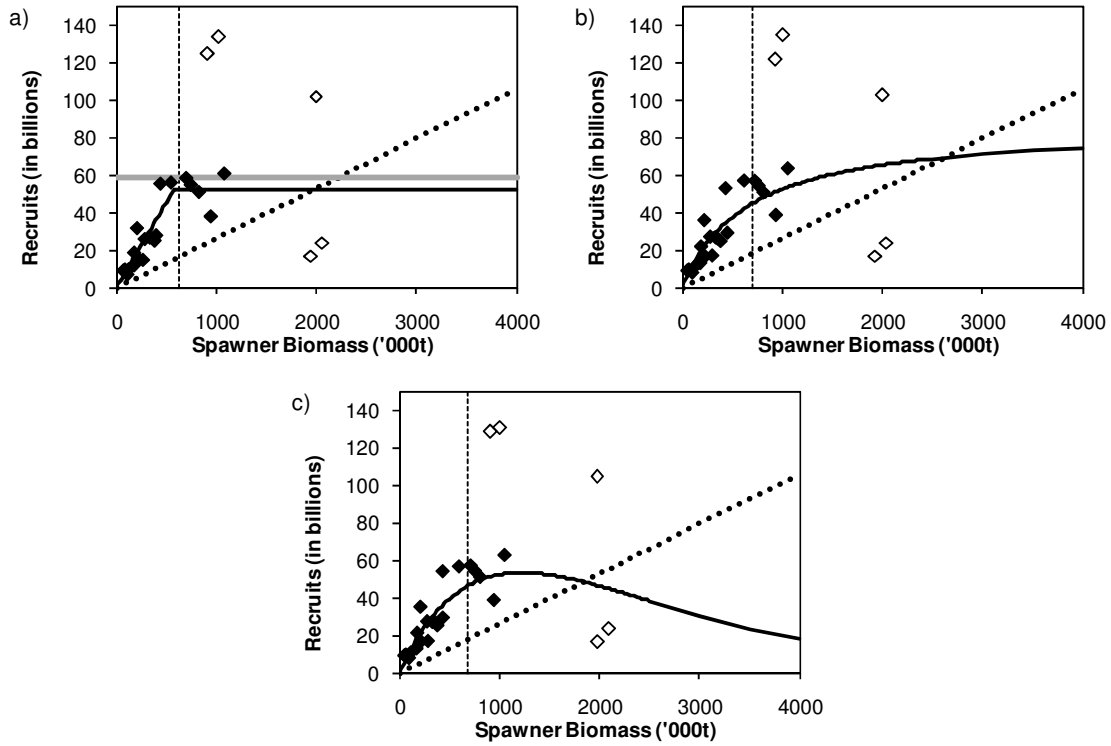
Figure 2. Stock-recruit relationships for a) A_{2HS} (with the grey line showing the median 2000-2004 recruitment), b) S_{BH} and c) S_R.
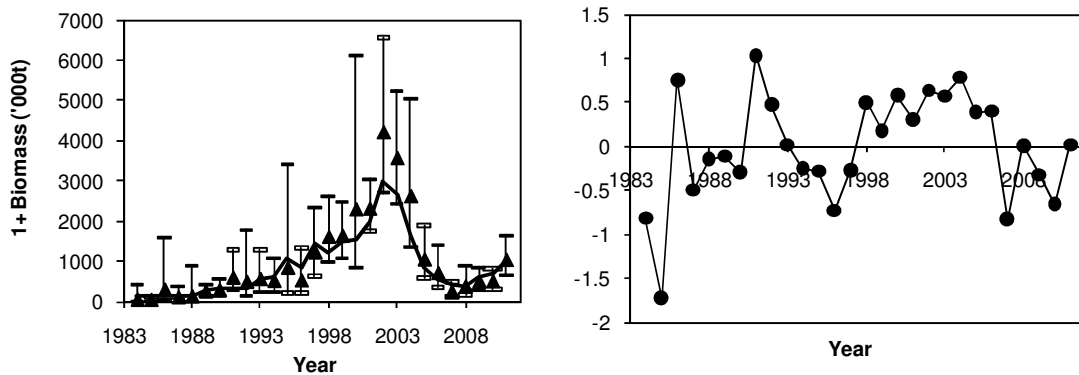
Figure 3. Acoustic survey estimated and model predicted November sardine 1+ biomass from 1984 to 2011 for S_{HS}. The survey indices are shown with 95% confidence intervals. The standardised residuals (i.e. the residual divided by the corresponding standard deviation, including additional variance where appropriate, as indicated in equation (A.26)) from the fit are given in the right hand plot.
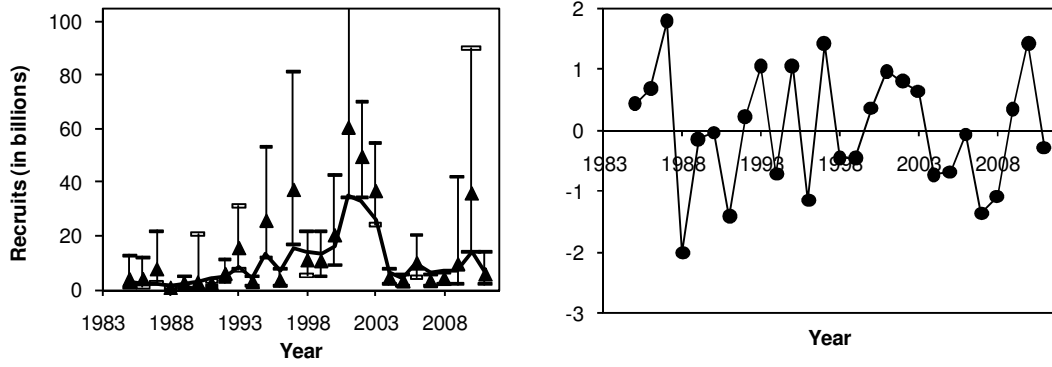
Figure 4. Acoustic survey estimated and model predicted sardine recruitment numbers from May 1985 to May 2011 for S_{HS}. The survey indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plot.
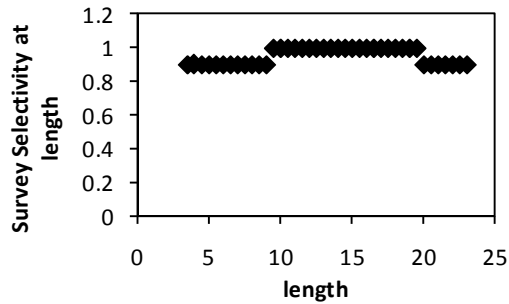

Figure 5. The model estimated November survey selectivity at length for S_{HS}.
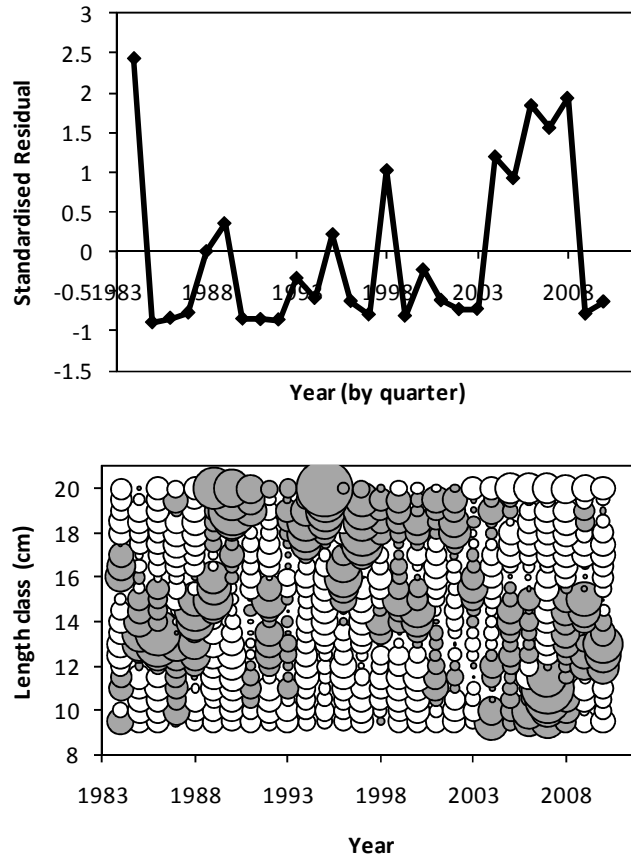
Figure 6. Residuals from the fit of the model predicted proportion-at-length in the November survey to the hydroacoustic survey estimated proportions for S_{HS}. The upper panel show the residuals for the minus length class (9cm) and the lower panel show the residuals for the remaining length classes.
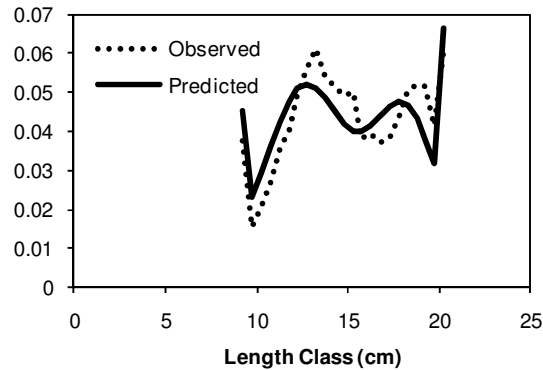

Figure 7. Average (over all years) model predicted and observed proportions-at-length in the November survey.
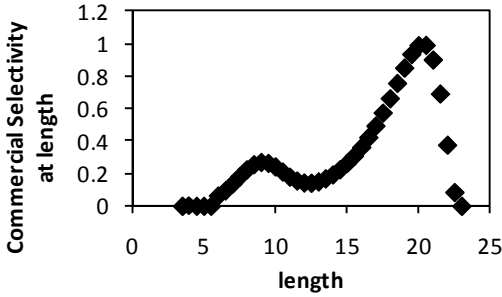
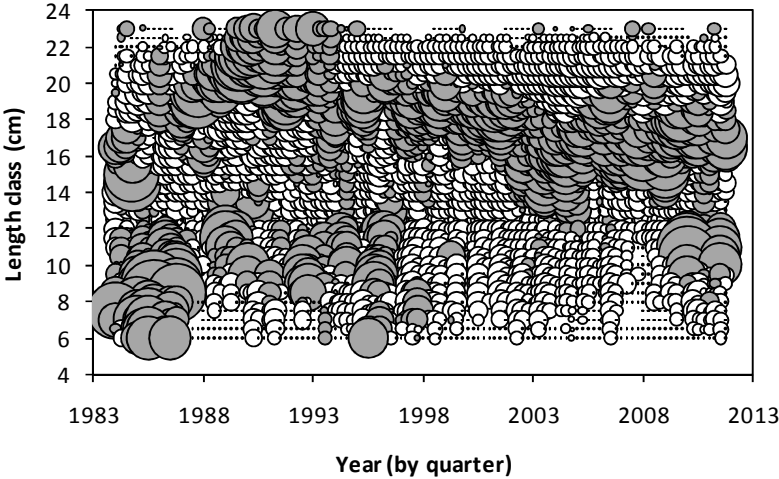**Figure 8.** The model estimated commercial selectivity at length for S_{HS}.

Figure 9. Residuals from the fit of the model predicted proportions-at-length in the commercial catch to the observed proportions for S_{HS}.
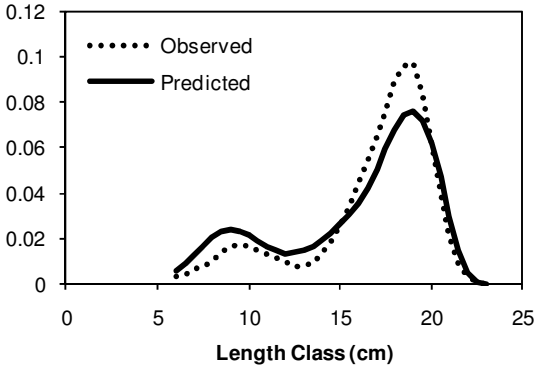

Figure 10. Average (over all quarters and years) model predicted and observed proportions-at-length in the commercial catch.
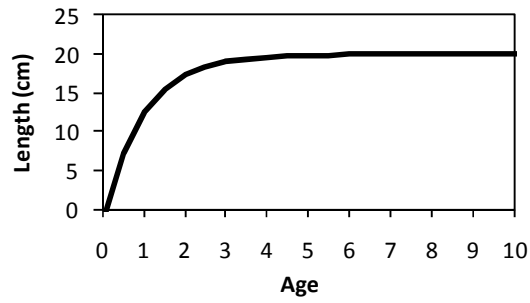
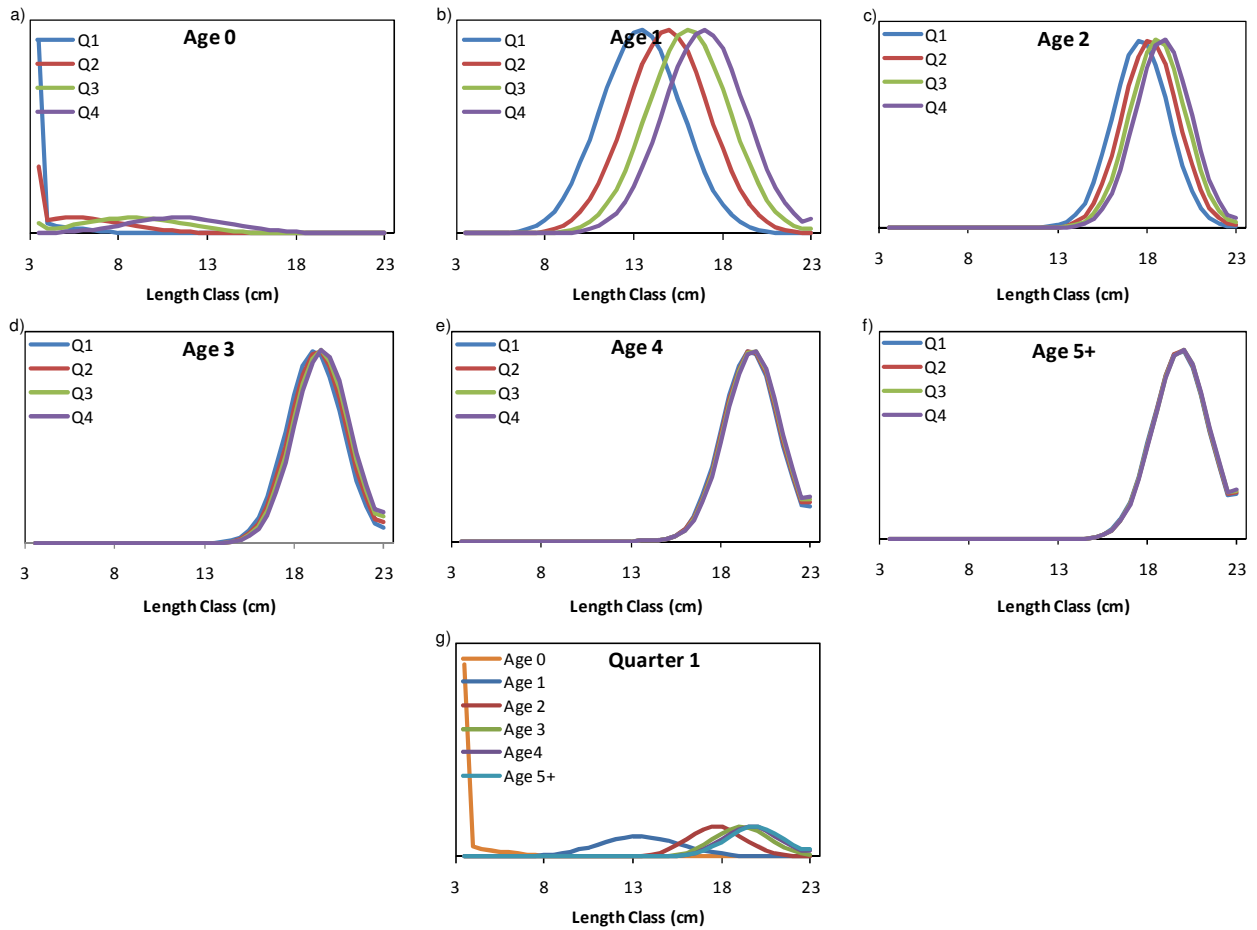**Figure 11**. The von Bertalanffy growth curve estimated for S_{HS}.

Figure 12. The model estimated distributions of proportions-at-length for each age, given at the middle of each quarter of the year (corresponding to the times commercial catch is modelled to be taken). Panel g) compares the distributions for all ages at the middle of quarter 1.

Appendix A: Bayesian age-structured operating model for the South African sardine resource

Base Case Model Assumptions

1) All fish have a birthdate of 1 November.

2) Sardine spawn for the first time when they turn two years old.

3) A plus group of age five is assumed.

4) Two surveys are held each year: the first takes place in November (known as the November survey) and surveys the adult (1+) stock (but see de Moor *et al*. 2012b); the second is in May/June (known as the recruit survey) and surveys juvenile (0-year-old) sardine (also called recruits).

5) The November survey provides a relative index of abundance of unknown bias.

6) The recruit survey provides a relative index of abundance of unknown bias.

7) The survey strategy is such that it results in surveys of invariant bias over time.

8) Pulse fishing occurs four times a year, in the middle of each quarter after the birthdate.

9) Natural mortality is age-invariant for adult fish.

Population Dynamics

The basic dynamic equations for sardine, based on Pope's approximation (Pope, 1984), are as follows, where $y_1 = 1984$ and $y_n = 2011$. The numbers-at-age are modelled at 1 November each year.

Catch is taken at four intervals during the year where $q = 1$ is from November $y-1$ to January y, $q = 2$ from February to April y, $q = 3$ from May to July y and $q = 4$ from August to October y:

Numbers-at-age at 1 November

$$N_{j,y,a}^S = \left(\left(\left(\left(\left(N_{j,y-1,a-1}^S e^{-M_{a-1,y}^S/8} - C_{j,y,1,a-1}^S\right)e^{-M_{a-1,y}^S/4}\right) - C_{j,y,2,a-1}^S\right)e^{-M_{a-1,y}^S/4} - C_{j,y,3,a-1}^S\right)e^{-M_{a-1,y}^S/4} - C_{j,y,4,a-1}^S\right)e^{-M_{a-1,y}^S/8}$$

$$y = y_1,\ldots,y_n,\ a = 1,\ldots,4$$

$$N_{j,y,5+}^S = \left(\left(\left(\left(\left(N_{j,y-1,4}^S e^{-M_{4,y}^S/8} - C_{j,y,1,4}^S\right)e^{-M_{4,y}^S/4}\right) - C_{j,y,2,4}^S\right)e^{-M_{4,y}^S/4} - C_{j,y,3,4}^S\right)e^{-M_{4,y}^S/4} - C_{j,y,4,4}^S\right)e^{-M_{4,y}^S/8}$$

$$+ \left(\left(\left(\left(\left(N_{j,y-1,5+}^S e^{-M_{5+,y}^S/8} - C_{j,y,1,5+}^S\right)e^{-M_{5+,y}^S/4}\right) - C_{j,y,2,5+}^S\right)e^{-M_{5+,y}^S/4} - C_{j,y,3,5+}^S\right)e^{-M_{5+,y}^S/4} - C_{j,y,4,5+}^S\right)e^{-M_{5+,y}^S/8}$$

$$y = y_1,\ldots,y_n \qquad\qquad \text{(A.1)}$$

where

$N_{j,y,a}^S$ is the model predicted number (in billions) of sardine of age a at the beginning of November in year y of stock j;

$C_{j,y,a,q}^S$ is the model predicted number (in billions) of sardine of age a of stock j caught during quarter q of year y;

$M^S_{a,y}$ is the rate of natural mortality (in year^{-1}) of sardine of age a in year y.

Natural mortality

Adult natural mortality varies around a median as follows:

$$M^S_{a,y} = \overline{M}^S_{ad} e^{\varepsilon_{ad,y}} \text{ for } a = 1,\ldots,5+ \text{ with } \varepsilon^{ad}_y = p\varepsilon^{ad}_{y-1} + \sqrt{1-p^2}\eta^{ad}_y \tag{A.2}$$

where $\eta^{ad}_y \sim N\left(0, \sigma^2_{ad}\right)$ and

Similarly, juvenile natural mortality varies around a median as follows:

$$M^S_{j,y} = \overline{M}^S_j e^{\varepsilon_{j,y}} \text{ with } \varepsilon^j_y = p\varepsilon^j_{y-1} + \sqrt{1-p^2}\eta^j_y \tag{A.3}$$

where $\eta^j_y \sim N\left(0, \sigma^2_j\right)$ and

$\overline{M}^S_{ad}$ - median adult rate of natural mortality

σ_{ad} - is the standard deviation in the annual residuals about adult natural mortality;

$\overline{M}^S_j$ - median juvenile rate of natural mortality

σ_j - is the standard deviation in the annual residuals about juvenile natural mortality; and

p - is the annual autocorrelation coefficient.

Movement

In the two stock hypothesis, movement of west stock ($j=1$) recruits to the east stock ($j=2$) at the beginning of November, i.e. when the recruits turn age 1, is modelled as follows:

$$N^S_{1,y,1} = \left(1 - move_y\right)N^{S*}_{1,y,1}$$

$$N^S_{2,y,1} = N^{S*}_{2,y,1} + move_y N^{S*}_{1,y,1} \qquad\qquad y = y_1,\ldots,y_n \tag{A.4}$$

where $N^{S*}_{j,y,1}$ is simply the numbers-at-age 1 given by equation (A.1) prior to movement, and

$move_y$ is the proportion of west stock recruits which migrate to the east stock at the beginning of November of year

y.

Biomass associated with the November survey

$$B^S_{j,y} = k^S_{j,N} \sum_{a=1}^{5+} N^S_{j,y,a} w^S_{j,y,a} \qquad\qquad y = y_1,\ldots,y_n \tag{A.5}$$

where

$B^S_{j,y}$ is the model predicted biomass (in thousand tonnes) of adult sardine of stock j at the beginning of November in year *y*, associated with the November survey;

$k^S_{j,N}$ is the constant of proportionality (multiplicative bias) associated with the November survey of stock j; and

$w_{j,y,a}^{S}$ is the mean mass (in grams) of sardine of age a of stock j sampled during the November survey of year y, calculated as follows:

$$w_{j,y,1}^{S} = \frac{\left(\sum_{a=1}^{5+} N_{j,y,a}^{S}\right) w_{j,y}^{S1+}}{\left(N_{j,y,1}^{S} + \frac{\overline{w_{j,2}^{S}}}{\overline{w_{j,1}^{S}}} N_{j,y,2}^{S} + \frac{\overline{w_{j,3}^{S}}}{\overline{w_{j,1}^{S}}} N_{j,y,3}^{S} + \frac{\overline{w_{j,4}^{S}}}{\overline{w_{j,1}^{S}}} N_{j,y,4}^{S} + \frac{\overline{w_{j,5+}^{S}}}{\overline{w_{j,1}^{S}}} N_{j,y,5+}^{S}\right)} \tag{A.6}$$

$$w_{j,y,a}^{S} = w_{j,y,1}^{S} \frac{\overline{w_{j,a}^{S}}}{\overline{w_{j,1}^{S}}}, \quad a = 2,\ldots,5+, \text{ where}$$

$w_{j,y}^{S1+}$ is the total (1+) mean mass (in grams) of sardine of stock j sampled during the November survey of year y (de Moor *et al.* 2012a); and

$\dfrac{\overline{w_{j,a}^{S}}}{\overline{w_{j,1}^{S}}}$ is the average ratio of mean mass (in grams) of sardine of stock j aged a to age 1 obtained from the growth curve.

The multiplicative bias in the November survey is assumed to be equal to that resulting from the acoustic survey only; hence it is assumed that the full distribution of sardine is covered by the survey, i.e.

$$k_{j,N}^{S} = k_{ac}^{S}$$

where

k_{ac}^{S} is the multiplicative bias associated with the acoustic survey (see Appendix B).

Sardine are assumed to mature at age two and thus the spawning stock biomass is:

$$SSB_{j,y}^{S} = \sum_{a=2}^{5+} N_{j,y,a}^{S} w_{j,y,a}^{S} \qquad\qquad y = y_1,\ldots,y_n \tag{A.7}$$

Proportion at length associated with the November survey

The model predicted numbers-at-length in the survey are:

$$N_{j,y,l}^{S} = \sum_{a=1}^{5+} A_{j,a,l}^{sur} N_{y,j,a}^{S} \qquad\qquad y = y_1,\ldots,y_n \,, \; l = 3.5cm,\ldots,23cm \tag{A.8}$$

and the model predicted proportion-at-length associated with the November survey is:

$$p_{j,y,l\min}^{S} = \frac{\sum_{l \le l\min} N_{j,y,l}^{S} S_{j,l}^{survey}}{\sum_{l} N_{j,y,l}^{S} S_{j,l}^{survey}} \qquad\qquad y = y_1,\ldots,y_n \tag{A.9}$$

$$p_{j,y,l}^{S} = \frac{N_{j,y,l}^{S} S_{j,l}^{survey}}{\sum_{l} N_{j,y,l}^{S} S_{j,l}^{survey}} \qquad\qquad y = y_1,\ldots,y_n \,, \; l = l\min+1,\ldots,l\max-1 \tag{A.10}$$

$$p^{S}_{j,y,l\max} = \frac{\sum\limits_{l \geq l\max} N^{S}_{j,y,l} S^{survey}_{j,l}}{\sum\limits_{l} N^{S}_{j,y,l} S^{survey}_{j,l}} \qquad\qquad y = y_1,\ldots, y_n \quad (A.11)$$

where

$S^{survey}_{j,l}$ is the survey selectivity at length l in the November survey for stock j;

$A^{sur}_{j,a,l}$ is the proportion of sardine of age a in stock j that fall in the length group l in November;

$l\min = 9\mathrm{cm}$ is the minus length class used when fitting the model to survey proportion-at-length data; and

$l\max = 20\mathrm{cm}$ is the plus length class used when fitting the model to survey proportion-at-length data.

The matrix $A^{sur}_{j,a,l}$ is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A^{sur}_{j,a,l} \sim N\!\left(L_{j,\infty}\!\left(1 - e^{-\kappa_j\left(a - t_{0,j}\right)}\right),\vartheta^2_{j,a}\right) \qquad a = 1,\ldots,5+,\; l = 3.5cm,\ldots,23cm \quad (A.6)$$

where

$L_{j,\infty}$ denotes the maximum length (in expectation) of sardine of stock j;

κ_j denotes the annual growth rate of sardine of stock j;

$t_{0,j}$ denotes the age at which the length (in expectation) of sardine of stock j is zero; and

$\vartheta_{j,a}$ denotes the standard deviation of the distribution about the mean length for age a of stock j.

Commercial selectivity

Commercial selectivity-at-length is assumed to follow the combined shape of a normal and inverted lognormal curve, with the second curve reaching a maximum of 1 corresponding to the fully selected length class. Commercial selectivity is assumed to remain unchanged over time. Selectivity-at-lengths less than and equal to the minus length class is taken to be zero (see footnote 7). Thus we have:

$$S_{j,y,l} = \begin{cases} 0 & l \leq 5 \\ \chi_j \exp\!\left\{-\dfrac{\left(l_{mid} - \bar{l}_{1,j}\right)^2}{\left(\sigma^{sel}_{1,j}\right)^2}\right\} + \exp\!\left\{-\dfrac{\left[\ln\!\left(\left(l_{mid} - l_{\max}\right)/\left(\bar{l}_{2,j} - l_{\max}\right)\right)\right]^2}{\left(\sigma^{sel}_{2,j}\right)^2}\right\} & 6 \leq l \leq 40 \end{cases}, \qquad y = y_1,\ldots, y_n$$

where

χ_j denotes the height of the normal curve component for stock j relative to the height of the other component;

l_{mid} is the midpoint (in cm) of length class l;

$l_{\max} = 23.5cm$ is one length class above the maximum for which observations can be predicted;

$\bar{l}_{1,j}$ is the mean of the normal distribution for stock j;

$\bar{l}_{2,j}$ is the median of the lognormal distribution for stock j;

$\left(\sigma^{sel}_{1,j}\right)^2$ is the variance parameter of the normal distribution for stock j; and

$\left(\sigma_{2,j}^{sel}\right)^2$ is the variance parameter of the lognormal distribution for stock j.

Catch

Sardine are landed by three major fisheries: the sardine-directed fishery (*fleet*=1), the red-eye-directed fishery (*fleet*=2), and the anchovy-directed fishery (*fleet*=3). Landings from the former two fisheries comprise mainly adult sardine while bycatch from the anchovy-directed fishery is primarily juvenile sardine. In the anchovy-directed fishery, the assumption is made that all sardine smaller than a pre-determined cut-off length are 0-year-olds, and the remaining bycatch from this fishery are assumed to be 1-year olds:

$$C_{j,y,1,0}^{bycatch} = \sum_{m=11}^{12} \sum_{l<lcut_{y,m}} C_{j,y-1,m,l}^{RLF,fleet=3} + \sum_{l<lcut_{y,1}} C_{j,y,1,l}^{RLF,fleet=3} \qquad C_{j,y,1,1}^{bycatch} = \sum_{m=11}^{12} \sum_{l>=lcut_{y,m}} C_{j,y-1,m,l}^{RLF,fleet=3} + \sum_{l>=lcut_{y,1}} C_{j,y,1,l}^{RLF,fleet=3}$$

$$C_{j,y,2,0}^{bycatch} = \sum_{m=2}^{4} \sum_{l<lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3} \qquad C_{j,y,2,1}^{bycatch} = \sum_{m=2}^{4} \sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3}$$

$$C_{j,y,3,0}^{bycatch} = \sum_{m=5}^{7} \sum_{l<lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3} \qquad C_{j,y,3,1}^{bycatch} = \sum_{m=5}^{7} \sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3}$$

$$C_{j,y,4,0}^{bycatch} = \sum_{m=8}^{10} \sum_{l<lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3} \qquad C_{j,y,4,1}^{bycatch} = \sum_{m=8}^{10} \sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3} , \quad y = y_1,\ldots, y_n \qquad (A.7)$$

$$C_{j,y,q,a}^{bycatch} = 0 \qquad\qquad y = y_1,\ldots, y_n , \; q = 1,\ldots, 4 , \; a = 2,\ldots,5+, \; (A.14)$$

where

$C_{j,y,m,l}^{RLF,fleet}$ is the number of fish landed by *fleet* in length class l landed in month m of year y of stock j (the 'raised length frequency'); and

$lcut_{y,m}$ is the cut off length for recruits in month m of year y (see de Moor *et al*. (2012a) for details).

In the directed sardine and redeye bycatch fisheries, sardine are split between ages using a model estimated selectivity:

$$C_{j,y,1,a}^{dir} = \left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch}\right) S_{j,y,1,a} F_{j,y,1}$$

$$C_{j,y,2,a}^{dir} = \left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch}\right) S_{j,y,2,a} F_{j,y,2}$$

$$C_{j,y,3,a}^{dir} = \left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,3,a}^{bycatch}\right) S_{j,y,3,a} F_{j,y,3}$$

$$C_{j,y,4,a}^{dir} =$$
$$\left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,3,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,4,a}^{bycatch}\right) S_{j,y,4,a} F_{j,y,4}$$

Finally:

$$C_{j,y,q,a}^{S} = C_{j,y,q,a}^{bycatch} + C_{j,y,q,a}^{dir} \qquad\qquad y = y_1,\ldots, y_n , \; q = 1,\ldots, 4 , \; a = 0,\ldots,5+ \qquad (A.15)$$

$F_{j,y,q}$ is the fished proportion in quarter q of year y for a fully selected age class a of stock j, by the directed and redeye bycatch fisheries.

In the equations above the difference in the year subscript between the catch-at-age and initial numbers-at-age is because these numbers-at-age pertain to November of the previous year.

The fished proportion of the available biomass from the directed and redeye bycatch fisheries is estimated by:

$$F_{j,y,1} = \frac{\sum\limits_{fleet=1}^{2} \sum\limits_{m=11}^{12} \sum\limits_{l>=6cm} C_{j,y-1,m,l}^{RLF,fleet} + \sum\limits_{fleet=1}^{2} \sum\limits_{l>=6cm} C_{j,y,1,l}^{RLF,fleet}}{\sum\limits_{a=0}^{5+} \left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} \right) S_{j,y,1,a}}$$

$$F_{j,y,2} = \frac{\sum\limits_{fleet=1}^{2} \sum\limits_{m=2}^{4} \sum\limits_{l>=6cm} C_{j,y,m,l}^{RLF,fleet}}{\sum\limits_{a=0}^{5+} \left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch} \right) S_{j,y,2,a}}$$

$$F_{j,y,3} = \frac{\sum\limits_{fleet=1}^{2} \sum\limits_{m=5}^{7} \sum\limits_{l>=6cm} C_{j,y,m,l}^{RLF,fleet}}{\sum\limits_{a=0}^{5+} \left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,2,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,3,a}^{bycatch} \right) S_{j,y,3,a}}$$

$$F_{j,y,4} = \frac{\sum\limits_{fleet=1}^{2} \sum\limits_{m=8}^{10} \sum\limits_{l>=6cm} C_{j,y,m,l}^{RLF,fleet}}{\sum\limits_{a=0}^{5+} \left(\left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,2,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,3,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,4,a}^{bycatch} \right) S_{j,y,4,a}} \quad (A.16)$$

A penalty is imposed within the model to ensure that $S_{j,y,q,a} F_{j,y,q} < 0.95$.

Recruitment

For the base case assessment of a single stock hypothesis, a Hockey Stick stock-recruitment curve is assumed. Recruitment at the beginning of November is assumed to fluctuate lognormally about the stock-recruitment curve:

$$N_{j,y,0}^{S} = f\left(SSB_{j,y}^{S}\right) e^{\varepsilon_{j,y}^{S}} \qquad\qquad y = y_1,\ldots, y_n \quad (A.8)$$

where

$\varepsilon_{j,y}^{S}$ is the annual lognormal deviation of sardine recruitment.

Number of recruits at the time of the recruit survey

The number of recruits at the time of the recruit survey is calculated taking into account the recruit catch during quarters 1 and 2 (November to April) and an estimate of the recruit catch between 1 May and the start of the survey:

$$N_{j,y,r}^{S} = k_{j,r}^{S} \left(\left(\left(N_{j,y-1,0}^{S} e^{-M_0^S/8} - C_{j,y,1,0}^{S} \right) e^{-M_0^S/4} - C_{j,y,2,0}^{S} \right) e^{-0.5t_y^S \times M_0^S/12} - \tilde{C}_{j,y,0bs}^{S} \right) e^{-0.5t_y^S \times M_0^S/12} \quad y = y_1,\ldots, y_n \quad (A.18)$$

where

$N_{j,y,r}^{S}$ is the model predicted number (in billions) of juvenile sardine of stock j at the time of the recruit survey in

year y;

$k_{j,r}^{S}$ is the constant of proportionality (multiplicative bias) associated with the recruit survey;

$\tilde{C}_{j,y,0bs}^{S}$ is the number (in billions) of juvenile sardine of stock j caught between 1 May and the day before the start of the recruit survey (see de Moor *et al*. 2012a); and

t_{y}^{S} is the time lapsed (in months) between 1 May and the start of the recruit survey in year y (see de Moor *et al*. 2012a).

The multiplicative bias in the recruit survey is assumed to be equal to that resulting from the acoustic survey as well as the proportion of the recruit abundance which the survey covers in comparison to the November survey. In addition, for the two stock hypothesis, the proportion of the east stock recruit abundance covered compared to that of the west stock abundance is also required. Thus

$$k_{1,r}^{S} = k_{\text{cov}}^{S} \times k_{ac}^{S}$$

and for the two stock hypothesis, $k_{2,r}^{S} = k_{\text{cov}\,E}^{S} \times k_{\text{cov}}^{S} \times k_{ac}^{S}$

where

k_{cov}^{S} is the multiplicative bias associated with the coverage of the recruits by the recruit survey compared to the 1+ biomass by the November survey; and

$k_{\text{cov}\,E}^{S}$ is the multiplicative bias associated with the coverage of the east stock recruits by the recruit survey compared to the west stock recruits during the same survey.

Proportion at length associated with the commercial catch

The commercial catch-at-length from the directed and redeye bycatch fisheries is:

$$C_{j,y,1,l}^{dir} = \sum_{a=0}^{5+} \left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch} \right) A_{j,1,a,l}^{com} S_{j,y,l} F_{j,y,1}$$

$$C_{j,y,2,l}^{dir} = \sum_{a=0}^{5+} \left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch} \right) A_{j,2,a,l}^{com} S_{j,y,l} F_{j,y,2}$$

$$C_{j,y,3,l}^{dir} = \sum_{a=0}^{5+} \left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,2,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,3,a}^{bycatch} \right) A_{j,3,a,l}^{com} S_{j,y,l} F_{j,y,3}$$

$$C_{j,y,4,l}^{dir} = \sum_{a=0}^{5+} \left(\left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,2,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,3,a}^{S} \right) e^{-M_a^S/4} - C_{j,y,4,a}^{bycatch} \right) A_{j,4,a,l}^{com} S_{j,y,l} F_{j,y,4}$$

$$y = y_1, \ldots, y_n, \; l = 3.5cm, \ldots, 23cm \quad (A.19)$$

The model predicted proportion-at-length in the commercial catch from the directed and redeye bycatch fisheries is:

$$p_{j,y,q,l}^{coml,S} = \frac{C_{j,y,q,l}^{dir}}{\sum_{l} C_{j,y,q,l}^{dir}} \,^5 \qquad\qquad y = y_1,\ldots, y_n , \; q = 1,\ldots,4 , \; l = 3.5cm,\ldots,23cm \quad (A.20)$$

where

$A_{j,q,a,l}^{com}$ is the proportion of sardine of age a in stock j that fall in the length group l in quarter q.

The matrix $A_{j,q,a,l}^{com}$ is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A_{j,1,a,l}^{com} \sim N\!\left(L_{j,\infty}\!\left(1 - e^{-\kappa_j\left(a+1/8-t_{0,j}\right)}\right), \vartheta_{j,a}^{\,2}\right) \qquad a = 0,\ldots,5+, \; l = 3.5cm,\ldots,23cm \quad (A.21)$$

$$A_{j,2,a,l}^{com} \sim N\!\left(L_{j,\infty}\!\left(1 - e^{-\kappa_j\left(a+3/8-t_{0,j}\right)}\right), \vartheta_{j,a}^{\,2}\right) \qquad a = 0,\ldots,5+, \; l = 3.5cm,\ldots,23cm \quad (A.22)$$

$$A_{j,3,a,l}^{com} \sim N\!\left(L_{j,\infty}\!\left(1 - e^{-\kappa_j\left(a+5/8-t_{0,j}\right)}\right), \vartheta_{j,a}^{\,2}\right) \qquad a = 0,\ldots,5+, \; l = 3.5cm,\ldots,23cm \quad (A.23)$$

$$A_{j,4,a,l}^{com} \sim N\!\left(L_{j,\infty}\!\left(1 - e^{-\kappa_j\left(a+7/8-t_{0,j}\right)}\right), \vartheta_{j,a}^{\,2}\right) \qquad a = 0,\ldots,5+, \; l = 3.5cm,\ldots,23cm \quad (A.24)$$

<u>Fitting the Model to Observed Data (Likelihood)</u>

The survey observations are assumed to be lognormally distributed. The standard errors of the log-distributions for the survey observations of adult biomass and recruitment numbers are approximated by the CVs of the untransformed distributions and a further additional variance parameter. The estimated proportions-at-length are also assumed to be lognormally distributed, with the variance inversely proportional to the observed proportion. Thus the negative log-likelihood function is given by:

$$-\ln L = -\ln L^{Nov} - \ln L^{rec} - \ln L^{sur\,propl\,min} - \ln L^{sur\,propl} - \ln L^{com\,propl} \qquad\qquad (A.25)$$

where

$$-\ln L^{Nov} = \frac{1}{2}\sum_{j}\sum_{y=y1}^{yn}\left\{ \frac{\left(\ln\!\left(\hat{B}_{j,y}^{S}\right) - \ln(B_{j,y}^{S})\right)^2}{(\sigma_{j,y,Nov}^{S})^2 + \left(\phi_{ac}^{S}\right)^2 + (\lambda_{j,N}^{S})^2} + \ln\!\left[2\pi\!\left((\sigma_{j,y,Nov}^{S})^2 + \left(\phi_{ac}^{S}\right)^2 + (\lambda_{j,N}^{S})^2\right)\right] \right\}$$

$$-\ln L^{rec} = \frac{1}{2}\sum_{j}\sum_{y=y1+1}^{yn}\left\{ \frac{\left(\ln\!\left(\hat{N}_{j,y,r}^{S}\right) - \ln(N_{j,y,r}^{S})\right)^2}{(\sigma_{j,y,rec}^{S})^2 + \left(\phi_{ac}^{S}\right)^2 + (\lambda_{j,r}^{S})^2} + \ln\!\left[2\pi\!\left((\sigma_{j,y,rec}^{S})^2 + \left(\phi_{ac}^{S}\right)^2 + (\lambda_{j,r}^{S})^2\right)\right] \right\}$$

$$-\ln L^{sur\,propl\,min} = w_{propl\,min}^{sur}\,\frac{1}{2}\sum_{j}\sum_{y=y1}^{yn}\left\{ \frac{\hat{p}_{j,y,l\,min}^{S}\left(\ln\!\left(\hat{p}_{j,y,l\,min}^{S}\right) - \ln\!\left(p_{j,y,l\,min}^{S}\right)\right)^2}{2\left(\sigma_j^{S,surl\,min}\right)^2} + \ln\!\left(\frac{\sigma_j^{S,surl\,min}}{\sqrt{\hat{p}_{j,y,l\,min}^{S}}}\right) \right\}$$

$$-\ln L^{sur\,propl} = w_{propl}^{sur}\,\frac{1}{2}\sum_{j}\sum_{y=y1}^{yn}\sum_{l=1\,min+1}^{l\,max}\left\{ \frac{\hat{p}_{j,y,l}^{S}\left(\ln\!\left(\hat{p}_{j,y,l}^{S}\right) - \ln\!\left(p_{j,y,l}^{S}\right)\right)^2}{2\left(\sigma_j^{S,surl}\right)^2} + \ln\!\left(\frac{\sigma_j^{S,surl}}{\sqrt{\hat{p}_{j,y,l}^{S}}}\right) \right\}\,^6$$

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[5] See footnote 7 and the section "Fixed Parameters" for the base case hypotheses. Commercial selectivity at length is fixed = 0 for length classes <6cm, and thus the commercial proportions-at-length in length classes < 6cm in equation (A.20) are not used in fitting the model.

[6] Although strictly there may be bias in the proportions of length-at-age data, no bias is assumed in this assessment. The effect of such a bias is assumed to be small.

$$-\ln L^{com\,propl} = w_{propl}^{com} \frac{1}{2} \sum_j \sum_{y=y1}^{yn} \sum_{q=1}^{4} \sum_{l>=6cm} \left\{ \frac{\hat{p}_{j,y,q,l}^{S,coml}\left(\ln\left(\hat{p}_{j,y,q,l}^{S,coml}\right) - \ln\left(p_{j,y,q,l}^{coml}\right)\right)^2}{2\left(\sigma_j^{S,coml}\right)^2} + \ln\left(\frac{\sigma_j^{S,coml}}{\sqrt{\hat{p}_{j,y,q,l}^{S,coml}}}\right) \right\}^{78}$$ (A.26)

Here

$\hat{B}_{j,y}^S$ is the acoustic survey estimate (in thousands of tonnes) of adult sardine biomass of stock j from the

November survey in year y, with associated CV $\sigma_{j,y,Nov}^S$;

$\hat{N}_{j,y,r}^S$ is the acoustic survey estimate (in billions) of sardine recruitment numbers of stock j from the recruit survey

in year y, with associated CV $\sigma_{j,y,rec}^S$; and

ϕ_{ac}^S is the CV associated with the factors which cause bias in the acoustic survey estimates and which vary inter-

annually rather than remain fixed over time;

$(\lambda_{j,N/r}^S)^2$ is the additional variance (over and above the squares of the survey sampling CV $\sigma_{y,Nov/rec}^S$ that reflects

survey inter-transect variance and of the CV ϕ_{ac}^S associated with the annually   varying factors causing bias

in the acoustic survey estimates) associated with the November/recruit surveys of stock j;

$\hat{p}_{j,y,l}^S$ is the observed proportion (by number) of sardine in length group l in the November survey of year y;

$w_{propl\,min}^{sur}$ is the weighting applied to the survey proportion at length data for the minus length class;

w_{propl}^{sur} is the weighting applied to the remaining survey proportion at length data;

$\sigma_j^{S,surl\,min}$ is the variance-related parameter for the log-transformed survey proportion-at-length data of the minus

length class, which is estimated in the fitting procedure by the closed form solution:

$$\sigma_j^{S,surl\,min} = \sqrt{\left. \sum_{y=y1}^{yn} \hat{p}_{j,y,l\,min}^S \left(\ln \hat{p}_{j,y,l\,min}^S - \ln p_{j,y,l\,min}^S\right)^2 \middle/ \sum_{y=y1}^{yn} 1 \right.}\,; \text{ and}$$

$\sigma_j^{S,surl}$ is the variance-related parameter for the log-transformed survey proportion-at-length data, which is estimated

in the fitting procedure by the closed form solution:

$$\sigma_j^{S,surl} = \sqrt{\left. \sum_{y=y1}^{yn} \sum_{l=1\,min+1}^{l\,max} \hat{p}_{j,y,l}^S \left(\ln \hat{p}_{j,y,l}^S - \ln p_{j,y,l}^S\right)^2 \middle/ \sum_{y=y1}^{yn} \sum_{l=1\,min+1}^{l\,max} 1 \right.}\,.$$

$\hat{p}_{j,y,q,l}^{S,coml}$ is the observed proportion (by number) of the directed and redeye bycatch commercial catch in length group

l of during quarter q ($q=1$ for Nov-Jan, $q=2$ for Feb-Apr, $q=3$ for May-Jul, $q=4$ for Aug-Oct) of year

y;

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[7] In only 11 out of 112 year-quarters were fish of length <6cm observed in the directed sardine and redeye bycatch fisheries. Due to the large variance associated with a minus group of 5.5cm in earlier models, and the small occurrence and small proportion-at-length of these fish, the directed sardine and redeye bycatch fisheries are modeled to cover length classes 6cm and larger only, and the 6cm length class is not treated as a minus class.

[8] The sum is over all quarters for which the catch is non-zero.

w_{propl}^{com} is the weighting applied to the commercial proportion at length data; and

$\sigma_j^{S,coml}$ is the variance-related parameter for the log-transformed commercial proportion-at-length data, which is estimated in the fitting procedure by the closed form solution:

$$\sigma_j^{S,coml} = \sqrt{\sum_{y=y1}^{yn}\sum_{q=1}^{4}\sum_{l>=6cm} \hat{p}_{j,y,q,l}^{S,coml}\left(\ln \hat{p}_{j,y,q,l}^{S,coml} - \ln p_{j,y,q,l}^{coml}\right)^2 \Bigg/ \sum_{y=y1}^{yn}\sum_{q=1}^{4}\sum_{l>=6cm}1}\ ^8.$$

Fixed Parameters for the Base Case Hypotheses

The following parameters were fixed externally in the model:

In the base case assessment, natural mortality is assumed to be time-invariant, thus $\sigma_j = \sigma_{ad} = 0$, giving $\varepsilon_y^j = \varepsilon_y^{ad} = 0$.

$S_{j,y,l} = 0$, $\quad l = 1,\ldots,5$, $y = y_1,\ldots,y_n$, see footnote 7.

Sardine of length 9.5-19.5cm are taken to be fully selected in the survey trawls: $S_{j,l}^{survey} = 1$, $l = 13,\ldots,33$.

$L_{1,\infty} = 19.928$, for the single stock hypothesis and $L_{1,\infty} = 19.458$ and $L_{2,\infty} = 20.579$ for the two stock hypothesis (Durholtz and Mtengwane pers. Commn; see Appendix D).

The weighting on the commercial proportions-at-length data should be about $1/(4 \times 6) \approx 0.04$ of that on the commercial proportions-at-age data, where the weighting is first reduced by a quarter due to the fact that there are four (quarterly) data points to every annual survey estimate of abundance, and also reduced by a sixth as 35 length classes are fit in the likelihood in comparison to 6 ages if proportions-at-age data had been available, with the two carrying essentially the same information content. Thus a value of $w_{propl}^{com} = 0.04$ is set. Similarly the weighting for the survey proportions-at-length data is set at one sixth, $w_{propl}^{sur} = 0.167$.

The CV associated with factors causing bias in the acoustic survey estimated which vary inter-annually is fixed at the CV of the posterior distribution calculated in Figure B.2, i.e. $\phi_{ac}^S = 0.215/0.969 = 0.222$.

From the von Bertalanffy growth curve (Durholtz and Mtengwane pers. comm.), $\dfrac{\overline{w_{j,2}^S}}{w_{j,1}^S} = 1.40$, $\dfrac{\overline{w_{j,3}^S}}{w_{j,1}^S} = 1.69$, $\dfrac{\overline{w_{j,4}^S}}{w_{j,1}^S} = 1.88$, and $\dfrac{\overline{w_{j,5+}^S}}{w_{j,1}^S} = 2.00$ for the single stock hypothesis. For the two stock hypothesis, $\dfrac{\overline{w_{1,2}^S}}{w_{1,1}^S} = 1.39$, $\dfrac{\overline{w_{1,3}^S}}{w_{1,1}^S} = 1.65$, $\dfrac{\overline{w_{1,4}^S}}{w_{1,1}^S} = 1.80$, $\dfrac{\overline{w_{1,5+}^S}}{w_{1,1}^S} = 1.89$, $\dfrac{\overline{w_{2,2}^S}}{w_{2,1}^S} = 1.39$, $\dfrac{\overline{w_{2,3}^S}}{w_{2,1}^S} = 1.68$, $\dfrac{\overline{w_{2,4}^S}}{w_{2,1}^S} = 1.89$, and $\dfrac{\overline{w_{2,5+}^S}}{w_{2,1}^S} = 2.02$.

Estimable Parameters and Prior Distributions for the Base Case Hypotheses

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve. For the single stock hypothesis, the variance about the stock recruitment curve is assumed to differ between peak and non-peak years, i.e.the prior pdfs for the recruitment residuals are given by:

$$\varepsilon_{j,y}^{S} \sim N\left(0,\left(\sigma_{r}^{S}\right)^{2}\right) \ , \qquad y = y_{1},\ldots,1999, 2005,\ldots, y_{n-1}$$

$$\varepsilon_{j,y}^{S} \sim N\left(0,\left(\sigma_{r,peak}^{S}\right)^{2}\right) \ , \qquad y = 2000,\ldots, 2004$$

while for the two stock hypothesis, the variance about the stock recruitment curves is assumed to differ between stocks, but not over years, i.e.

$$\varepsilon_{j,y}^{S} \sim N\left(0,\left(\sigma_{j,r}^{S}\right)^{2}\right) \ , \qquad y = y_{1},\ldots, y_{n-1}$$

$$k_{ac}^{S} \sim N\left(0.714, 0.077^{2}\right), \text{ see Appendix B}$$

The remaining estimable parameters are defined as having the following near non-informative prior distributions:

$move_{y} \sim U(0,1)$, $y = y_{1},\ldots, y_{n}$, for the two stock hypothesis only

$$k_{cov}^{S} \sim U(0.3,1)$$

$$k_{cov E}^{S} \sim U(0,1)$$

$$\left(\lambda_{j,N/r}^{S}\right)^{2} \sim U(0,10)$$

Initial results indicated that survey selectivity-at-length could be reasonably well reflected by these constant levels:

$$S_{j,l}^{survey} = S_{j}^{survey1} \sim U(0.9,1.1)^{9}, \ l = 1,\ldots,12$$

$$S_{j,l}^{survey} = S_{j}^{survey5} \sim U(0.9,1.1), \ l = 34,\ldots,40$$

While the priors for the commercial selectivity-at-length parameters are:

$$\chi_{j} \sim U(0,1)$$

$$\bar{l}_{1,j} \sim U(5cm,15cm)$$

$$\bar{l}_{2,j} - \bar{l}_{1,j} \sim U(0cm,15cm)$$

$$\left(\sigma_{1,j}^{sel}\right)^{2} \sim U(2,10)$$

$$\left(\sigma_{2,j}^{sel}\right)^{2} \sim U(0,2)$$

For the single stock hypothesis: $\left(\sigma_{r}^{S}\right)^{2} \sim U(0.16,10)$ and $\left(\sigma_{r,peak}^{S}\right)^{2} \sim U(0.16,10)$

While for the two stock hypothesis: $\left(\sigma_{j,r}^{S}\right)^{2} \sim U(0.16,10)$

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[9] By design, surveys aim to achieve equal selectivity over all ages. Age 1 sardine distributed inshore may be under caught in comparison to other ages. On the other hand older, faster fish may be more able to avoid day-time trawls and thus be under

$$N_{j,1983,a}^{S} \sim U(0,50) \text{ billion } a = 0,1,2$$

$$N_{j,1983,a}^{S} = N_{j,1983,a-1}^{S} e^{-\left(Finit+\overline{M}_a^S\right)} \quad a = 3,4$$

$$N_{j,1983,5+}^{S} = N_{j,1983,4}^{S} \frac{e^{-\left(Finit+\overline{M}_{ad}^S\right)}}{1-e^{-Finit-\overline{M}_{ad}^S}}, \text{ with } Finit \sim U(0,1)$$

$\vartheta_{j,a} \sim U(0.01,3)$, for $a = 1,\ldots,5+$ These parameters are assumed to be the same for the "west" and "south" stocks.

$\kappa_1 \times L_{1,\infty} = 10.176$ for the single stock hypothesis and $\kappa_1 \times L_{1,\infty} = 11.685$ and $\kappa_2 \times L_{2,\infty} = 9.599$ for the two stock hypothesis (Durholtz and Mtengwane pers. comm.)   ½ parameters

$t_{0,j} \sim U(-4,4)$ These parameters are assumed to be the same for the "west" and "south" stocks

Further Outputs

Recruitment serial correlation:

$$s_{j,cor}^{S} = \frac{\sum_{y=y1}^{yn-2} \varepsilon_{j,y} \varepsilon_{j,y+1}}{\sqrt{\left(\sum_{y=y1}^{yn-2} \varepsilon_{j,y}^2\right)\left(\sum_{y=y1}^{yn-2} \varepsilon_{j,y+1}^2\right)}} \tag{A.27}$$

and the standardised recruitment residual value for 2011:

$$\eta_{j,2010}^{S} = \frac{\varepsilon_{j,2010}^{S}}{\sigma_{j,r}^{S}} \tag{A.28}$$

are also required as input into the OM.

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represented in any day-time (about ½) trawl samples. It is, however, most likely that selectivity of ages 3 to 5+ is flat (Coetzee pers. comm.).

Appendix B: Calculating the bias in estimates of sardine from the May and November hydro-acoustic surveys

A probability density function (pdf) for the bias in the May and November survey that relate directly to the acoustic survey, rather than, for example the coverage of the stock, k_{ac}^{S}, was calculated as follows. Ten thousand samples were drawn from the individual pdfs for each source of constant error, together with the median values of the individual pdfs of each source of variable error (see Table B.1, Anon. 2000). Constant error relates to a factor whose value is not known exactly, but whatever it is, it is the same for each year. In contrast variable errors relate to a factor whose true value will change from one year to the next. A second pdf of the factors causing bias in the acoustic survey estimated which vary inter-annually, ϕ_{ac}^{S}, was then calculated by drawing ten thousand samples from the individual pdfs for each source of variable error. The resultant pdfs on the model predicted biomass (i.e. the inverse of the pdf calculated using the errors provided), together with normal distributions fitted to these pdfs are given in Figures B.1 and B.2. A prior distribution for the multiplicative bias associated with the acoustic survey, k_{ac}^{S}, is then the normal distribution obtained in Figure B.1, with the mean multiplied by the mean of the normal distribution obtained in Figure B.2, i.e. $k_{ac}^{S} \sim N\left(0.969 \times 0.737, 0.077^{2}\right)$. The reason to include the 0.969 mean from Figure B.2 here is that the distribution of the annually varying bias factors in combination is not centred on 1; this then takes account of the formulation of equation A.21 treating the impact of these factors as a symmetric variance. There may, however, still be systematic errors relating to the target strength that are unaccounted for in these pdfs. These could be taken into account through sensitivity tests using alternative k_{ac}^{S} values.

Table B.1. Individual error factors for hydro-acoustic surveys of sardine biomass, where the values define trapezium form pdfs. Note that these error factors apply to the observed biomass, i.e. they reflect the inverse of the multiplicative bias (applied to the model predicted biomass) in this document.

Error	Minimum	Likely (lower)	Likely (midpoint)	Likely (upper)	Maximum	Nature
Calibration						
(On-axis sensitivity)	0.90	0.95	1.00	1.05	1.10	Variable[11]
(Beam factor)	0.75[10]	0.90	1.00	1.10	1.25	Constant
Surface Schooling	1.00	1.05	1.075	1.10	1.15	Variable
Target Identification	0.50	0.90	1.00	1.10	1.50	Variable[8]
Weather Effects	1.01	1.05	1.15	1.25	2.00	Variable

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[10] This was originally reported as 0.8 in Anon 2000, but subsequently corrected (I. Hampton pers. Comm.).

[11] This was recorded in Anon. (2000) as random error denoting that it would be positive or negative rather than purely positive or negative.

**Multiplicative bias in the estimate of sardine
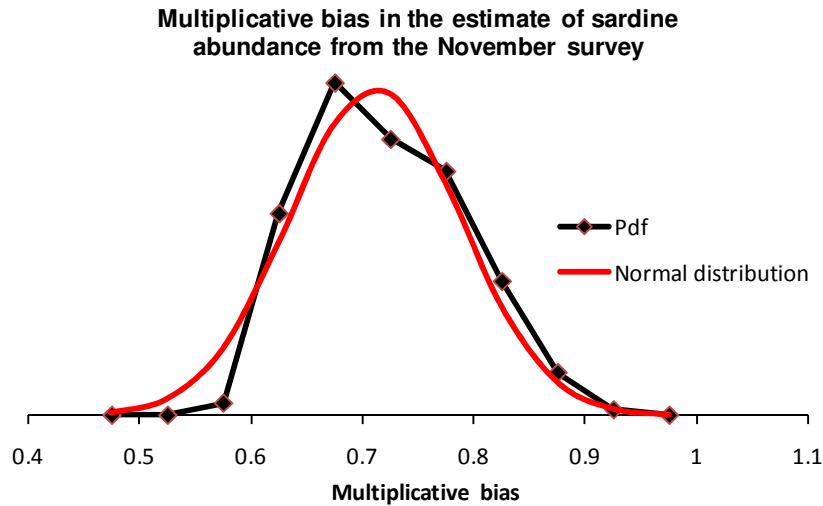abundance from the November survey**

Figure B.1. The probability density function for the overall bias in the estimate of sardine abundance from the November survey, calculated by drawing 10 000 samples from the individual probability distribution functions for each source of constant error, together with the median values of the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $N(0.737, 0.077^2)$.

**Additional inter-annually variable bias in the estimate
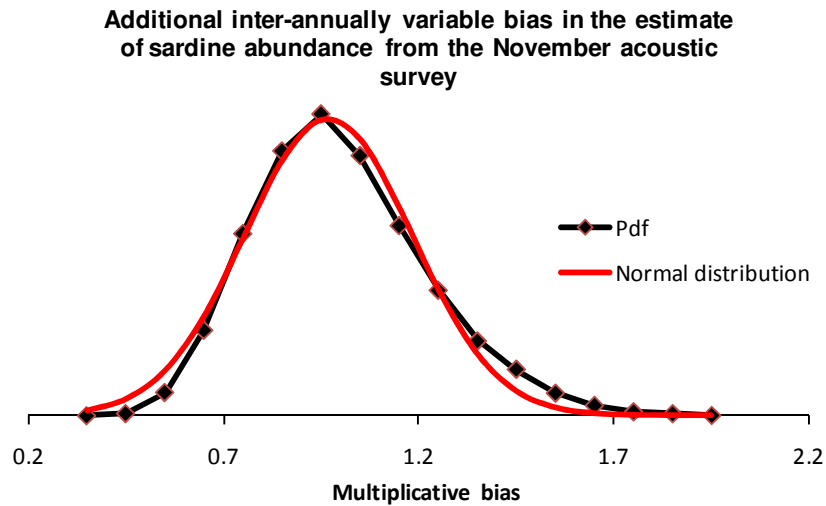of sardine abundance from the November acoustic
survey**

Figure B.2. The probability density function for the factors which cause bias in the sardine acoustic survey estimates and which vary inter-annually, calculated by drawing 10 000 samples from the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $N(0.969, 0.215^2)$. The CV of this distribution is thus $\phi_{ac}^{S} = 0.215/0.969 = 0.222$.

Appendix C: Glossary of parameters used in this document

Annual numbers and biomass:

$N_{j,y,a}^{S}$ - model predicted number (in billions) of sardine of age a at the beginning of November in year y of stock j

$B_{j,y}^{S}$ - model predicted biomass (in thousand tonnes) of adult sardine of stock j at the beginning of November in year y, associated with the November survey

$SSB_{j,y}^{S}$ - model predicted spawning stock biomass (in thousand tonnes) of stock j at the beginning of November in year y

$w_{j,y,a}^{S}$ - mean mass (in grams) of sardine of age a of stock j sampled during the November survey of year y

$w_{j,y}^{S1+}$ is the total (1+) mean mass (in grams) of sardine of stock j sampled during the November survey of year y

$\dfrac{\overline{w_{j,a}^{S}}}{w_{j,1}^{S}}$ is the average ratio of mean mass (in grams) of sardine of stock j aged a to age 1 obtained from the growth curve

$N_{j,y,r}^{S}$ - model predicted number (in billions) of juvenile sardine of stock j at the time of the recruit survey in year y

t_{y}^{S} - time lapsed (in months) between 1 May and the start of the recruit survey in year y

$move_{y}$ - proportion of west stock recruits which migrate to the east stock at the beginning of November of year y

Natural mortality:

$M_{a,y}^{S}$ - rate of natural mortality (in year^{-1}) of sardine of age a in year y

$\overline{M}_{ju}^{S}$ - median juvenile rate of natural mortality (in year^{-1})

$\overline{M}_{ad}^{S}$ - median adult rate of natural mortality (in year^{-1})

ε_{y}^{ad} - annual residuals about adult natural mortality

η_{y}^{ad} - normally distributed error used in calculating ε_{y}^{ad}

σ_{ad} - standard deviation in the annual residuals about adult natural mortality

σ_{j} - standard deviation in the annual residuals about juvenile natural mortality

p - annual autocorrelation coefficient in annual residuals about adult natural mortality

Commercial selectivity

$S_{j,y,l}$ - commercial selectivity at length l during year y of stock j

χ_{j} - denotes the height of the normal curve component for stock j relative to the height of the other component

l_{mid} - the midpoint (in cm) of length class l

$l_{max} = 23.5cm$ - one length class above the maximum for which observations can be predicted

$\bar{l}_{1,j}$ - the mean of the normal distribution for stock j

$\bar{l}_{2,j}$ - the median of the lognormal distribution for stock j

$\left(\sigma_{1,j}^{sel}\right)^2$ - the variance parameter of the normal distribution for stock j

$\left(\sigma_{2,j}^{sel}\right)^2$ - the variance parameter of the lognormal distribution for stock j

$S_{j,y,q,a}$ - commercial selectivity at age a during quarter q of year y of stock j

Catch:

$C_{j,y,a,q}^{S}$ - model predicted umber (in billions) of sardine of age a of stock j caught during quarter q of year y

$C_{j,y,m,l}^{RLF}$ - number of fish in length class l landed in month m of year y of stock j (the 'raised length frequency')

$lcut_{y,m}$ - cut off length for recruits in month m of year y

$C_{j,y,q,a}^{bycatch}$ - the number of fish of age $a \geq 1$ from the anchovy-directed fishery in quarter q of year y

$F_{j,y,q}$ - fished proportion in quarter q of year y for a fully selected age class a of stock j, by the   directed and

redeye bycatch fisheries

$\tilde{C}_{j,y,0bs}^{S}$ - number (in billions) of juvenile sardine of stock j caught between 1 May and   the day before the start of

the recruit survey

Proportions at age:

$p_{j,y,a}^{S}$ - model predicted proportion-at-age a of stock j in the November survey of year y

$S_{j,a}^{survey}$ - survey selectivity at age a in the November survey for stock j

$p_{j,y,q,a}^{com,S}$ - model predicted proportion-at-age a of stock j in the directed and redeye bycatch commercial catch of

quarter q of year y

Recruitment:

h_{j}^{S} - "steepness" of the stock-recruitment relationship for stock j

K_{j}^{S} - carrying capacity for stock j

K_{peak}^{S} - carrying capacity during peak years (only for single stock hypothesis)

a_{j}^{S} - maximum recruitment of stock j in the hockey stick model;

b_{j}^{S} - spawner biomass for stock j below which the expectation for recruitment is reduced below the maximum

c^{S} - constant recruitment (distribution median) during the "peak" years of 2000 to 2004 (only for single stock

hypothesis)

$\varepsilon_{j,y}^{S}$ - annual lognormal deviation of sardine recruitment.

$\sigma^{S}_{j,r}$ - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve of stock j

$\sigma^{S}_{r,peak}$ - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve during peak years in the single stock hypothesis

Proportions at length and growth curve:

$p^{S}_{j,y,l}$ - model predicted proportion-at-length l of stock j associated with the November survey in year y

$A^{sur}_{j,a,l}$ - proportion of sardine of age a of stock j that fall in the length group l in November

$p^{coml,S}_{j,y,q,l}$ - model predicted proportion-at-length l of stock j in the directed and redeye bycatch commercial catch of quarter q of year y

$A^{com}_{j,q,a,l}$ - proportion of sardine of age a of stock j that fall in the length group l in quarter q

$L_{j,\infty}$ - maximum length of sardine of stock j

κ_{j} - annual growth rate of sardine of stock j

$t_{0,j}$ - age at which the length of sardine of stock j is zero

$\vartheta_{j,a}$ - standard deviation about the mean length for age a of sardine of stock j

Likelihoods:

$-\ln L^{Nov}$ - contribution to the negative log likelihood from the model fit to the November 1+ biomass data

$-\ln L^{rec}$ - contribution to the negative log likelihood from the model fit to the May recruit data

$-\ln L^{sur\ propl\ \min}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data for the minus length class only

$-\ln L^{sur\ propl}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data for the minus length class only

$-\ln L^{com\ propl}$ - contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data for the remaining length classes

$\hat{B}^{S}_{j,y}$ - acoustic survey estimate (in thousands of tonnes) of adult sardine biomass of stock j from the November survey in year y

$\sigma^{S}_{j,y,Nov}$ - survey sampling CV associated with $\hat{B}^{S}_{j,y}$ that reflects survey inter-transect variance

$k^{S}_{j,N}$ - constant of proportionality (multiplicative bias) associated with the November survey of stock j

k^{S}_{ac} - multiplicative bias associated with the acoustic survey

$\hat{N}^{S}_{j,y,r}$ - acoustic survey estimate (in billions) of sardine recruitment numbers of stock j from the recruit survey in year y

$\sigma^{S}_{j,y,rec}$ - survey sampling CV associated with $\hat{N}^{S}_{j,y,r}$ that reflects survey inter-transect variance

$k_{j,r}^{S}$ - constant of proportionality (multiplicative bias) associated with the recruit survey of stock j

k_{cov}^{S} - multiplicative bias associated with the coverage of the recruits by the recruit survey compared to the 1+ biomass by the November survey

$k_{\text{cov}E}^{S}$ - multiplicative bias associated with the coverage of the east stock recruits by the recruit survey compared to the west stock recruits during the same survey

ϕ_{ac}^{S} - the CV associated with factors which cause bias in the acoustic survey estimates and which vary inter-annually;

$(\lambda_{j,N/r}^{S})^{2}$ - additional variance (over and above $\sigma_{y,Nov/rec}^{S}$ and ϕ_{ac}^{S}) associated with the November/recruit surveys of stock j;

$\hat{p}_{j,y,l}^{S}$ - observed proportion (by number) of sardine from stock j in length group l in the November survey of year y;

$w_{propl\,\min}^{sur}$ - weighting applied to the survey proportion at length data for the minus length class;

w_{propl}^{sur} - weighting applied to the remaining survey proportion at length data;

$\sigma_{j}^{S,surl\,\min}$ - variance-related parameter for the log-transformed survey proportion-at-length data for the minus length class;

$\sigma_{j}^{S,surl}$ - variance-related parameter for the log-transformed survey proportion-at-length data;

$\hat{p}_{j,y,q,l}^{S,coml}$ - observed proportion (by number) of the directed and redeye bycatch commercial catch in length group l of during quarter q of year y;

w_{propl}^{com} - weighting applied to the commercial proportion at length data

$\sigma_{j}^{S,coml}$ - variance-related parameter for the log-transformed commercial proportion-at-length data

Other:

F_{init} - rate of fishing mortality assumed in the initial year

$s_{j,cor}^{S}$ - recruitment serial correlation for stock j

$\eta_{j,2009}^{S}$ - standardised recruitment residual value for 2009 for stock j

$\overline{w}_{j,a}^{S}$ - mean mass (in grams) of sardine of age a from stock j sampled during each November survey, averaged over all years

$w_{j,y,a}^{catch}$ - mean mass (in grams) in the catch of sardine of age a from stock j in year y (from de Moor *et al*. 2012a).

Appendix D: Sardine Growth Estimation

by Deon Durholtz

Age data

Otoliths were collected from sardine during November spawner biomass surveys. Each fish (of known length) was assigned to an age group based on the number of putative annual growth increments visible in the otolith. Fish otoliths, when viewed with a stereo microscope using reflected light against a dark background display patterns of alternating "light" (opaque) and "dark" (hyaline) rings/zones. An opaque and the following hyaline zone are assumed to reflect a year of growth (i.e. an annual growth increment), and the count of the total number of completed hyaline zones are therefore assumed to reflect the age of the fish at an annual resolution. It is important to note that this approach only allows the assignment of a fish to an age group, and is not an estimate of the true age of the fish. For example, a fish with no hyaline zones visible in the otolith will be assigned to the 0+ age group, indicating the fish is in its first year of growth. The true age of the fish could be anywhere between 0 and just less than 1 year. Similarly, a fish with one hyaline zone visible in its otolith will be assigned to the 1+ age group, and its true age could be anywhere between 1 and just less than 2 years (i.e. the fish is in its second year of growth). A source of error that manifests at this stage (particularly in cases where the hyaline zone is on the edge of the otolith) is the subjective interpretation of whether or not the most recent (outermost) hyaline zone is completed or not, as this will influence to which age group the fish will be assigned. Generally, it is only possible to establish that the outermost hyaline zone is complete when the following opaque zone is already being deposited. In cases where this cannot be reliably established (i.e. when the outermost hyaline zone is near or on the edge of the otolith), the fish may be assigned to the incorrect age group. This source of error is particularly apparent in young fish, and often results in relatively large fish being assigned to the 0+ age group when they should in fact be assigned to the 1+ age group. Interpretation of otolith structure to obtain an estimate of the number of annual increments in each otolith was conducted by one reader (Cynthia Mtengwane).

Growth modeling

The age data collected from the samples was assigned to either the "west" stock (samples collected in strata A – C) or the "south" stock (samples collected in strata D and E). The numbers of samples for each stock for each year:

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YEAR	ALL	WEST	SOUTH
1993	560	255	305
1994	138	138	0
1996	338	174	164
2001	564	284	280
2003	142	87	55
2004	327	137	190
2006	396	216	180
2007	176	96	80
2008	227	127	100
2009	507	288	219
2010	527	135	392

Von Bertalanffy growth models were fitted the size-at-age data using the "Solver" facility of MS Excel. Essentially, the L_∞, k and t_0 parameters of the von Bertalanffy growth function are estimated by an iterative process that minimizes the sum of squares of the residuals. Growth models were fitted to the "south" and "west" data separately (the two-stock hypothesis), or to all data combined (one-stock hypothesis). The parameter estimates obtained from the model fitting procedure:

Parameter	ALL	WEST	SOUTH
L_∞	19.928	19.458	20.579
k	0.511	0.601	0.466
t_0	-1.893	-1.650	-2.093

Appendix E: Calculation of Annual Total Proportion Fished and Loss to Predation of Sardine

The assessment model assumes catch is taken in four pulses during the year. For simplicity, this catch is totalled and assumed to be taken mid-year when calculating the loss of sardine to predation. The loss in numbers of age a of stock j in year y is calculated by:

$$P_{j,y,a}^{S} = N_{j,y-1,a-1}^{S}\left(1-e^{-M_{a-1,y}^{S}/2}\right) + \left(N_{j,y-1,a-1}^{S}e^{-M_{a-1,y}^{S}/2} - C_{j,y,a-1}^{S}\right)\left(1-e^{-M_{a-1,y}^{S}/2}\right) \qquad y=y_1,\ldots,y_n$$

$$P_{j,y,5+}^{S} = N_{j,y-1,4}^{S}\left(1-e^{-M_{4,y}^{S}/2}\right) + \left(N_{j,y-1,4}^{S}e^{-M_{4,y}^{S}/2} - C_{j,y,4}^{S}\right)\left(1-e^{-M_{4,y}^{S}/2}\right)$$
$$+ N_{j,y-1,5+}^{S}\left(1-e^{-M_{5+,y}^{S}/2}\right) + \left(N_{j,y-1,5+}^{S}e^{-M_{5+,y}^{S}/2} - C_{j,y,5+}^{S}\right)\left(1-e^{-M_{5+,y}^{S}/2}\right) \qquad y=y_1,\ldots,y_n$$

Where $C_{j,y,a}^{S} = \sum_q C_{j,y,q,a}^{S}$.

The loss in biomass of fish of age a of stock j to predation in year y is therefore given by:

$$P_{j,y,a}^{S} = \left[N_{j,y-1,a-1}^{S}\left(1-e^{-M_{a-1,y}^{S}/2}\right) + \left(N_{j,y-1,a-1}^{S}e^{-M_{a-1,y}^{S}/2} - C_{j,y,a-1}^{S}\right)\left(1-e^{-M_{a-1}^{S}/2}\right)\right]\frac{1}{2}\left(w_{j,y-1,a-1} + w_{j,y,a}\right)$$
$$y=y_1,\ldots,y_n$$

$$P_{j,y,5+}^{S} = N_{j,y-1,4}^{S}\left(1-e^{-M_{4,y}^{S}/2}\right) + \left(N_{j,y-1,4}^{S}e^{-M_{4,y}^{S}/2} - C_{j,y,4}^{S}\right)\left(1-e^{-M_{4,y}^{S}/2}\right)\frac{1}{2}\left(w_{j,y-1,4} + w_{j,y,5+}\right)$$
$$+ N_{j,y-1,5+}^{S}\left(1-e^{-M_{5+,y}^{S}/2}\right) + \left(N_{j,y-1,5+}^{S}e^{-M_{5+,y}^{S}/2} - C_{j,y,5+}^{S}\right)\left(1-e^{-M_{5+,y}^{S}/2}\right)\frac{1}{2}\left(w_{j,y-1,5+} + w_{j,y,5+}\right)$$
$$y=y_1,\ldots,y_n$$

The assumption is made that $w_{j,1983,a} = w_{j,1984,a}$, $a=0,\ldots,5+$.

The total loss in sardine biomass of stock j to predation in year y is then given by:

$$P_{j,y}^{S} = \sum_{a=1}^{5+} P_{j,y,a}^{S}.$$

The sardine biomass mid-way through the year is given by:

$$B_{j,y,a}^{Mid-year} = N_{j,y-1,a-1}^{S}e^{-M_{a-1,y}^{S}/2}\frac{w_{j,y-1,a-1}^{S} + w_{j,y,a}^{S}}{2} \qquad y=1984,\ldots,1998, \ a=1,\ldots,4$$

$$B_{j,y,5+}^{Mid-year} = N_{j,y-1,4}^{S}e^{-M_{4,y}^{S}/2}\frac{w_{j,y-1,4}^{S} + w_{j,y,5}^{S}}{2} + N_{j,y-1,5}^{S}e^{-M_{5,y}^{S}/2}\frac{w_{j,y-1,5}^{S} + w_{j,y,5}^{S}}{2} \qquad y=1984,\ldots,1998$$

The annual total proportion fished (catch/biomass) is thus given by:

$$F_{j,y}^{S} = \frac{\sum_{a=0}^{5+} C_{j,y,a}^{S} \times \frac{1}{2}\left(w_{j,y-1,a-1}^{S} + w_{j,y,a}^{S}\right)}{\sum_{a=1}^{5+} B_{a}^{Mid-year}}.$$