Assessment of the South African sardine resource using data from 1984-2011: results for a two stock hypothesis at the posterior mode

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Introduction

As part of the process of updating the assessment of the South African sardine resource, a model of a sardine two mixing-stock hypothesis has been developed. This hypothesis postulates a "west" stock distributed west of Cape Agulhas and a "south" stock distributed south-east of Cape Agulhas, with movement from the "west" to the "south" stock in November as recruits age to 1 year olds. de Moor and Butterworth (2012a, 2013) presented some initial results for a two mixing-stock model.

Work on modelling this hypothesis has continued to try to ascertain in which areas the model may be over-parameterised and which "unimportant" parameters can be fixed at constant values without unduly influencing key results. Further testing of model assumptions has also been undertaken. In this document further results for the model of a two sardine mixing-stock hypothesis are presented and a base case operating model is proposed.

Population Dynamics Model

The generalised operating model for the South African sardine resource, which can apply to either the single or two mixing-stock hypotheses, is detailed in Appendix A, and the data used in this assessment are listed in de Moor *et al*. (2012). The particular difference when fitting the two-stock model to the data compared to the single-stock model of de Moor and Butterworth (2012b) is that both abundance index and proportions-at-length data are divided west and south of Cape Agulhas, and the negative log likelihoods include terms for each of these spatially separate components. A glossary of terms used in this model is repeated from de Moor and Butterworth (2012b) in Appendix B of this document for ease of reference.

Key differences in the model used to calculate the results presented in this document compared to those of de Moor and Butterworth (2013) are as follows:

- A hockey-stick stock-recruitment curve is estimated for the "west" stock within the model, with a fixed variance about the curve.

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- The slope of the hockey-stick stock-recruitment curve for the "south" stock is estimated (previously fixed at $b_2^S / K_2^S = 0.2$).
- Initial (November 1983) numbers at ages 0, 1 and 2 are estimated for the west stock and at age 0 for the south stock. The remaining numbers at age are calculated from natural mortality and an estimated initial fishing mortality parameter. The initial fishing mortality parameter now differs by stock.
- The likelihoods used to condition the model to the May recruitment and November biomass data been modified to "robustify" the likelihood against undue influence from any extreme (outlying) values for residuals (see equation A.26 in Appendix A).
- The November stock weight-at-age is estimated using equation (A.6), but the average of these weights is fixed at the estimated average to obtain convergence more easily.

Other features of the proposed base case model which remain unchanged from de Moor and Butterworth (2012a) include:

- Hockey stick stock recruitment relationships are assumed for the "south" stock and now also for the "west" stock, in line with the choice for the current base case single stock hypothesis.
- In line with the current base case single stock hypothesis, the juvenile and adult natural mortality rates are assumed time invariant, i.e. $\varepsilon_y^{ad} = \varepsilon_y^j = 0$, with median values for these two mortality rates of 1.0year^{-1} and 0.8year^{-1}, respectively.
- The additional variance parameters, $(\lambda_{j,N/r}^S)^2$, are fixed = 0. (This has little impact on the results.)
- Annual movement is estimated from 1994 to 2011 only, corresponding to years for which survey estimates of recruitment exist for the "south" stock, and set equal to zero in earlier years.
- The two variance parameters in the commercial selectivity-at-length function are set the same for both stocks.
- The age at which the length of sardine of is zero, $t_{0,j}$, and the product of the maximum length and annual growth rate of the stocks, $\kappa_j L_{j,\infty}$, are set to be the same for both stocks.

<u>Stock recruitment relationship</u>

The following alternative stock recruitment relationships have been considered (Table 1):

S^{noSR} – hockey stick stock-recruitment curve for the "south" stock, with uniform priors on the log of the maximum recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity, and no relationship for the "west" stock (similar to de Moor and Butterworth 2013)

S^{HS} – hockey stick stock-recruitment curve for the "west" and "south" stocks, with uniform priors on the log of the maximum recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity

S^{BH} – Beverton Holt stock-recruitment curve, with uniform priors on steepness and carrying capacity

Natural mortality

A number of combinations of juvenile and median adult natural mortality values are tested, covering the range 0.6 to 1.2 year^{-1}, for the case where a Hockey Stick stock recruitment relationship is assumed for both the "west" and "south" stocks. For realism, only combinations with $\overline{M}_j^S \geq \overline{M}_{ad}^S$ were tested.

Three alternatives were tested which allowed juvenile and/or adult natural mortality to change in 2000; with the amount of change being an estimable parameter. Eight alternatives were tested which allowed "south" stock juvenile or adult natural mortality to be ±0.1 or ±0.2 that of the "west" stock.

Robustness Tests

The base case hypothesis estimates the initial (November 1983) numbers at ages 0, 1, and 2 for the "west" stock and initial numbers at ages 3+ are calculated from natural mortality and an estimated initial "west" stock fishing mortality. For the "south" stock, only age 0 is estimated and ages 1+ are calculated from natural mortality and an estimated initial "south" stock fishing mortality (see the Estimable Parameters section of Appendix A). Alternatives tested included:

S^{init1} – only ages 0 and 1 are estimated independently for "west" stock

S^{init2} – only age 0 is estimated independently for "west" stock

S^{init3} – ages 0 and 1 are estimated independently for the "south" stock

S^{init4} – ages 0, 1 and 2 are estimated independently for the "south" stock

Alternative assumptions about the commercial selectivity-at-length were tested as follows:

S^{sel1} – "west" stock commercial selectivity-at-length was modelled to change in 2000

S^{sel2} – S^{sel1} assumptions, and the "south" stock commercial selectivity-at-length was modelled to change in 1998

S^{sel3} – S^{sel2} assumptions and "south" stock commercial selectivity-at-length was modified to exclude the small normal distribution: $S_{j=2,y,l} = \begin{cases} 0 & l \leq 5 \\ \exp\left\{-\dfrac{\left[\ln\left(\left(l_{mid} - l_{\max}\right)/\left(\bar{l}_{2,j} - l_{\max}\right)\right)\right]^2}{\left(\sigma_{2,j=2}^{sel}\right)^2}\right\} & 6 \leq l \leq 40 \end{cases}$

The base case hypothesis assumes the growth curves for the "west" and "south" stocks differ only in their $L_{j,\infty}$ values. Alternatives tested included:

S[growth1] – the same $L_{j,\infty}$ for both "west" and "south" stocks

S[growth2] – $t_{0,j}$ differs between "west" and "south" stocks

S[growth3] – $\kappa_j \times L_{j,\infty}$ differs between "west" and "south" stocks

S[growth4] – $\vartheta_{j,a}$ differs between "west" and "south" stocks

The base case hypothesis assumes a fixed variability about the "west" stock recruitment relationship ($\sigma^S_{j=1,r} = 0.5$). Alternatives tested included:

S[sigma1] – $\sigma^S_{j=1,r} = 0.4$

S[sigma2] – $\sigma^S_{j=1,r} = 0.6$

S[sigma3] – $\sigma^S_{j=1,r} = 0.7$

S[sigma4] – estimate $\left(\sigma^S_{j=1,r}\right)^2 \sim U(0.16,10)$

To test the robustness of the results to the estimated variability about the "south" stock recruitment relationship, the alternative assumed was:

S[sigma5] – $\sigma^S_{j=2,r} = 0.5$

Results and Discussion

Natural mortality

Table 2 lists the various contributions to the objective function at the posterior mode for the full range of combinations of juvenile and adult natural mortality tested. Given the choice of prior distributions, the ratio $k^S_{j,r} / k^S_{j,N}$ is by definition less than 1. All alternative combinations of time-invariant or changing natural mortality result in $k^S_r / k^S_N \geq 0.5$ i.e. within the plausible range.

There is little change in the joint posterior mode as $\overline{M}^S_j$ is changed for a given $\overline{M}^S_{ad}$ (<1.2 objective function points, improving as $\overline{M}^S_j$ decreases). Given $\overline{M}^S_j$, the posterior indicated a worse fit to the data for both increasing and decreasing $\overline{M}^S_{ad}$ away from 0.8. The lowest values for the negative log posterior mode were obtained for $\overline{M}^S_{ad} = 0.8$, with $\overline{M}^S_j = 0.8$ and $\overline{M}^S_j = 1.0$. To maintain consistency with previous assessments, the base case hypothesis currently assumes $\overline{M}^S_j = 1.0$ and $\overline{M}^S_{ad} = 0.8$.

Allowing either the juvenile or adult natural mortality (but not both) to change in 2002 results in a better fit (Table 2), with either an estimated decrease in juvenile natural mortality (to $\overline{M}^S_j = 0.6$) after 2002 or an estimated increase in adult natural mortality (to $\overline{M}_{ad}^{S}=1.06$). However, satisfactory convergence to the joint posterior mode has not been attained. The scenario where $\overline{M}_{j}^{S}$ is estimated to change in 2002 results in $\overline{M}_{j}^{S}<\overline{M}_{ad}^{S}$ from 2002 onwards which is considered unrealistic. The option where $\overline{M}_{ad}^{S}=1.06$ is estimated to change in 2002 also results in $\overline{M}_{j}^{S}<\overline{M}_{ad}^{S}$, though the difference is not as large.

There is little change in the joint posterior mode if the "south" stock juvenile natural mortality differs by ±-0.2 from that of the "west" stock (Table 2), and only a small improvement in the joint posterior mode if the "south" stock adult natural mortality is 0.1year^{-1} less than that of the "west" stock.

Stock recruitment relationship

Table 3 lists the various contributions to the negative log posterior pdf at the posterior mode for the alternative stock-recruitment relationships considered, as well as estimated parameter values and other key outputs at these posterior modes.

Model selection criteria slightly preferred the Hockey stick to the Beverton Holt stock recruitment relationship (Table 5).

Although the model which assumes no "west" stock recruitment relationship results in a better fit to the model, and thus a lower (i.e. better) AIC, the model with a hockey stick stock recruitment relationship has fewer "effective" parameters as the deviates about the stock recruitment curve are not independent parameters, but in a frequentist model would be considered "random effects". MMIC could be used to compare models with "random effects" (Cooke et al. 2003), but this method requires the posterior variance for each estimated parameter. S^{noSR} has not adequately converged to the joint posterior mode and thus an MMIC value could not be calculated for S^{noSR}, for comparison with S^{HS}. Comparison with an earlier model that also assumed no "west" stock recruitment relationship (de Moor and Butterworth 2013) indicates that MMIC model selection criteria would prefer S^{HS} to S^{noSR} because of the lower degrees of freedom of the "random effects" (Table 5). If S^{noSR} were chosen as a base case hypothesis, a stock recruitment relationship would need to be fit to the model outputs for use in future projections of the resource. Thus SHS was chosen as a preferred base case hypothesis.

The alternative stock recruitment relationships are plotted in Figure 1.

Base case (S_{HS}) results at the posterior mode

The population model fits to the time series of abundance estimates of November 1+ biomass are reasonable, though there is an under-prediction of the peak in the "south" stock in the early 2000s

(Figure 2). There is an associated series of positive residuals in the model fit to the “south” stock November 1+ biomass in the 1990-2000s. The corresponding model fits to the time series of May recruitments are plotted in Figure 3. The residual for the “south” stock in 2001 has less influence since de Moor and Butterworth (2013) due to the change in equation (A.26) to “robustify” the likelihood against outliers. A larger than observed recruitment is required by the model as a non-negligible 0-year-old catch occurred to the east of Cape Agulhas before November 2001.

As was the case for the single stock hypothesis, the model under-predicts recruitment to the “west” stock in May 2010 as it is unable to reconcile the conflicting data of an above average recruitment estimate in May 2010 with almost no increase in the November 1+ biomass estimate from 2009 to 2010. The bias associated with the hydroacoustic survey is estimated to be similar to that estimated by the single stock assessment (0.75 compared to 0.72), while the coverage of the May recruit survey in comparison to that of the November survey is estimated to be 67% (compared to 54% estimated for the single stock hypothesis).

The estimated initial (November 1983) fishing mortality on the “west” and “south” stocks is 0.50 and <0.001, respectively. This large difference corresponds to the observed negligible catch from the “south” stock in the early years of the time series of data, and results in a more stable estimation of the initial model parameters at the joint posterior mode than when a single initial fishing mortality was estimated for both stocks (<0.001, de Moor and Butterworth 2013).

The coverage of the recruits to the “south” stock relative to that for the “west” stock is estimated to be 100%, a large increase from the most recent estimate of 26% (de Moor and Butterworth 2013) and the first such estimate of 76% (de Moor and Butterworth 2012a). This estimate is also robust to the choice of “west” stock recruitment relationship (see S^{noSR}, Table 3). The multiplicative bias associated with the May survey estimate of recruitment is thus 0.50 for both the “west” and “south” stocks, compared to 0.39 estimated for the single stock hypothesis. This change has seemingly resulted primarily from the trade off between this bias parameter and the median maximum recruitment for the “south” stock. A lower maximum recruitment is now estimated (see Table 3, Figure 4 and immediately following paragraphs), but a greater proportion of this recruitment is assumed to be estimated by the May hydroacoustic survey.

The model estimated November recruitment is plotted in Figure 4 against Spawning Stock Biomass. This model now estimates separate “hockey stick” stock recruitment relationships for both the “west” and “south” stocks. The “west” stock is clearly estimated to be a far more productive stock than the “south” stock, with a median maximum recruitment estimated at 73 billion compared to 2.4 billion for the “south” stock at the joint posterior mode (Table 3). The estimated median maximum recruitment for the “south” stock has more than halved compared to de Moor and Butterworth (2013), but as mentioned above, a greater proportion of this recruitment is now estimated to be covered by the May survey. However, recruitment for the "west" stock is impaired as SSB decreases below a threshold (with a slope of $b_1^S / K_1^S = 0.2$), while this model estimates little impairment to median "south" stock recruitment at low SSB ($b_2^S / K_2^S = 0.04$).

Variability about the "west" stock recruitment curve is fixed ($\sigma_{j=1,r}^S = 0.5$), as the model is unable to accurately estimate this parameter (see alternatives below). Variability about the "south" stock recruitment curves is estimated at the lower bound of the prior distribution (Table 3). Most notably, the three highest years of survey estimated recruitment to the "south" stock are substantially under-predicted by this lognormal likelihood model (Figure 3).

More than 50% of the "west" stock recruits are modelled to move to the "south" stock in 9 out of 18 years, with the greatest movement between stocks in biomass terms occurring from the late 1990's to the early 2000s, and then again at the end of the time series (Figure 5), the former corresponding closely to the years of peak biomass in the "south" stock. The model has been configured to estimate movement of recruits from the "west" stock to the "south" stock only for years for which hydroacoustic estimates of recruitment are first available for the "south" stock, as there is little information to estimate the south-east movement of recruits in November precisely prior to 1994.

The model estimated survey selectivities-at-length, which are restricted to vary from 1 only for lengths contributing to the minus and plus length classes (see Appendix A for details), are shown in Figure 6. The residuals from the model fits to the survey proportions-at-length are given in Figure 7. Figure 8 shows the average (over all years) model predicted November survey proportions-at-length. Considering the restriction of survey selectivity to be 1 for all length classes other than the minus and plus length classes, the comparison in Figure 8 of the model predicted to observed averages is relatively good, though the model over-predicts the proportion-at-length in the minus and plus groups for the "west" stock, with the selectivity being as low as the prior will allow for the minus group (0.6; Figure 6), while a few large negative residuals appear to have overly influenced the estimate of the selectivity for the plus group.

The model estimated commercial selectivities-at-length are shown in Figure 9, with a higher selectivity about the lower lengths for the "west" stock than the "south" stock. Some non-random patterns in the residuals from the model fit to the commercial proportion-at-length data are evident (Figure 10), but given the assumption of constant selectivity over time, these are considered to be acceptable (but see below for alternative models). The average (over all years and quarters) model predicted commercial proportions-at-length matches the general pattern observed, although the peak at the higher lengths is under-predicted for both stocks (Figure 11). An attempt to force the model to fit this peak in the average commercial proportions-at-length by increasing the weight in the likelihood on length classes 15-20cm, resulted in a much worse fit to the survey abundance indices (results not shown).

A key factor in the model fits to the proportion-at-length data is the model estimated growth curves (Figure 12) and the variability about this curve (Figure 13).

The proportion of the "west" stock harvested is estimated to have decreased during the first half of the time series, and then increased during the late 1990s to early 2000s, the years over which the population as a whole peaked and high TACs were set (Figure 14). The proportion of the "south" stock harvested has increased since the turn of the century, peaking in the mid-2000s with the harvest proportion being similar for both stocks over 2007-2009 (Figure 14). Although the maximum harvest proportion for the whole population is estimated to have been 0.25 during the period when the population began to decline following the record peak in the early 2000s, if there are indeed two mixing-stocks of sardine, the harvest proportion of the "west" stock is estimated to have reached above 40% in the early 2000s.

West – south movement

In order to avoid possibly biasing the model estimation of the annual proportion of "west" recruits moving to the "south" stock, the model has estimated these proportions using uninformative prior distributions. However, relationships between the estimated proportions and abundance have been investigated for possible use in future projections of the resource.

Results show little relationship between the proportion of recruits moving and the "west" stock November 1+ biomass or May recruitment (Figure 15a,b,e), but some relationship appears to exist between the estimated proportions and the "south" stock November 1+ biomass (Figure 15c,d). The relationship between the proportion moving and the "south" stock November 1+ biomass in the same year is a better fit than that with the biomass in the previous year (Table 4). However, the current year relationship may be more a reflection of the "south" stock November 1+ biomass being dependent on the movement of recruits into the stock, and such a relationship would not be straightforward to use in future projections. The relationship between the proportion moving and the previous year's "south" stock November 1+ biomass may be a reflection of some "south" recruits being surveyed west of Cape Agulhus before completing their return migration to the south coast (i.e. Natal homing, Coetzee pers. comm.). In this case the time series of "west" recruitment data would actually contain "west" and "south" recruits. Though this is possible, such a hypothesis can not be further explored with the current model and would require further data analysis to be conducted in the future.

The final relationship considered is one with the relative “south” stock November 1+ biomass to that of the “west” stock (Figure 15f) and suggests that the proportion moving drops from a maximum as the “south” stock November 1+ biomass relative to that of the “west” stock decreases below a threshold of about 1.5. This could be indicative of entrainment, in which a higher relative “south” stock November 1+ biomass in the previous year (which could be considered a surrogate to the 2+ biomass in the current year) could better “facilitate” the movement of more recruits in the current year (Coetzee pers. comm; ICES 2007). Another possibility is that a higher ratio of “south” to “west” stock November 1+ biomass may be a proxy to indicate improved environmental suitability for both juvenile and adult sardine east rather than west of Cape Agulhas (Coetzee pers. comm.).

The autocorrelation between the estimated annual proportions moving is also high at 0.88.

Initial conditions

Three of the robustness tests concerning initial conditions were slightly preferred to the base case hypothesis by model selection criteria (Table 5). The model with the lowest AIC/MMIC is S^{init1}, in which the initial fishing mortality and natural mortality are used to estimate ages 2+ for the “west” stock. In this case $Finit_{j=1}$ is estimated close to the upper limit of 1, compared to the base case estimate of 0.5 (Table 3). Given the large difference between initial “west” stock numbers at age 1 and 2 estimated for the base case hypothesis, it was decided to rather estimate these parameters separately than use S^{init1} as a baseline. Model selection criteria did not support estimating only age 0 for both the “west” and “south” stocks (S^{init2}, Table 5)

Commercial selectivity

Estimating commercial selectivity-at-length for each year would over-parameterise the model. However the base case model assumption of time-invariant commercial selectivity may be too rigid (Figure 10). Allowing a single change in the commercial selectivity-at-length over the time period slightly improved the pattern in the residuals (Figures 9,16), and resulted in an improved overall fit to the data (Table 5), but convergence to this posterior mode was not attained. Model selection criteria also suggested a preference for S^{sel3}, a model in which the commercial selectivity-at-length on the “south” stock excluded a normally distributed peak at small lengths (Figure 9, Table 5), but again convergence to this posterior mode was not attained.

Growth curves

Model selection criteria only support $S^{growth1}$, in which a single $L_{j,\infty}$ is estimated for both "west" and "south" stocks, in preference to the base case hypothesis (Table 5). However, as the difference in AIC[1] is not large (less than 2) and adequate convergence to the joint posterior mode has not yet been achieved, $S^{growth1}$ has been maintained as a robustness test rather than the base case hypothesis.

<u>Variability about the stock recruitment relationships</u>

Model selection criteria support the alternatives of a higher variability about the "south" and "west" stock recruitment relationship when prior distributions are not included in the calculations (AIC,Table 5), but support a lower variability about the "west" stock recruitment relationship when prior distributions are included in the calculations (MMIC, Table 5). A higher fixed variability for the "west" stock recruitment relationship is a realistic precautionary approach to future simulations in a situation where the best estimate of the extent of variability may give a false impression of the certainty with which future recruitments can be predicted.

Summary

This document has given further results for a two stock hypothesis for the South African sardine resource. A Hockey Stick stock recruitment relationship is assumed for both the "west" and "south" stocks. Furthermore, time-invariant natural mortality, assumed to be the same for both stocks, has been used for the base case hypothesis. These results indicate the "west" stock resource abundance was around 280 thousand tons and the "south" stock resource abundance was 870 thousand tons in November 2011. This "west" stock abundance is well below its long-term average, while the "south" stock abundance is above its long-term average. Seven out of the last eight years have resulted in below average May recruitment for the "west" stock, while the "south" stock has experienced above average recruitment in nine out of thirteen years since 1999.

These results re-interate former results which showed that the "west" stock is substantially more productive than the "south" stock, with the model estimated movement of recruits from the "west" to the "south" stock having a greater impact on the "south" stock biomass than good "south" stock recruitment year classes. A lower median maximum "south" stock recruitment has been estimated by this model compared to earlier results, with a corresponding greater proportion of "south" stock recruits estimated to be covered by the survey. In other words, the model now interprets the survey indices as representing a higher proportion of the actual sardine "south" stock recruits, but this implies a lower actual recruitment of "south" stock sardines. The "south" stock recruitment relationship, however, now assumes that the SSB is required to fall very low before median recruitment is impaired.

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[1] MMIC can only be calculated from output generated when a positive definite Hessian is obtained.

Former results suggested a smaller fraction of the "south" recruits were covered by the May hydroacoustic survey, compared to the "west" recruits. These results were rationalised by the assumption that the winter spawning occurs for the "south" stock, but not for the "west" stock. The model now estimates that the coverage of recruits by the May hydroacoustic survey is similar for both the "west" and "south" stocks.

These results indicate that a relationship between the proportion of recruits moving from the "west" to the "south" stock, and the "south" stock November 1+ biomass may exist. If this is a reflection of natal homing, as proposed above, a possible way to test this hypothesis would be to fit a model to the combined ("west" and "south") survey estimates of recruitment. However, this would require other information (e.g. a movement-biomass relationship) to be available to estimate the annual proportions of recruits moving from "west" to "south".

An alternative relationship to use for future projections of the resource, which was explored above, would be with the relative "south" to "west" stock November 1+ biomass, and may be a indicative of entrainment or the ratio may be a proxy to indicate years of improved environmental suitability for sardines of the area east of Cape Agulhas.

Although the autocorrelation between model estimated annual proportions of "west" recruits moving to the "south" stock is high, use of the above mentioned relationships would likely be more useful and simple for future projections.

Although a better fit to the data was obtained when either juvenile or adult natural mortality was estimated to change in 2002, these options resulted in an unrealistic scenario of $\overline{M}_j^S < \overline{M}_{ad}^S$ from 2002. Further work could consider fixed changes in 2002 which ensure this constraint is not breached.

The results presented in this document are for model results at the joint posterior mode only. A separate document containing full posterior distributions of the chosen base case two sardine stock hypothesis and selected alternative hypotheses will be presented at a later date. As for the single stock hypothesis, Markov Chain Monte Carlo chains with alternative assumptions for fixed variability about the stock recruitment relationship(s) will be run. A single hypothesis for future projection could be drawn equally from more than one of these chains to account for a spread in the assumed stock recruitment variability. Other robustness tests which may prove useful alternative hypotheses to simulation test alternative MPs against would be one that assumes commercial selectivity has changed over time (S^{sel2} and/or S^{sel3}) and one which assumes the same growth curves for both stocks ($S^{growth1}$).

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Table 1. The alternative stock-recruitment relationships considered. The parameter h_j^S denotes the "steepness" of the stock-recruitment relationship for stock j, which is the proportion of the virgin recruitment that is realised at a spawning biomass level of 20% of average pre-exploitation (virgin) spawning biomass K_j^S (shown in units of thousands of tons). For the hockey stick model, $X_j = \sum_{a=1}^{4} \overline{w}_{j,a}^S e^{-M_j^S-(a-1)\overline{M}_{ad}^S} + \overline{w}_{j,5+} e^{-M_j^S-3\overline{M}_{ad}^S} \dfrac{1}{1-e^{-\overline{M}_{ad}^S}}$, where $\overline{w}_{j,a}^S$ is the average of $w_{j,y,a}^S$ as defined in Appendix A. For this same model, a_j^S denotes the maximum recruitment (in billions) and b_j^S denotes the spawner biomass below which the expectation for recruitment is reduced below the maximum.

Test	Stock recruitment relationship	$f\left(SSB_{y,N}^S\right)=$	Parameters
S^{BH}	Beverton Holt	$\dfrac{\alpha_j^S SSB_{j,y}^S}{\beta_j^S + SSB_{j,y}^S}$	$h_j^S \sim U(0.2,1)$ $K_j^S \sim U(0,10\,000)$ $\alpha_j^S = \dfrac{4h_j^S}{5h_j^S-1}\dfrac{K_j^S}{X_j}$ $\beta_j^S = \dfrac{K_j^S(1-h_j^S)}{5h_j^S-1}$
S^{HS}	Hockey stick	$\begin{cases} a_j^S & \text{,if SS}B_{j,y}^S \geq b_j^S \\ \dfrac{a_j^S}{b_j^S} SSB_{j,y}^S & \text{,if SS}B_{j,y}^S < b_j^S \end{cases}$	$\ln(a_j^S) \sim U(0,5.6)$ [2] $b_j^S / K_j^S \sim U(0,1)$ $K_j^S = a_j^S X_j$ [3]

[2] Given the lack of *a priori* information on the scale of a_j^S, a log-scale was used, with a maximum corresponding to about 10 million tons (9.7 million tons for "west" stock and 11.1 million tons for "south" stock).

[3] For consistency, *K* relates throughout to corresponding MLEs, i.e. the approach works with median rather than mean estimates of *K* and thus a bias correction factor for the log-normal distribution is not used. These values for *K* will be less than the corresponding average pre-exploitation levels because of the lognormal distributions assumed for recruitment.

Table 2. The contributions to the objective function at the posterior mode for a range of combinations of juvenile, $\overline{M}_j^S$, and adult, $\overline{M}_{ad}^S$, natural mortality for models assuming the Hockey Stick stock recruitment relationship for both the "west" and "south" stocks. The ratio of the multiplicative bias in the recruit survey to that in the November survey, $k_{j,r}^S / k_{j,N}^S$, is given for diagnostic purposes. Shaded rows represent what are considered unrealistic values for this ratio.

	$\overline{M}_j^S$	$\overline{M}_{ad}^S$	$-\ln L$	$-\ln L^{Nov}$	$-\ln L^{rec}$	$-\ln L^{sur\ propl}$	$-\ln L^{sur\ pr}$	$-\ln L^{com\ p}$	$-\ln prior($	$-\ln prior$	k_{ac}^S	k_{cov}^S	$k_{\mathrm{cov}\,E}^S$	$k_{j,r}^S / k_{j,N}^S$
Time invariant, same both stocks	0.6	0.6	684.89*	47.65	53.09	505.02	5.59	56.44	-1.50	18.59	0.76	0.99	1.0	0.99
	0.8	0.6	685.26*	47.65	53.15	505.23	5.58	56.54	-1.51	18.61	0.75	0.88	1.0	0.88
	0.8	0.8	684.35	47.70	53.50	506.80	5.45	54.08	-1.56	18.39	0.75	0.75	1.0	0.75
	1	0.6	685.63*	47.64	53.22	505.44	5.57	56.63	-1.51	18.65	0.75	0.79	1.0	0.79
	1	0.8	684.79	47.71	53.57	507.05	5.44	54.15	-1.56	18.44	0.75	0.67	1.0	0.67
	1	1	685.57*	47.81	54.01	508.96	5.33	53.03	-1.60	18.03	0.74	0.59	1.0	0.59
	1.2	0.6	686.01*	47.61	53.30	505.65	5.55	56.75	-1.52	18.66	0.75	0.71	1.0	0.71
	1.2	0.8	685.24	47.72	53.64	507.31	5.43	54.23	-1.57	18.48	0.74	0.60	1.0	0.60
	1.2	1	692.53*	48.81	55.77	509.11	5.21	50.30	-1.61	24.95	0.74	0.51	1.0	0.51
	1.2	1.2	710.93*	53.61	57.74	513.09	5.63	53.70	-1.57	28.73	0.75	0.52	1.0	0.52
Change in 2000	1.0+	0.8	682.51*	47.37	53.03	506.82	5.45	53.90	-1.54	17.47	0.75	0.72	1.0	0.72
	1.0	0.8+	682.47*	48.40	54.71	505.07	5.32	50.62	-1.61	19.95	0.74	0.62	1.0	0.62
	1.0+	0.8+	684.74*	48.23	53.63	506.78	5.37	53.16	-1.58	19.15	0.74	0.65	1.0	0.65
Time invariant, differ by stock	1.0; 0.8	0.8	684.96	47.92	53.60	506.98	5.44	54.08	-1.56	18.51	0.75	0.68	1.0	0.68
	1.0; 0.9	0.8	684.87	47.81	53.58	507.01	5.44	54.12	-1.56	18.47	0.75	0.68	1.0	0.68
	1.0; 1.1	0.8	684.73	47.62	53.56	507.09	5.44	54.19	-1.56	18.40	0.75	0.67	1.0	0.67
	1.0; 1.2	0.8	684.69	47.53	53.55	507.13	5.44	54.23	-1.56	18.37	0.75	0.66	1.0	0.66
	1.0	0.8; 0.6	685.13*	47.85	53.44	506.18	5.58	55.26	-1.54	18.36	0.75	0.73	1.0	0.73
	1.0	0.8;0.7	684.65	47.74	53.44	506.57	5.50	54.53	-1.55	18.42	0.75	0.70	1.0	0.70
	1.0	0.8;0.9	685.38*	47.71	53.79	507.54	5.38	54.12	-1.57	18.41	0.74	0.65	1.0	0.65
	1.0	0.8;1.0	686.27*	47.70	54.07	508.02	5.32	54.39	-1.58	18.34	0.74	0.63	1.0	0.63

* indicates satisfactory convergence to the posterior mode has not yet been achieved.

\+ indicates the natural mortality component that was estimated to change in 2002.

Table 3. Key model parameter values and model outputs estimated at the joint posterior mode. Results are given for S^{HS} as estimated by de Moor and Butterworth (2013), S^{HS} as estimated in this document and S^{noSR}. Values fixed on input are given in **bold**. Numbers are reported in billions and biomass in thousands of tonnes. $j=1$ denotes the "west" stock and $j=2$ denotes the "south" stock.

Parameter	S^{HS} (Feb13)	S^{HS} (Aug13)	S^{noSR}	Parameter	S^{HS} (Feb13)	S^{HS} (Aug13)	S^{noSR}
$-\ln(posterior)$	678.5	684.8	662.1	$a^S_{j=1}$	-	73.0	-
$-\ln L^{Nov}$	50.6	47.7	46.2	$a^S_{j=2}$	5.4	2.4	2.3
$-\ln L^{rec}$	59.0	53.6	51.9	$b^S_{j=1}$	-	540	-
$-\ln L^{coml\ propl}$	506.6	507.1	505.2	$b^S_{j=2}$	43	0.4	0.4
$-\ln L^{sur\ propl\ \min}$	5.4	5.4	5.3	$K^S_{j=1}$	-	2637	-
$-\ln L^{surl\ propl}$	49.2	54.2	52.1	$K^S_{j=2}$	216	97	36
$-\ln(priors)$	7.7	16.9	1.5	$\sigma^S_{j=1,r}$	-	**0.50**	**-**
$\overline{M}^S_j$	**1.0**	**1.0**	**1.0**	$\sigma^S_{j=2,r}$	0.40	0.40	0.40
$\overline{M}^S_{ad}$	**0.8**	**0.8**	**0.8**	$\overline{B}^S_{Nov}$ [4]	585	534	546
$k^S_{j=1,N}=k^S_{ac}$	0.79	0.75	0.75	$\overline{B}^S_{j=1,Nov}$	427	417	444
$k^S_{j=2,N}=k^S_{ac}$	0.79	0.75	0.75	$\overline{B}^S_{j=2,Nov}$	158	107	102
k^S_{cov}	0.76	0.67	0.66	$\eta^S_{j=1,2009}$	-	-0.13	
$k^S_{\text{cov}\,E}$	0.26	1.00	1.00	$\eta^S_{j=2,2009}$	-0.05	-0.01	
$k^S_{j=1,r}$	0.60	0.50	0.49	$s^S_{j=1,cor}$	-	0.29	
$k^S_{j=2,r}$	0.16	0.50	0.49	$s^S_{j=2,cor}$	-0.19	0.32	
$k^S_{j=1,r}/k^S_{j=1,N}$	0.76	0.67	0.66	$L_{j=1,\infty}$	19.0	19.1	19.0
$k^S_{j=2,r}/k^S_{j=2,N}$	0.20	0.67	0.66	$L_{j=2,\infty}$	19.6	19.7	19.8
$\left(\lambda^S_{j=1,N}\right)^2$	**0.00**	**0.00**	**0.00**	$\kappa_{j=1}$	1.20	1.22	1.22
$\left(\lambda^S_{j=2,N}\right)^2$	**0.00**	**0.00**	**0.00**	$\kappa_{j=2}$	1.17	1.18	1.17
$\left(\lambda^S_{j=1,r}\right)^2$	**0.00**	**0.00**	**0.00**	t_0	0.12	0.11	0.11
$\left(\lambda^S_{j=2,r}\right)^2$	**0.00**	**0.00**	**0.00**	ϑ_0	3.0	3.0	3.0
$N^S_{j=1,1983,0}$	**5.00**	5.56	3.95	ϑ_1	2.5	2.7	2.7
$N^S_{j=1,1983,1}$	**2.60**	2.43	1.32	ϑ_2	1.8	1.8	1.8

[4] This average it taken over 1991 to 1994. OMP-04 and OMP-08 were developed using Risk defined as "the probability that 1+ sardine biomass falls below the average 1+ sardine biomass between November 1991 and November 1994 at least once during the projection period of 20 years". Results are also presented for the individual "west" and "south" stocks, though the risk threshold to be used for a 2 stock hypothesis is yet to be finalised.

Table 3 (continued).

Parameter	S^{HS} (Feb13)	S^{HS} (Aug13)	S^{noSR}	Parameter	S^{HS} (Feb13)	S^{HS} (Aug13)	S^{noSR}
$N^S_{j=1,1983,2}$	**<0.001**	<0.001	<0.001	ϑ_3	1.8	1.8	1.8
$N^S_{j=2,1983,0}$	**0.001**	0.010	0.011	ϑ_4	1.8	1.8	1.8
$N^S_{j=2,1983,1}$	**0.022**	0.004	0.004	ϑ_{5+}	1.8	1.8	1.8
$N^S_{j=2,1983,2}$	**0.012**	0.002	0.002	$Finit_{j=1}$	-	0.50	0.50
Finit	<0.001	-		$Finit_{j=2}$	-	<0.001	<0.001

Table 4. The estimated relationships between model predicted proportion of "west" stock recruits moving to the "south" stock, $\hat{p}_y$, and abundance (X): $\hat{p}_y = a\left(1 - e^{-bX_y}\right)$.

Relationship	a	b	Residual standard deviation
$X = B^S_{j=1,y-1}$	0.523	0.005	0.24
$X = B^S_{j=1,y}$	0.471	0.381	0.25
$X = B^S_{j=2,y-1}$	0.634	0.004	0.18
$X = B^S_{j=2,y}$	0.685	0.002	0.16
$X = N^S_{j=1,y,r}$	0.524	0.141	0.24
$X = B^S_{j=2,y-1} \big/ B^S_{j=1,y-1}$	0.603	2.245	0.21

Table 5. Number of model parameters, and number of “random effect” parameters ($\varepsilon_{j,y}^{S}$ in this model), contributions to the negative log posterior at the estimated joint posterior mode, and the difference in the model selection criteria AIC and MMIC[5], where AIC for S^{HS} is 1545.8 and MMIC for S^{HS} is 1470.4.

	Total parameters	“Random effect” parameters	$-\ln L$	$-\ln L^{Nov}$	$-\ln L^{rec}$	$-\ln L^{sur\ propl\ min}$	$-\ln L^{sur\ propl}$	$-\ln L^{com\ propl}$	$-\ln prior(k_{ac}^{S})$	$-\ln prior(\varepsilon_{j,y}^{S})$	ΔAIC[6]	ΔMMIC[7]
S^{HS} (Feb13)	97	27	678.47	50.60	59.02	506.56	5.38	49.16	-1.20	8.95	-10.4	2.2
S^{HS}	105	54	684.79	47.71	53.57	507.05	5.44	54.15	-1.56	18.44		
S^{noSR}	103	27	662.12	46.19	51.92	505.16	5.26	52.08	-1.55	3.06	-18.6	-
S^{BH}	105	54	685.76	50.13	52.59	508.45	5.54	52.45	-1.53	18.14	2.5	0.9
S^{init1}	104	54	685.13	47.68	53.57	507.36	5.43	54.22	-1.56	18.43	-1.3	-1.4
S^{init2}	103	54	745.05*	67.47	77.83	506.37	4.79	54.28	-1.58	35.90	81.6	$^{+}$
S^{init3}	106	54	683.26	47.96	53.57	506.98	5.40	52.35	-1.56	18.56	-1.3	-0.9
S^{init4}	107	54	683.12	48.02	53.56	506.83	5.40	52.28	-1.56	18.59	0.3	0.8
S^{sel1}	110	54	674.04*	48.01	53.75	495.77	5.50	53.73	-1.55	18.84	-12.3	$^{+}$
S^{sel2}	113	54	662.96*	46.81	53.75	484.60	5.63	54.53	-1.58	19.22	-29.2	$^{+}$
S^{sel3}	109	54	663.82*	46.81	53.75	485.48	5.63	54.52	-1.58	19.22	-35.5	$^{+}$
$S^{growth1}$	104	54	685.01*	47.94	53.61	506.79	5.47	54.28	-1.53	18.46	-1.7	$^{+}$
$S^{growth2}$	106	54	684.25*	47.96	53.57	506.65	5.49	53.65	-1.56	18.50	0.8	$^{+}$
$S^{growth3}$	106	54	683.83*	47.88	53.55	506.25	5.52	53.71	-1.56	18.49	0.0	$^{+}$
$S^{growth4}$	108	54	683.02*	48.04	53.60	506.33	5.62	52.20	-1.57	18.79	1.8	$^{+}$
S^{sigma1}	105	54	682.76*	48.37	55.79	507.72	5.45	55.11	-1.54	11.86	9.0	$^{+}$
S^{sigma2}	105	54	686.36	47.25	51.93	506.54	5.42	53.49	-1.57	23.31	-6.6	7.4
S^{sigma3}	105	54	687.86	46.98	50.67	506.10	5.40	52.98	-1.58	27.31	-11.6	14.1
S^{sigma4}	106	54	682.76*	48.38	55.79	507.72	5.45	55.10	-1.54	11.86	11.0	$^{+}$
S^{sigma5}	104	54	689.26	47.13	52.15	507.14	5.43	53.77	-1.56	25.20	-6.6	10.3

* indicates satisfactory convergence to the posterior mode has not yet been achieved.

$^{+}$ MMIC can only be calculated from output generated when a positive definite Hessian is obtained.

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[5] Mixed Model Information Criterion (Cooke et al. 2003)

[6] AIC = -2lnL+2n, where L is taken to be the Likelihood

[7] MMIC = -2lnL + 2Σd_i, where L is taken to be the Posterior, d_i is the effective degrees of freedom associated with fitted effect (parameter) i; d_i = $1-\mathrm{var}(\hat{x}_i)/\sigma^2$, where σ^2 is the collective prior variance for the effects x_i and $\mathrm{var}(\hat{x}_i)\leq\sigma^2$ is the posterior variance of the individual effect.

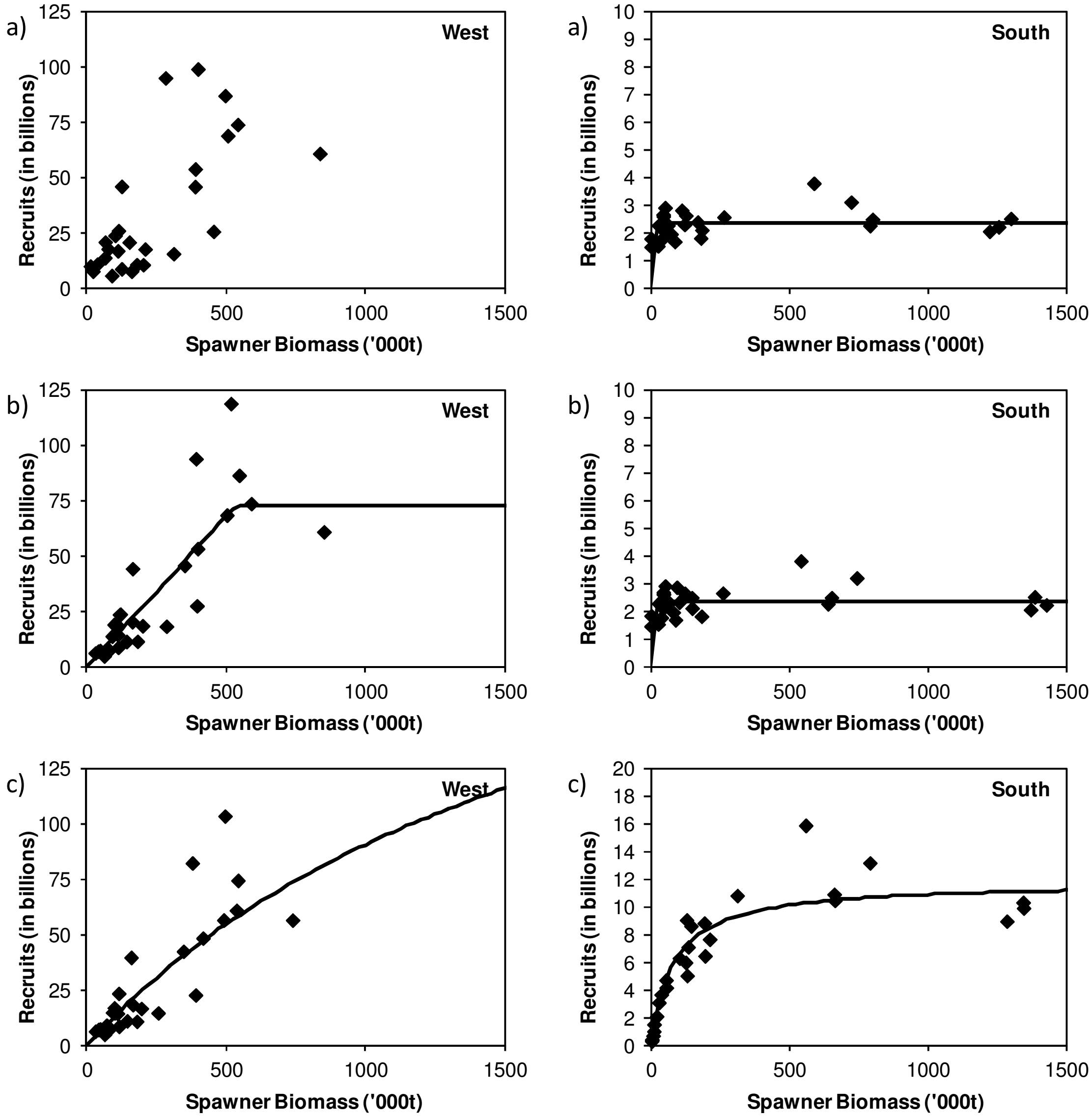

Figure 1. Model predicted sardine recruitment (in November) plotted against spawner biomass from November 1984 to November 2010 for a) S^{noSR}, a model which assumes no "west" stock recruitment relationship, b) S^{HS}, a model which assumes a hockey stick stock recruitment relationship for both the "west" and "south" stocks, and c) S^{BH}, a model which assumes a Beverton Holt stock recruitment relationship for both the "west" and "south" stocks.

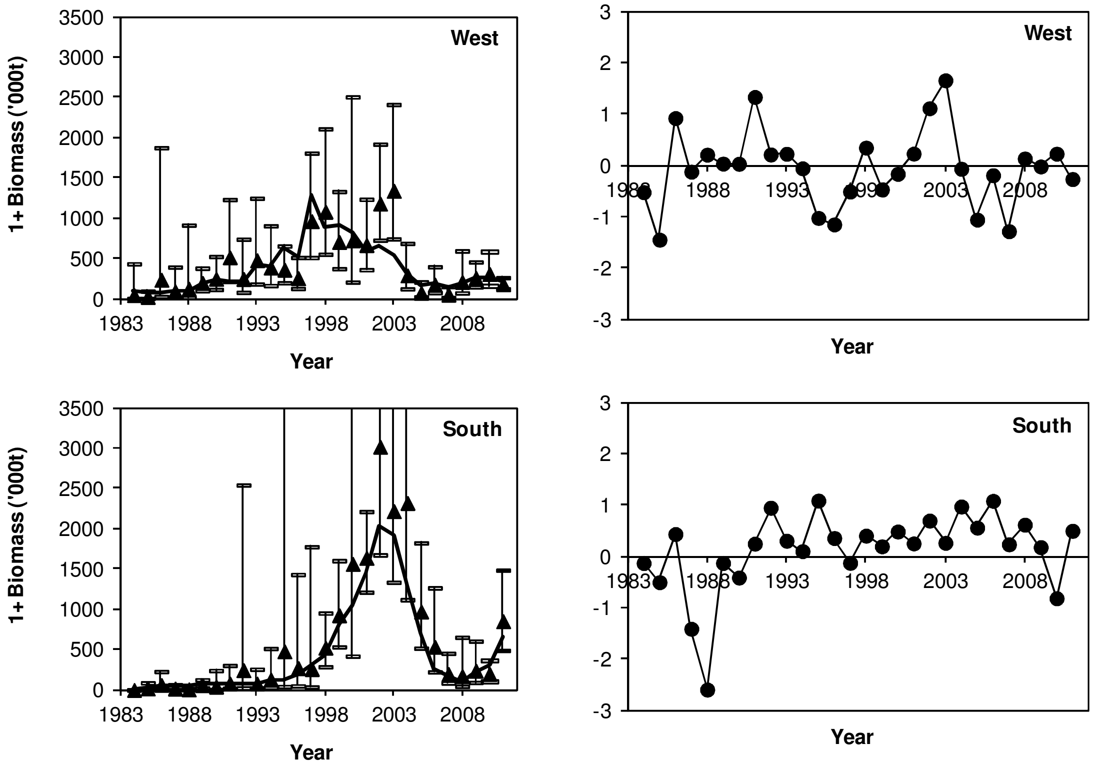

Figure 2. Acoustic survey estimated and model predicted November sardine 1+ biomass from 1984 to 2011. The observed indices are shown with 95% confidence intervals. The standardised residuals (i.e. the residual divided by the corresponding standard deviation, including additional variance where appropriate, as indicated in equation (A.26) of Appendix A) from the fits are given in the right hand plot.

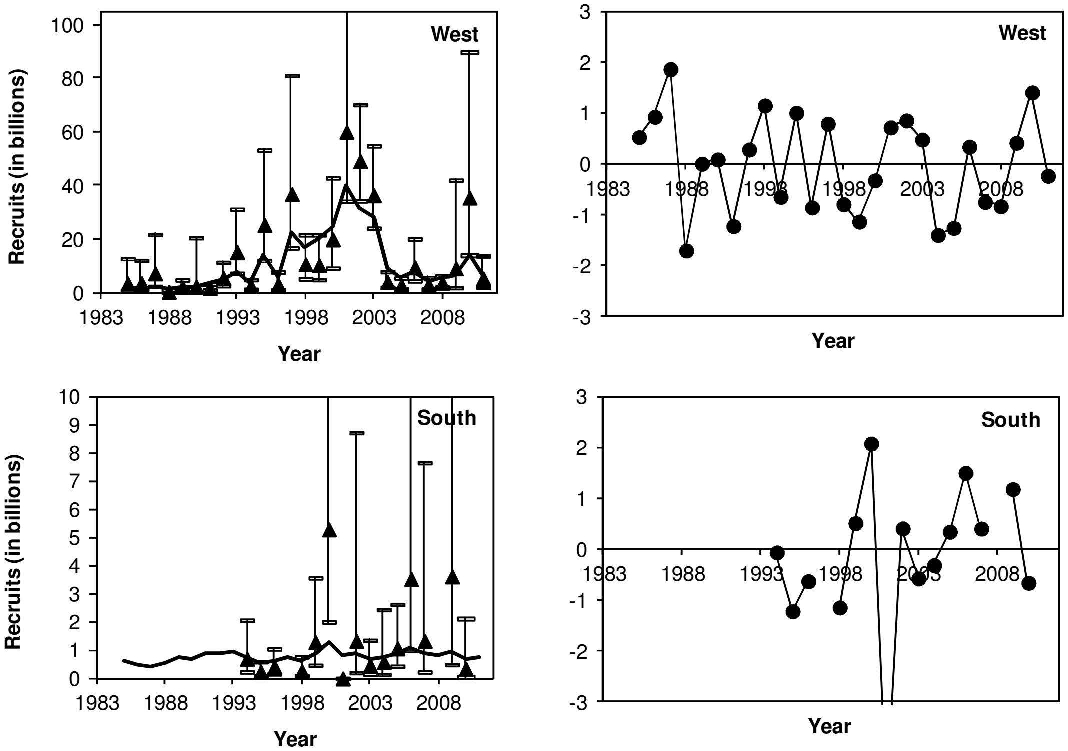

Figure 3. Acoustic survey estimated and model predicted sardine recruitment numbers from May 1985 to May 2011.The survey indices are shown with 95% confidence intervals. The standardised residuals from the fit are given in the right hand plot.

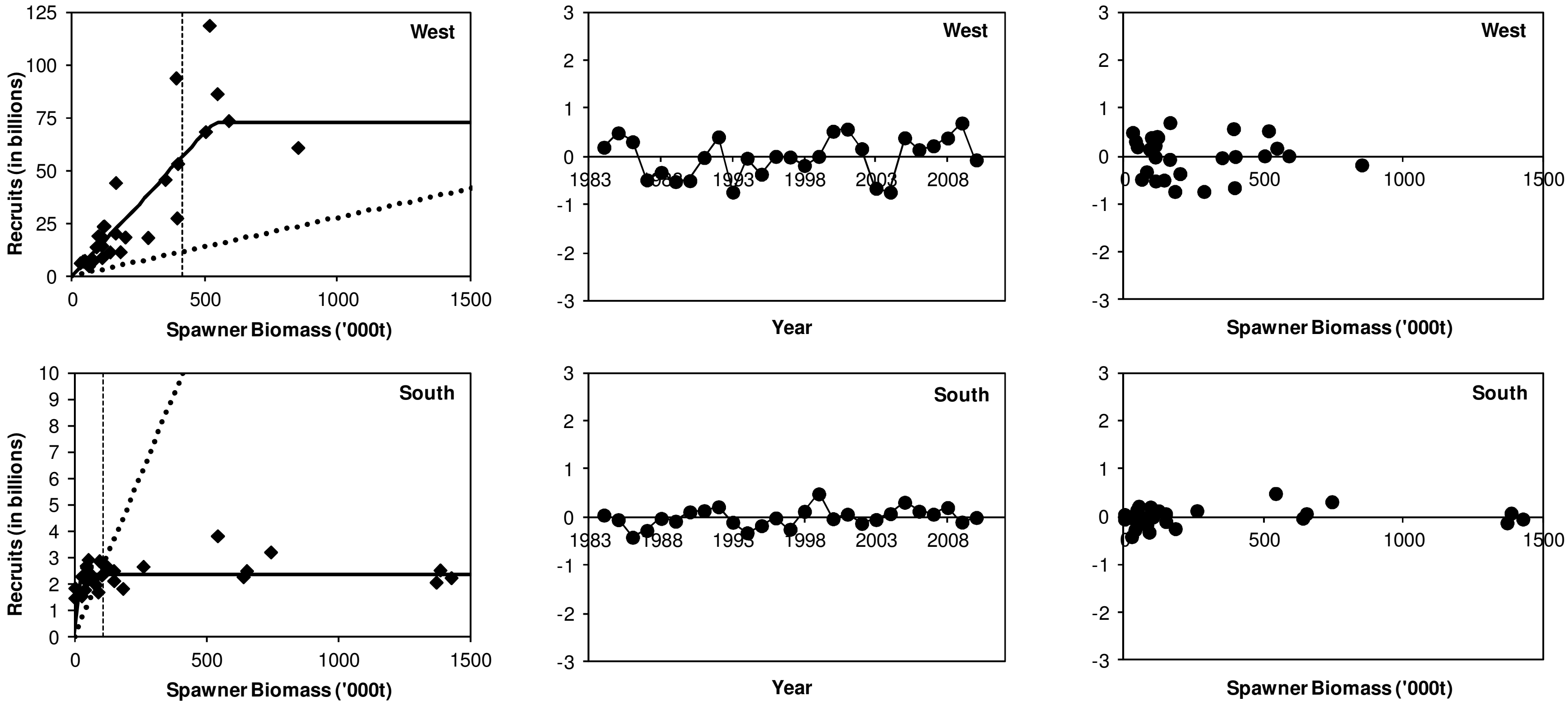

Figure 4. Model predicted sardine recruitment (in November) plotted against spawner biomass from November 1984 to November 2010 with the estimated Hockey st stock recruitment relationship. The vertical thin dashed line indicates the average 1991 to 1994 1+ biomass (the total population average was used in the definition of risk OMP-04 and OMP-08 for a single sardine stock). The dotted line indicates the replacement line. The standardised residuals from the fit are given in the lower plots, agai year and against spawner biomass.

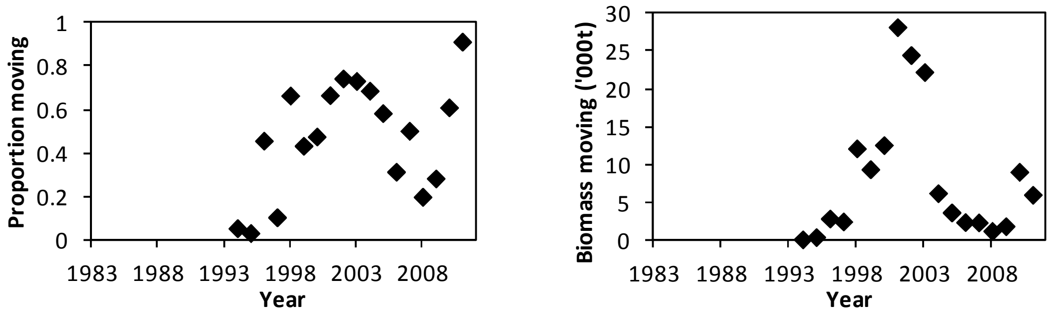

Figure 5. Model estimated proportion of recruits which move from the "west" stock to the "south" stock in November as they reach age 1 (no movement is modelled prior to 1994). The right hand plot shows rough[8] estimates of the biomass of recruits which move.

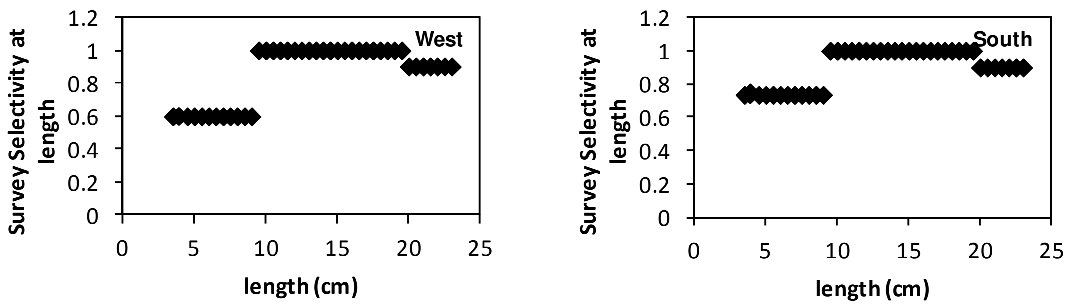

Figure 6. The model estimated November survey selectivity at length.

[8] Calculated using the average of "west" and "south" stock weights-at-age 1.

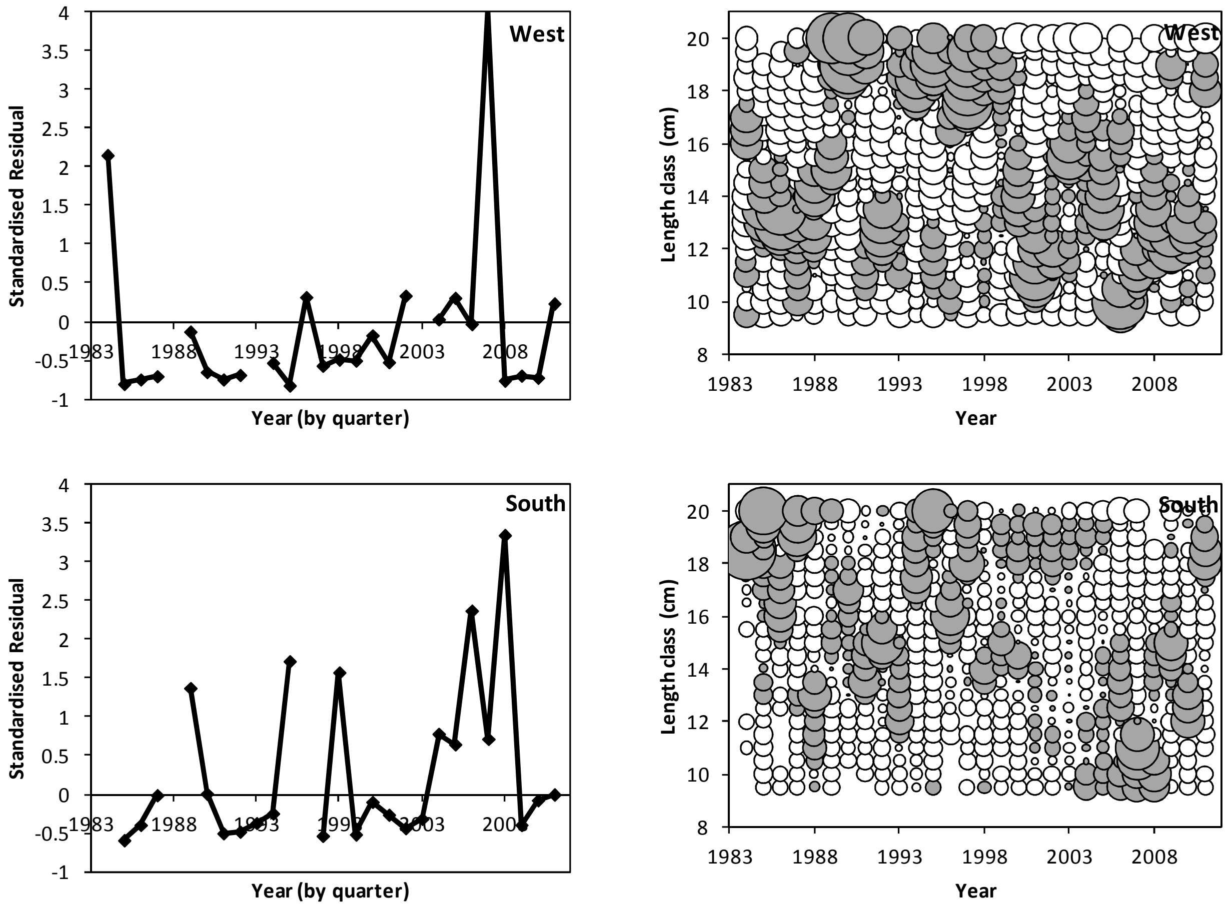

Figure 7. Residuals from the fit of the model predicted proportions-at-length in the November survey to the hydroacoustic survey estimated proportions. The left panels show the residuals for the minus length class (9cm) and the right panels show the residuals for the remaining length classes.

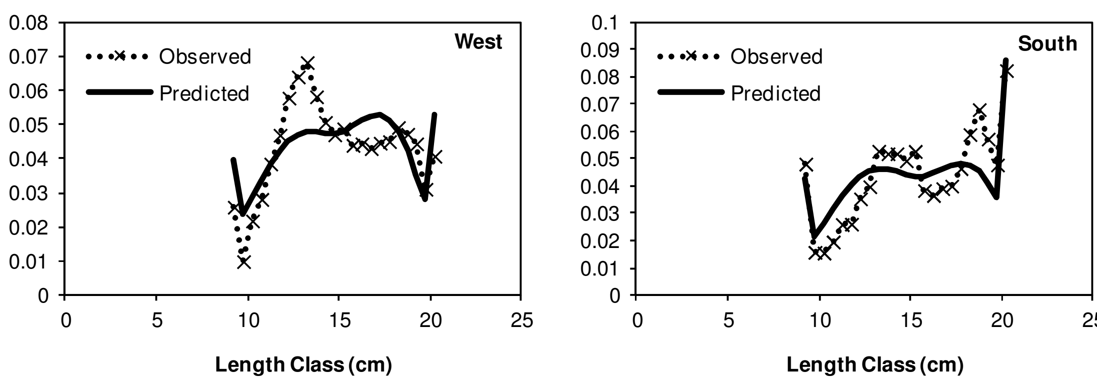

Figure 8. Average (over all years) model predicted and observed proportion-at-length in the November survey.

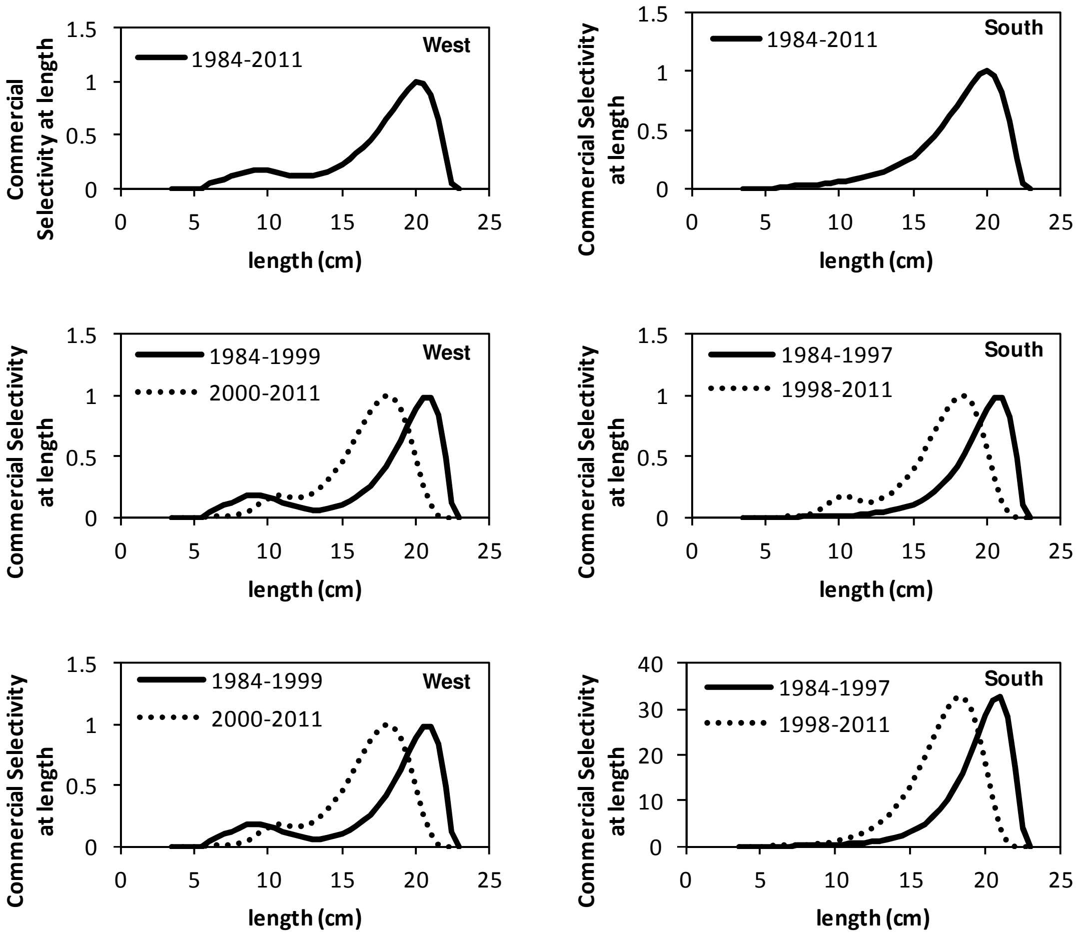

Figure 9. The model estimated commercial selectivity at age for S^{HS} (upper row), S^{sel2} (middle row) and S^{sel3} (lower row).

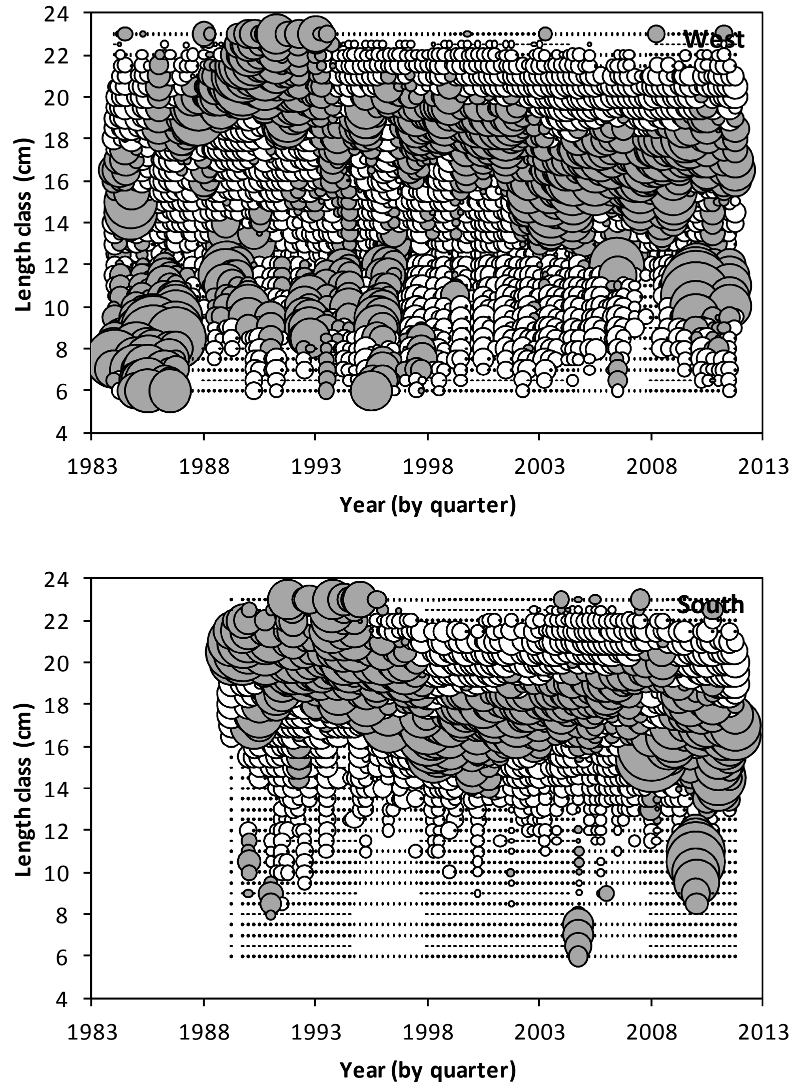

Figure 10. Residuals from the fit of the model predicted proportions-at-length in the commercial catch to the observed proportions for S^{HS}.

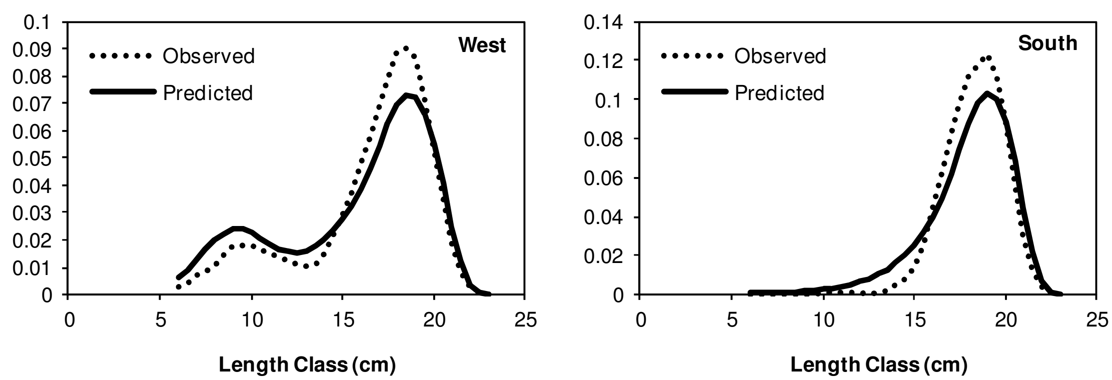

Figure 11. Average (over all quarters and years) model predicted and observed proportion-at-length in the commercial catch.

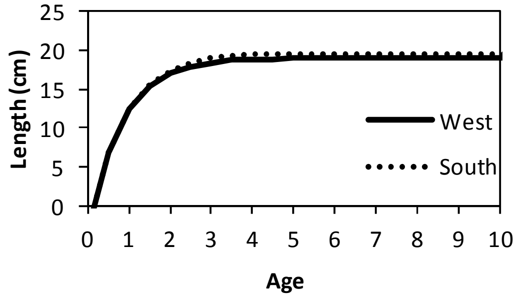

Figure 12. The von Bertalanffy growth curves estimated.

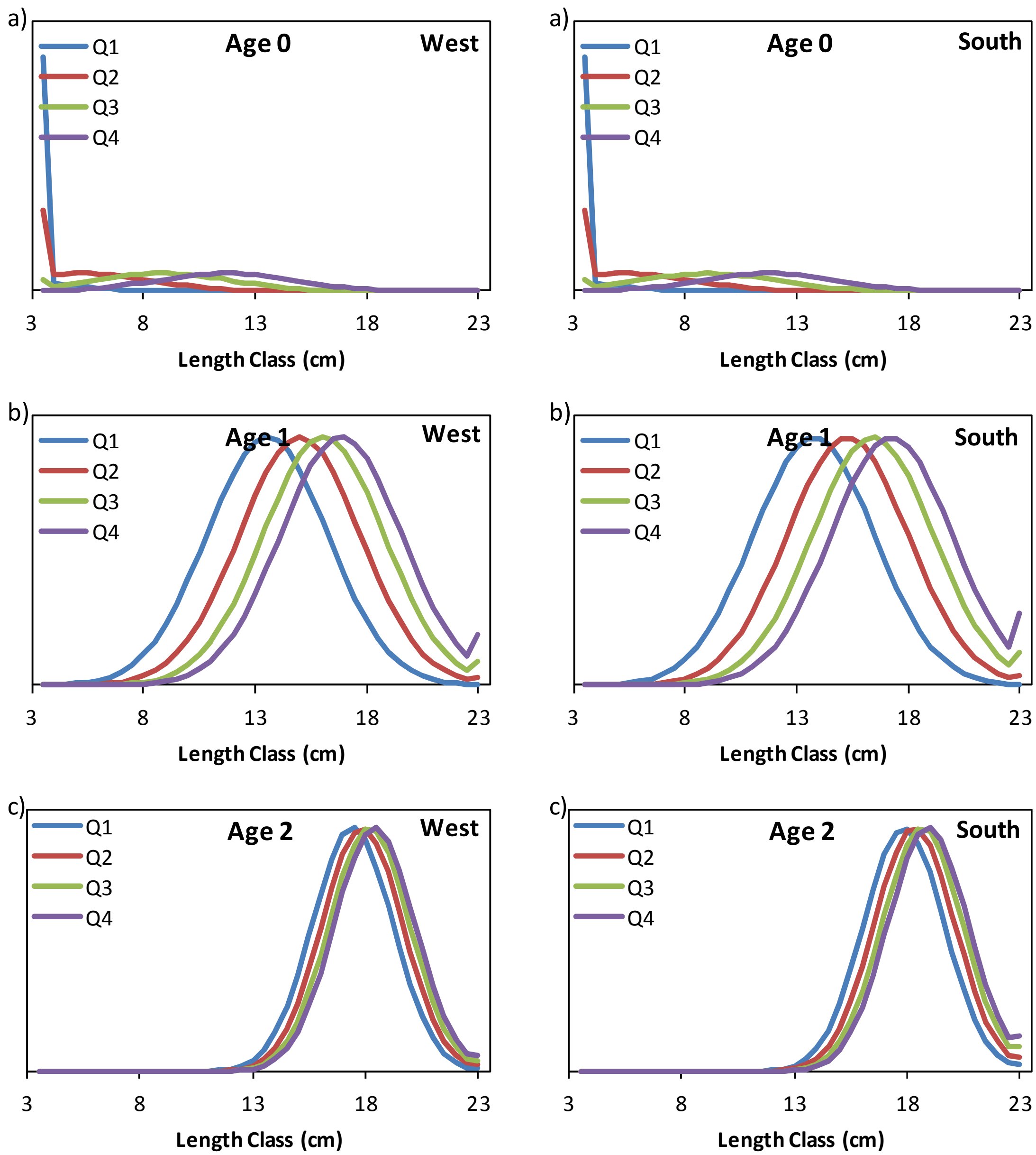

Figure 13. The model estimated distributions of proportions-at-length for each age, given at the middle of each quarter of the year (corresponding to the times commercial catch is modelled to be taken). The lowest plot compares the distributions for all ages at the middle of quarter 1.

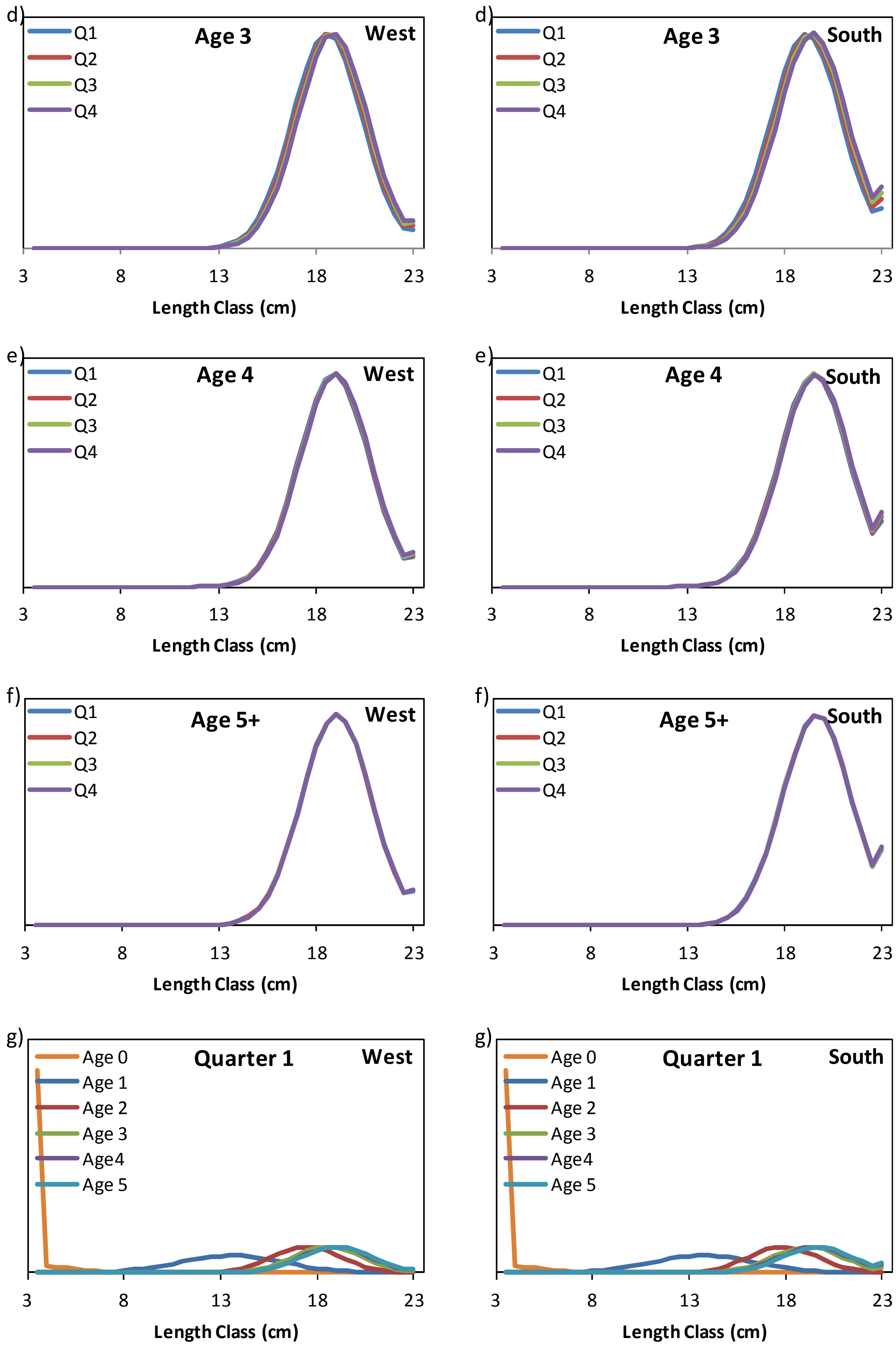

Figure 13 (continued).

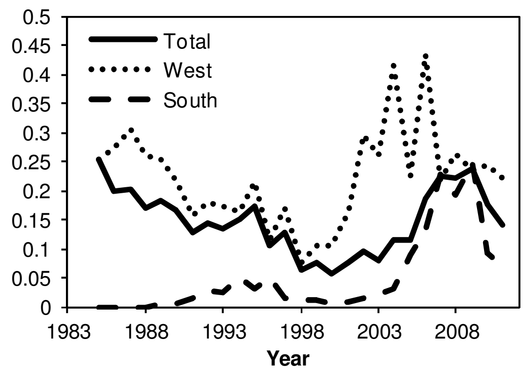

Figure 14. The harvest proportion of the "west" and "south" stocks and the total population.

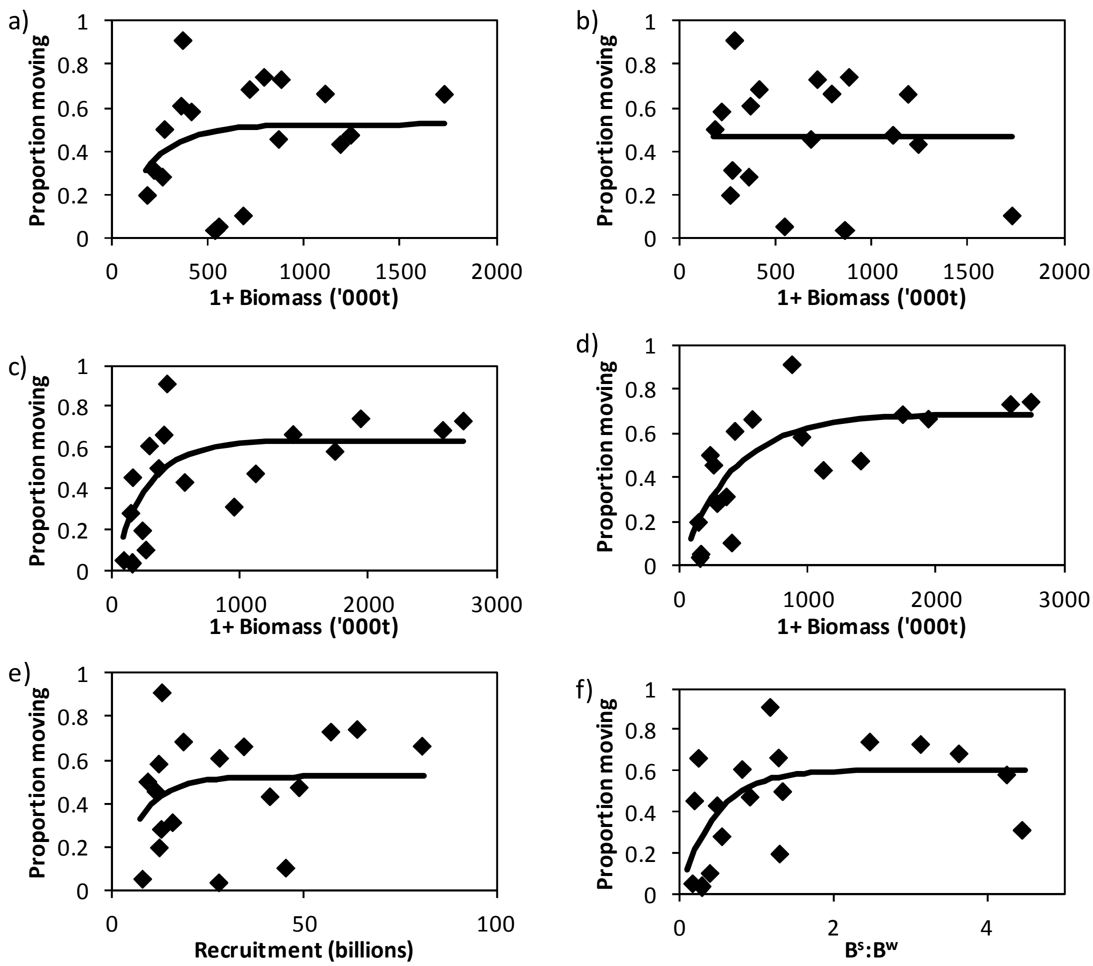

Figure 15. The model estimated proportion of recruits which move from the "west" stock to the "south" stock in November 1994 to 2011 as they reach age 1, plotted against a) "west" stock November 1+ biomass of the previous year, b) "west" stock November 1+ biomass of the current year, c) "south" stock November 1+ biomass of the previous year, d) "south" stock November 1+ biomass of the current year, e) "west" stock May recruitment of the current year, and f) the ratio of "south" to "west" stock November 1+ biomass of the previous year.

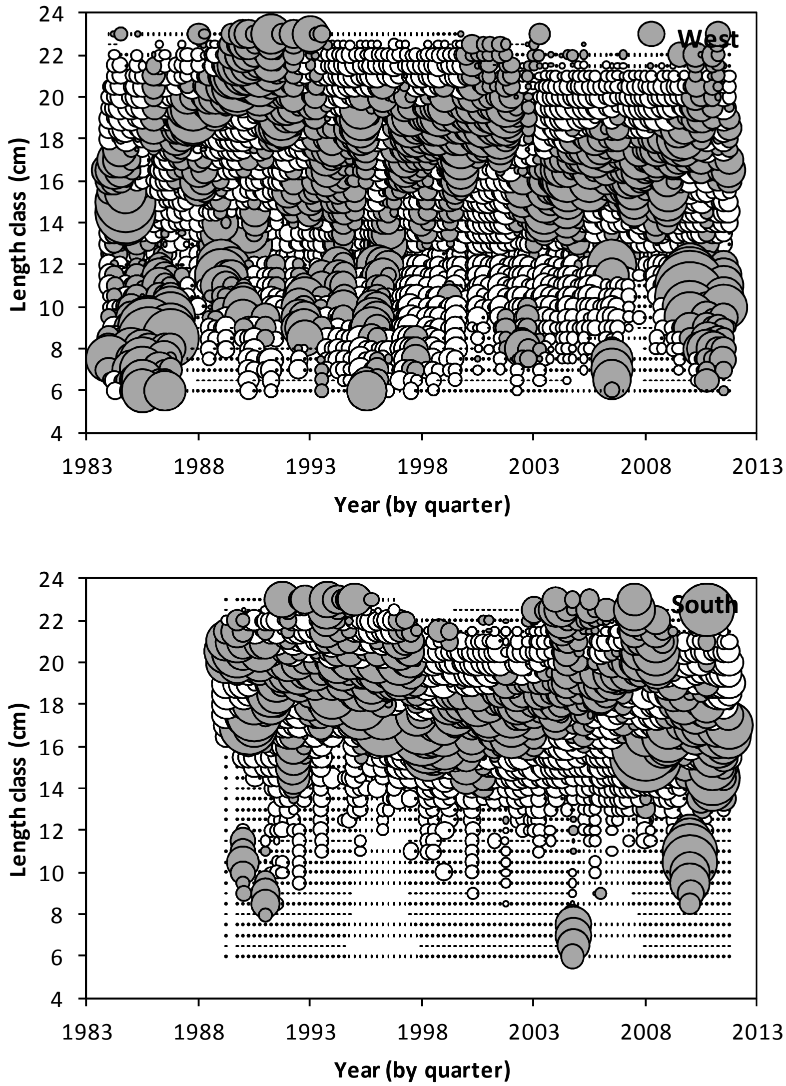

Figure 16. Residuals from the fit of the model predicted proportions-at-length in the commercial catch to the observed proportions for S^{Sel2}.

Appendix A: Bayesian age-structured operating model for the South African sardine resource, updated from de Moor and Butterworth (2012b) with yellow highlights

Base Case Model Assumptions

1) All fish have a birthdate of 1 November.
2) Sardine spawn for the first time when they turn two years old.
3) A plus group of age five is assumed.
4) Two surveys are held each year: the first takes place in November (known as the November survey) and surveys the adult (1+) stock (but see de Moor *et al*. 2012b); the second is in May/June (known as the recruit survey) and surveys juvenile (0-year-old) sardine (also called recruits).
5) The November survey provides a relative index of abundance of unknown bias.
6) The recruit survey provides a relative index of abundance of unknown bias.
7) The survey strategy is such that it results in surveys of invariant bias over time.
8) Pulse fishing occurs four times a year, in the middle of each quarter after the birthdate.
9) Natural mortality is age-invariant for adult fish.

Population Dynamics

The basic dynamic equations for sardine, based on Pope's approximation (Pope, 1984), are as follows, where $y_1 = 1984$ and $y_n = 2011$. The numbers-at-age are modelled at 1 November each year.

Catch is taken at four intervals during the year where $q = 1$ is from November $y-1$ to January y, $q = 2$ from February to April y, $q = 3$ from May to July y and $q = 4$ from August to October y:

Numbers-at-age at 1 November

$$N^S_{j,y,a} = \left(\left(\left(\left((N^S_{j,y-1,a-1} e^{-M^S_{a-1,y}/8} - C^S_{j,y,1,a-1}) e^{-M^S_{a-1,y}/4}\right) - C^S_{j,y,2,a-1}\right) e^{-M^S_{a-1,y}/4} - C^S_{j,y,3,a-1}\right) e^{-M^S_{a-1,y}/4} - C^S_{j,y,4,a-1}\right) e^{-M^S_{a-1,y}/8}$$

$$y = y_1, \ldots, y_n,\ a = 1, \ldots, 4$$

$$\begin{aligned} N^S_{j,y,5+} = & \left(\left(\left(\left((N^S_{j,y-1,4} e^{-M^S_{4,y}/8} - C^S_{j,y,1,4}) e^{-M^S_{4,y}/4}\right) - C^S_{j,y,2,4}\right) e^{-M^S_{4,y}/4} - C^S_{j,y,3,4}\right) e^{-M^S_{4,y}/4} - C^S_{j,y,4,4}\right) e^{-M^S_{4,y}/8} \\ & + \left(\left(\left(\left((N^S_{j,y-1,5+} e^{-M^S_{5+,y}/8} - C^S_{j,y,1,5+}) e^{-M^S_{5+,y}/4}\right) - C^S_{j,y,2,5+}\right) e^{-M^S_{5+,y}/4} - C^S_{j,y,3,5+}\right) e^{-M^S_{5+,y}/4} - C^S_{j,y,4,5+}\right) e^{-M^S_{5+,y}/8} \end{aligned}$$

$$y = y_1, \ldots, y_n \qquad \text{(A.1)}$$

where

$N^S_{j,y,a}$ is the model predicted number (in billions) of sardine of age a at the beginning of November in year y of stock j;

$C^S_{j,y,a,q}$ is the model predicted number (in billions) of sardine of age a of stock j caught during quarter q of year y;

$M_{a,y}^{S}$ is the rate of natural mortality (in year^{-1}) of sardine of age a in year y.

Natural mortality

Adult natural mortality varies around a median as follows:

$$M_{a,y}^{S} = \overline{M}_{ad}^{S} e^{\varepsilon_{ad,y}} \text{ for } a = 1,\ldots,5+ \text{ with } \varepsilon_{y}^{ad} = p\varepsilon_{y-1}^{ad} + \sqrt{1-p^{2}}\,\eta_{y}^{ad} \tag{A.2}$$

where $\eta_{y}^{ad} \sim N\left(0,\sigma_{ad}^{2}\right)$ and

Similarly, juvenile natural mortality varies around a median as follows:

$$M_{j,y}^{S} = \overline{M}_{j}^{S} e^{\varepsilon_{j,y}} \text{ with } \varepsilon_{y}^{j} = p\varepsilon_{y-1}^{j} + \sqrt{1-p^{2}}\,\eta_{y}^{j} \tag{A.3}$$

where $\eta_{y}^{j} \sim N\left(0,\sigma_{j}^{2}\right)$ and

$\overline{M}_{ad}^{S}$ - median adult rate of natural mortality

σ_{ad} - is the standard deviation in the annual residuals about adult natural mortality;

$\overline{M}_{j}^{S}$ - median juvenile rate of natural mortality

σ_{j} - is the standard deviation in the annual residuals about juvenile natural mortality; and

p - is the annual autocorrelation coefficient.

Movement

In the two stock hypothesis, movement of west stock ($j=1$) recruits to the east stock ($j=2$) at the beginning of November, i.e. when the recruits turn age 1, is modelled as follows:

$$N_{1,y,1}^{S} = \left(1 - move_{y}\right) N_{1,y,1}^{S*}$$

$$N_{2,y,1}^{S} = N_{2,y,1}^{S*} + move_{y} N_{1,y,1}^{S*} \qquad y = y_{1},\ldots,y_{n} \tag{A.4}$$

where $N_{j,y,1}^{S*}$ is simply the numbers-at-age 1 given by equation (A.1) prior to movement, and

$move_{y}$ is the proportion of west stock recruits which migrate to the east stock at the beginning of November of year y.

Biomass associated with the November survey

$$B_{j,y}^{S} = k_{j,N}^{S} \sum_{a=1}^{5+} N_{j,y,a}^{S} w_{j,y,a}^{S} \qquad y = y_{1},\ldots,y_{n} \tag{A.5}$$

where

$B_{j,y}^{S}$ is the model predicted biomass (in thousand tonnes) of adult sardine of stock j at the beginning of November in year *y*, associated with the November survey;

$k_{j,N}^{S}$ is the constant of proportionality (multiplicative bias) associated with the November survey of stock j; and

$w_{j,y,a}^{S}$ is the mean mass (in grams) of sardine of age *a* of stock j sampled during the November survey of year *y*, calculated as follows:

$$w^S_{j,y,1} = \frac{\left(\sum_{a=1}^{5+} N^S_{j,y,a}\right) w^{S1+}_{j,y}}{\left(N^S_{j,y,1} + \overline{\frac{w^S_{j,2}}{w^S_{j,1}}} N^S_{j,y,2} + \overline{\frac{w^S_{j,3}}{w^S_{j,1}}} N^S_{j,y,3} + \overline{\frac{w^S_{j,4}}{w^S_{j,1}}} N^S_{j,y,4} + \overline{\frac{w^S_{j,5+}}{w^S_{j,1}}} N^S_{j,y,5+}\right)} \qquad \text{(A.6)}$$

$w^S_{j,y,a} = w^S_{j,y,1} \overline{\frac{w^S_{j,a}}{w^S_{j,1}}}$, $a = 2,\ldots,5+$, where

$w^{S1+}_{j,y}$ is the total (1+) mean mass (in grams) of sardine of stock j sampled during the November survey of year y (de Moor *et al*. 2012a); and

$\overline{\frac{w^S_{j,a}}{w^S_{j,1}}}$ is the average ratio of mean mass (in grams) of sardine of stock j aged a to age 1 obtained from the growth curve.

The multiplicative bias in the November survey is assumed to be equal to that resulting from the acoustic survey only; hence it is assumed that the full distribution of sardine is covered by the survey, i.e.

$k^S_{j,N} = k^S_{ac}$

where

k^S_{ac} is the multiplicative bias associated with the acoustic survey (see Appendix B).

Sardine are assumed to mature at age two and thus the spawning stock biomass is:

$$SSB^S_{j,y} = \sum_{a=2}^{5+} N^S_{j,y,a} w^S_{j,y,a} \qquad y = y_1,\ldots,y_n \qquad \text{(A.7)}$$

Proportion at length associated with the November survey

The model predicted numbers-at-length in the survey are:

$$N^S_{j,y,l} = \sum_{a=1}^{5+} A^{sur}_{j,a,l} N^S_{y,j,a} \qquad y = y_1,\ldots,y_n,\ l = 3.5cm,\ldots,23cm \qquad \text{(A.8)}$$

and the model predicted proportion-at-length associated with the November survey is:

$$p^S_{j,y,l\min} = \frac{\sum_{l \le l\min} N^S_{j,y,l} S^{survey}_{j,l}}{\sum_l N^S_{j,y,l} S^{survey}_{j,l}} \qquad y = y_1,\ldots,y_n \qquad \text{(A.9)}$$

$$p^S_{j,y,l} = \frac{N^S_{j,y,l} S^{survey}_{j,l}}{\sum_l N^S_{j,y,l} S^{survey}_{j,l}} \qquad y = y_1,\ldots,y_n,\ l = l\min+1,\ldots,l\max-1 \qquad \text{(A.10)}$$

$$p^S_{j,y,l\max} = \frac{\sum_{l \geq l\max} N^S_{j,y,l} S^{survey}_{j,l}}{\sum_{l} N^S_{j,y,l} S^{survey}_{j,l}} \qquad y = y_1, \ldots, y_n \quad \text{(A.11)}$$

where

$S^{survey}_{j,l}$ is the survey selectivity at length l in the November survey for stock j;

$A^{sur}_{j,a,l}$ is the proportion of sardine of age a in stock j that fall in the length group l in November;

$l\min = 9\text{cm}$ is the minus length class used when fitting the model to survey proportion-at-length data; and

$l\max = 20\text{cm}$ is the plus length class used when fitting the model to survey proportion-at-length data.

The matrix $A^{sur}_{j,a,l}$ is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A^{sur}_{j,a,l} \sim N\left(L_{j,\infty}\left(1 - e^{-\kappa_j\left(a - t_{0,j}\right)}\right), \vartheta^2_{j,a}\right) \qquad a = 1, \ldots, 5+, \ l = 3.5cm, \ldots, 23cm \quad \text{(A.6)}$$

where

$L_{j,\infty}$ denotes the maximum length (in expectation) of sardine of stock j;

κ_j denotes the annual growth rate of sardine of stock j;

$t_{0,j}$ denotes the age at which the length (in expectation) of sardine of stock j is zero; and

$\vartheta_{j,a}$ denotes the standard deviation of the distribution about the mean length for age a of stock j.

Commercial selectivity

Commercial selectivity-at-length is assumed to follow the combined shape of a normal and inverted lognormal curve, with the second curve reaching a maximum of 1 corresponding to the fully selected length class. Commercial selectivity is assumed to remain unchanged over time. Selectivity-at-lengths less than and equal to the minus length class is taken to be zero (see footnote 7). Thus we have:

$$S_{j,y,l} = \begin{cases} 0 & l \leq 5 \\ \chi_j \exp\left\{-\dfrac{\left(l_{mid} - \bar{l}_{1,j}\right)^2}{\left(\sigma^{sel}_{1,j}\right)^2}\right\} + \exp\left\{-\dfrac{\left[\ln\left(\left(l_{mid} - l_{\max}\right)/\left(\bar{l}_{2,j} - l_{\max}\right)\right)\right]^2}{\left(\sigma^{sel}_{2,j}\right)^2}\right\} & 6 \leq l \leq 40 \end{cases}, \qquad y = y_1, \ldots, y_n$$

where

χ_j denotes the height of the normal curve component for stock j relative to the height of the other component;

l_{mid} is the midpoint (in cm) of length class l;

$l_{\max} = 23.5cm$ is one length class above the maximum for which observations can be predicted;

$\bar{l}_{1,j}$ is the mean of the normal distribution for stock j;

$\bar{l}_{2,j}$ is the median of the lognormal distribution for stock j;

$\left(\sigma^{sel}_{1,j}\right)^2$ is the variance parameter of the normal distribution for stock j; and

$\left(\sigma_{2,j}^{sel}\right)^2$ is the variance parameter of the lognormal distribution for stock j.

Catch

Sardine are landed by three major fisheries: the sardine-directed fishery (*fleet*=1), the red-eye-directed fishery (*fleet*=2), and the anchovy-directed fishery (*fleet*=3). Landings from the former two fisheries comprise mainly adult sardine while bycatch from the anchovy-directed fishery is primarily juvenile sardine. In the anchovy-directed fishery, the assumption is made that all sardine smaller than a pre-determined cut-off length are 0-year-olds, and the remaining bycatch from this fishery are assumed to be 1-year olds:

$$C_{j,y,1,0}^{bycatch} = \sum_{m=11}^{12} \sum_{l<lcut_{y,m}} C_{j,y-1,m,l}^{RLF,fleet=3} + \sum_{l<lcut_{y,m}} C_{j,y,1,l}^{RLF,fleet=3} \qquad C_{j,y,1,1}^{bycatch} = \sum_{m=11}^{12} \sum_{l>=lcut_{y,m}} C_{j,y-1,m,l}^{RLF,fleet=3} + \sum_{l>=lcut_{y,m}} C_{j,y,1,l}^{RLF,fleet=3}$$

$$C_{j,y,2,0}^{bycatch} = \sum_{m=2}^{4} \sum_{l<lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3} \qquad C_{j,y,2,1}^{bycatch} = \sum_{m=2}^{4} \sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3}$$

$$C_{j,y,3,0}^{bycatch} = \sum_{m=5}^{7} \sum_{l<lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3} \qquad C_{j,y,3,1}^{bycatch} = \sum_{m=5}^{7} \sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3}$$

$$C_{j,y,4,0}^{bycatch} = \sum_{m=8}^{10} \sum_{l<lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3} \qquad C_{j,y,4,1}^{bycatch} = \sum_{m=8}^{10} \sum_{l>=lcut_{y,m}} C_{j,y,m,l}^{RLF,fleet=3}, \quad y = y_1, \ldots, y_n \qquad \text{(A.7)}$$

$$C_{j,y,q,a}^{bycatch} = 0 \qquad y = y_1, \ldots, y_n,\ q = 1, \ldots, 4,\ a = 2, \ldots, 5+, \qquad \text{(A.14)}$$

where

$C_{j,y,m,l}^{RLF,fleet}$ is the number of fish landed by *fleet* in length class l landed in month m of year y of stock j (the 'raised length frequency'); and

$lcut_{y,m}$ is the cut off length for recruits in month m of year y (see de Moor *et al*. (2012a) for details).

In the directed sardine and redeye bycatch fisheries, sardine are split between ages using a model estimated selectivity:

$$C_{j,y,1,a}^{dir} = \left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch}\right) S_{j,y,1,a} F_{j,y,1}$$

$$C_{j,y,2,a}^{dir} = \left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch}\right) S_{j,y,2,a} F_{j,y,2}$$

$$C_{j,y,3,a}^{dir} = \left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,3,a}^{bycatch}\right) S_{j,y,3,a} F_{j,y,3}$$

$$C_{j,y,4,a}^{dir} = \left(\left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,3,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,4,a}^{bycatch}\right) S_{j,y,4,a} F_{j,y,4}$$

Finally:

$$C_{j,y,q,a}^{S} = C_{j,y,q,a}^{bycatch} + C_{j,y,q,a}^{dir} \qquad y = y_1, \ldots, y_n,\ q = 1, \ldots, 4,\ a = 0, \ldots, 5+ \qquad \text{(A.15)}$$

$F_{j,y,q}$ is the fished proportion in quarter q of year y for a fully selected age class a of stock j, by the directed and redeye bycatch fisheries.

In the equations above the difference in the year subscript between the catch-at-age and initial numbers-at-age is because these numbers-at-age pertain to November of the previous year.

The fished proportion of the available biomass from the directed and redeye bycatch fisheries is estimated by:

$$F_{j,y,1} = \frac{\sum_{fleet=1}^{2}\sum_{m=11}^{12}\sum_{l>=6cm} C_{j,y-1,m,l}^{RLF,fleet} + \sum_{fleet=1}^{2}\sum_{l>=6cm} C_{j,y,1,l}^{RLF,fleet}}{\sum_{a=0}^{5+}\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{bycatch}\right) S_{j,y,1,a}}$$

$$F_{j,y,2} = \frac{\sum_{fleet=1}^{2}\sum_{m=2}^{4}\sum_{l>=6cm} C_{j,y,m,l}^{RLF,fleet}}{\sum_{a=0}^{5+}\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{bycatch}\right) S_{j,y,2,a}}$$

$$F_{j,y,3} = \frac{\sum_{fleet=1}^{2}\sum_{m=5}^{7}\sum_{l>=6cm} C_{j,y,m,l}^{RLF,fleet}}{\sum_{a=0}^{5+}\left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,3,a}^{bycatch}\right) S_{j,y,3,a}}$$

$$F_{j,y,4} = \frac{\sum_{fleet=1}^{2}\sum_{m=8}^{10}\sum_{l>=6cm} C_{j,y,m,l}^{RLF,fleet}}{\sum_{a=0}^{5+}\left(\left(\left(\left(N_{j,y-1,a}^{S} e^{-M_a^S/8} - C_{j,y,1,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,2,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,3,a}^{S}\right) e^{-M_a^S/4} - C_{j,y,4,a}^{bycatch}\right) S_{j,y,4,a}} \qquad \text{(A.16)}$$

A penalty is imposed within the model to ensure that $S_{j,y,q,a} F_{j,y,q} < 0.95$.

Recruitment

For the base case assessment of a single stock hypothesis, a Hockey Stick stock-recruitment curve is assumed. Recruitment at the beginning of November is assumed to fluctuate lognormally about the stock-recruitment curve:

$$N_{j,y,0}^{S} = f\left(SSB_{j,y}^{S}\right) e^{\varepsilon_{j,y}^{S}} \qquad y = y_1, \ldots, y_n \quad \text{(A.8)}$$

where

$\varepsilon_{j,y}^{S}$ is the annual lognormal deviation of sardine recruitment.

Number of recruits at the time of the recruit survey

The number of recruits at the time of the recruit survey is calculated taking into account the recruit catch during quarters 1 and 2 (November to April) and an estimate of the recruit catch between 1 May and the start of the survey:

$$N_{j,y,r}^{S} = k_{j,r}^{S}\left(\left((N_{j,y-1,0}^{S} e^{-M_0^S/8} - C_{j,y,1,0}^{S}) e^{-M_0^S/4} - C_{j,y,2,0}^{S}\right) e^{-0.5 t_y^S \times M_0^S/12} - \tilde{C}_{j,y,0bs}^{S}\right) e^{-0.5 t_y^S \times M_0^S/12} \qquad y = y_1, \ldots, y_n \quad \text{(A.18)}$$

where

$N_{j,y,r}^{S}$ is the model predicted number (in billions) of juvenile sardine of stock j at the time of the recruit survey in year y;

$k^S_{j,r}$ is the constant of proportionality (multiplicative bias) associated with the recruit survey;

$\tilde{C}^S_{j,y,0bs}$ is the number (in billions) of juvenile sardine of stock j caught between 1 May and the day before the start of the recruit survey (see de Moor *et al*. 2012a); and

t^S_y is the time lapsed (in months) between 1 May and the start of the recruit survey in year y (see de Moor *et al*. 2012a).

The multiplicative bias in the recruit survey is assumed to be equal to that resulting from the acoustic survey as well as the proportion of the recruit abundance which the survey covers in comparison to the November survey. In addition, for the two stock hypothesis, the proportion of the east stock recruit abundance covered compared to that of the west stock abundance is also required. Thus

$k^S_{1,r} = k^S_{\mathrm{cov}} \times k^S_{ac}$

and for the two stock hypothesis, $k^S_{2,r} = k^S_{\mathrm{cov}\,E} \times k^S_{\mathrm{cov}} \times k^S_{ac}$

where

k^S_{cov} is the multiplicative bias associated with the coverage of the recruits by the recruit survey compared to the 1+ biomass by the November survey; and

$k^S_{\mathrm{cov}\,E}$ is the multiplicative bias associated with the coverage of the east stock recruits by the recruit survey compared to the west stock recruits during the same survey.

Proportion at length associated with the commercial catch

The commercial catch-at-length from the directed and redeye bycatch fisheries is:

$$C^{dir}_{j,y,1,l} = \sum_{a=0}^{5+} \left(N^S_{j,y-1,a} e^{-M^S_a/8} - C^{bycatch}_{j,y,1,a} \right) A^{com}_{j,1,a,l} S_{j,y,l} F_{j,y,1}$$

$$C^{dir}_{j,y,2,l} = \sum_{a=0}^{5+} \left(\left(N^S_{j,y-1,a} e^{-M^S_a/8} - C^S_{j,y,1,a} \right) e^{-M^S_a/4} - C^{bycatch}_{j,y,2,a} \right) A^{com}_{j,2,a,l} S_{j,y,l} F_{j,y,2}$$

$$C^{dir}_{j,y,3,l} = \sum_{a=0}^{5+} \left(\left(\left(N^S_{j,y-1,a} e^{-M^S_a/8} - C^S_{j,y,1,a} \right) e^{-M^S_a/4} - C^S_{j,y,2,a} \right) e^{-M^S_a/4} - C^{bycatch}_{j,y,3,a} \right) A^{com}_{j,3,a,l} S_{j,y,l} F_{j,y,3}$$

$$C^{dir}_{j,y,4,l} = \sum_{a=0}^{5+} \left(\left(\left(\left(N^S_{j,y-1,a} e^{-M^S_a/8} - C^S_{j,y,1,a} \right) e^{-M^S_a/4} - C^S_{j,y,2,a} \right) e^{-M^S_a/4} - C^S_{j,y,3,a} \right) e^{-M^S_a/4} - C^{bycatch}_{j,y,4,a} \right) A^{com}_{j,4,a,l} S_{j,y,l} F_{j,y,4}$$

$$y = y_1, \ldots, y_n, \; l = 3.5cm, \ldots, 23cm \quad \text{(A.19)}$$

The model predicted proportion-at-length in the commercial catch from the directed and redeye bycatch fisheries is:

$$p^{coml,S}_{j,y,q,l} = \frac{C^{dir}_{j,y,q,l}}{\sum_l C^{dir}_{j,y,q,l}} \; {}^{9} \qquad y = y_1, \ldots, y_n, \; q = 1, \ldots, 4, \; l = 3.5cm, \ldots, 23cm \quad \text{(A.20)}$$

where

[9] See footnote 7 and the section "Fixed Parameters" for the base case hypotheses. Commercial selectivity at length is fixed = 0 for length classes <6cm, and thus the commercial proportions-at-length in length classes < 6cm in equation (A.20) are not used in fitting the model.

$A^{com}_{j,q,a,l}$ is the proportion of sardine of age a in stock j that fall in the length group l in quarter q.

The matrix $A^{com}_{j,q,a,l}$ is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A^{com}_{j,1,a,l} \sim N\left(L_{j,\infty}\left(1-e^{-\kappa_j\left(a+1/8-t_{0,j}\right)}\right), \vartheta_{j,a}{}^2\right) \qquad a=0,\ldots,5+,\ l=3.5cm,\ldots,23cm \qquad \text{(A.21)}$$

$$A^{com}_{j,2,a,l} \sim N\left(L_{j,\infty}\left(1-e^{-\kappa_j\left(a+3/8-t_{0,j}\right)}\right), \vartheta_{j,a}{}^2\right) \qquad a=0,\ldots,5+,\ l=3.5cm,\ldots,23cm \qquad \text{(A.22)}$$

$$A^{com}_{j,3,a,l} \sim N\left(L_{j,\infty}\left(1-e^{-\kappa_j\left(a+5/8-t_{0,j}\right)}\right), \vartheta_{j,a}{}^2\right) \qquad a=0,\ldots,5+,\ l=3.5cm,\ldots,23cm \qquad \text{(A.23)}$$

$$A^{com}_{j,4,a,l} \sim N\left(L_{j,\infty}\left(1-e^{-\kappa_j\left(a+7/8-t_{0,j}\right)}\right), \vartheta_{j,a}{}^2\right) \qquad a=0,\ldots,5+,\ l=3.5cm,\ldots,23cm \qquad \text{(A.24)}$$

Fitting the Model to Observed Data (Likelihood)

The survey observations are assumed to be lognormally distributed. The standard errors of the log-distributions for the survey observations of adult biomass and recruitment numbers are approximated by the CVs of the untransformed distributions and a further additional variance parameter. The estimated proportions-at-length are also assumed to be lognormally distributed, with the variance inversely proportional to the observed proportion. Thus the negative log-likelihood function is given by:

$$-\ln L = -\ln L^{Nov} - \ln L^{rec} - \ln L^{sur\ propl\min} - \ln L^{sur\ propl} - \ln L^{com\ propl} \qquad \text{(A.25)}$$

where

$$-\ln L^{Nov} = \tfrac{1}{2}\sum_j \sum_{y=y1}^{yn}\left\{\left\{\frac{5^5\left(\dfrac{\left|\ln\left(\hat{B}^S_{j,y}\right)-\ln(B^S_{j,y})\right|}{\sqrt{(\sigma^S_{j,y,Nov})^2+\left(\phi^S_{ac}\right)^2+(\lambda^S_{j,N})^2}}\right)^5}{5^5+\left(\dfrac{\left|\ln\left(\hat{B}^S_{j,y}\right)-\ln(B^S_{j,y})\right|}{\sqrt{(\sigma^S_{j,y,Nov})^2+\left(\phi^S_{ac}\right)^2+(\lambda^S_{j,N})^2}}\right)^5}\right\}^{2/5} + \ln\left[2\pi\left((\sigma^S_{j,y,Nov})^2+\left(\phi^S_{ac}\right)^2+(\lambda^S_{j,N})^2\right)\right]\right\}$$

$$-\ln L^{rec} = \tfrac{1}{2}\sum_j \sum_{y=y1+1}^{yn}\left\{\left\{\frac{5^5\left(\dfrac{\left|\ln\left(\hat{N}^S_{j,y,r}\right)-\ln(N^S_{j,y,r})\right|}{\sqrt{(\sigma^S_{j,y,rec})^2+\left(\phi^S_{ac}\right)^2+(\lambda^S_{j,r})^2}}\right)^5}{5^5+\left(\dfrac{\left|\ln\left(\hat{N}^S_{j,y,r}\right)-\ln(N^S_{j,y,r})\right|}{\sqrt{(\sigma^S_{j,y,rec})^2+\left(\phi^S_{ac}\right)^2+(\lambda^S_{j,r})^2}}\right)^5}\right\}^{2/5} + \ln\left[2\pi\left((\sigma^S_{j,y,rec})^2+\left(\phi^S_{ac}\right)^2+(\lambda^S_{j,r})^2\right)\right]\right\}$$

$$-\ln L^{sur\ propl\min} = w^{sur}_{propl\min}\,\frac{1}{2}\sum_j \sum_{y=y1}^{yn}\left\{\frac{\hat{p}^S_{j,y,l\min}\left(\ln\left(\hat{p}^S_{j,y,l\min}\right)-\ln\left(p^S_{j,y,l\min}\right)\right)^2}{2\left(\sigma^{S,surl\min}_j\right)^2} + \ln\left(\frac{\sigma^{S,surl\min}_j}{\sqrt{\hat{p}^S_{j,y,l\min}}}\right)\right\}$$

$$-\ln L^{sur\ propl} = w^{sur}_{propl}\frac{1}{2}\sum_{j}\sum_{y=y1}^{yn}\sum_{l=l\min+1}^{l\max}\left\{\frac{\hat{p}^{S}_{j,y,l}\left(\ln\left(\hat{p}^{S}_{j,y,l}\right)-\ln\left(p^{S}_{j,y,l}\right)\right)^2}{2\left(\sigma^{S,surl}_{j}\right)^2}+\ln\left(\frac{\sigma^{S,surl}_{j}}{\sqrt{\hat{p}^{S}_{j,y,l}}}\right)\right\}$$ [10]

$$-\ln L^{com\ propl} = w^{com}_{propl}\frac{1}{2}\sum_{j}\sum_{y=y1}^{yn}\sum_{q=1}^{4}\sum_{l>=6cm}\left\{\frac{\hat{p}^{S,coml}_{j,y,q,l}\left(\ln\left(\hat{p}^{S,coml}_{j,y,q,l}\right)-\ln\left(p^{coml}_{j,y,q,l}\right)\right)^2}{2\left(\sigma^{S,coml}_{j}\right)^2}+\ln\left(\frac{\sigma^{S,coml}_{j}}{\sqrt{\hat{p}^{S,coml}_{j,y,q,l}}}\right)\right\} \quad \text{(A.26)}$$ [11][12]

where a "robustified likelihood" is used for the contributions from the first two surveys (the functional form chosen to robustify makes negligible difference for standardised residuals of magnitude 3 or less, but essentially treats large standardised residuals as if they do not exceed 5 in magnitude).

Here

$\hat{B}^{S}_{j,y}$ is the acoustic survey estimate (in thousands of tonnes) of adult sardine biomass of stock j from the November survey in year y, with associated CV $\sigma^{S}_{j,y,Nov}$;

$\hat{N}^{S}_{j,y,r}$ is the acoustic survey estimate (in billions) of sardine recruitment numbers of stock j from the recruit survey in year y, with associated CV $\sigma^{S}_{j,y,rec}$; and

ϕ^{S}_{ac} is the CV associated with the factors which cause bias in the acoustic survey estimates and which vary inter-annually rather than remain fixed over time;

$(\lambda^{S}_{j,N/r})^2$ is the additional variance (over and above the squares of the survey sampling CV $\sigma^{S}_{y,Nov/rec}$ that reflects survey inter-transect variance and of the CV ϕ^{S}_{ac} associated with the annually varying factors causing bias in the acoustic survey estimates) associated with the November/recruit surveys of stock j;

$\hat{p}^{S}_{j,y,l}$ is the observed proportion (by number) of sardine in length group l in the November survey of year y;

$w^{sur}_{propl\min}$ is the weighting applied to the survey proportion at length data for the minus length class;

w^{sur}_{propl} is the weighting applied to the remaining survey proportion at length data;

$\sigma^{S,surl\min}_{j}$ is the variance-related parameter for the log-transformed survey proportion-at-length data of the minus length class, which is estimated in the fitting procedure by the closed form solution:

$$\sigma^{S,surl\min}_{j}=\sqrt{\sum_{y=y1}^{yn}\hat{p}^{S}_{j,y,l\min}\left(\ln\hat{p}^{S}_{j,y,l\min}-\ln p^{S}_{j,y,l\min}\right)^2\Big/\sum_{y=y1}^{yn}1}\text{; and}$$

[10] Although strictly there may be bias in the proportions of length-at-age data, no bias is assumed in this assessment. The effect of such a bias is assumed to be small.

[11] In only 11 out of 112 year-quarters were fish of length <6cm observed in the directed sardine and redeye bycatch fisheries. Due to the large variance associated with a minus group of 5.5cm in earlier models, and the small occurrence and small proportion-at-length of these fish, the directed sardine and redeye bycatch fisheries are modeled to cover length classes 6cm and larger only, and the 6cm length class is not treated as a minus class.

[12] The sum is over all quarters for which the catch is non-zero.

$\sigma_j^{S,surl}$ is the variance-related parameter for the log-transformed survey proportion-at-length data, which is estimated in the fitting procedure by the closed form solution:

$$\sigma_j^{S,surl} = \sqrt{\sum_{y=y1}^{yn} \sum_{l=1\min+1}^{l\max} \hat{p}_{j,y,l}^{S} \left(\ln \hat{p}_{j,y,l}^{S} - \ln p_{j,y,l}^{S}\right)^2 \Big/ \sum_{y=y1}^{yn} \sum_{l=1\min+1}^{l\max} 1} .$$

$\hat{p}_{j,y,q,l}^{S,coml}$ is the observed proportion (by number) of the directed and redeye bycatch commercial catch in length group l of during quarter q ($q=1$ for Nov-Jan, $q=2$ for Feb-Apr, $q=3$ for May-Jul, $q=4$ for Aug-Oct) of year y;

w_{propl}^{com} is the weighting applied to the commercial proportion at length data; and

$\sigma_j^{S,coml}$ is the variance-related parameter for the log-transformed commercial proportion-at-length data, which is estimated in the fitting procedure by the closed form solution:

$$\sigma_j^{S,coml} = \sqrt{\sum_{y=y1}^{yn} \sum_{q=1}^{4} \sum_{l>=6cm} \hat{p}_{j,y,q,l}^{S,coml} \left(\ln \hat{p}_{j,y,q,l}^{S,coml} - \ln p_{j,y,q,l}^{coml}\right)^2 \Big/ \sum_{y=y1}^{yn} \sum_{q=1}^{4} \sum_{l>=6cm} 1} \text{[8]}.$$

<u>Fixed Parameters for the Base Case Hypotheses</u>

The following parameters were fixed externally in the model:

In the base case assessment, natural mortality is assumed to be time-invariant, thus $\sigma_j = \sigma_{ad} = 0$, giving $\varepsilon_y^j = \varepsilon_y^{ad} = 0$.

$S_{j,y,l} = 0$, $l = 1,\ldots,5$, $y = y_1,\ldots,y_n$, see footnote 7.

Sardine of length 9.5-19.5cm are taken to be fully selected in the survey trawls: $S_{j,l}^{survey} = 1$, $l = 13,\ldots,33$.

The weighting on the commercial proportions-at-length data should be about $1/(4\times 6)\approx 0.04$ of that on the commercial proportions-at-age data, where the weighting is first reduced by a quarter due to the fact that there are four (quarterly) data points to every annual survey estimate of abundance, and also reduced by a sixth as 35 length classes are fit in the likelihood in comparison to 6 ages if proportions-at-age data had been available, with the two carrying essentially the same information content. Thus a value of $w_{propl}^{com} = 0.04$ is set. Similarly the weighting for the survey proportions-at-length data is set at one sixth, $w_{propl}^{sur} = 0.167$.

The CV associated with factors causing bias in the acoustic survey estimated which vary inter-annually is fixed at the CV of the posterior distribution calculated in Figure B.2, i.e. $\phi_{ac}^{S} = 0.215/0.969 = 0.222$.

From the von Bertalanffy growth curve (Durholtz and Mtengwane pers. comm.), $\frac{\overline{w_{j,2}^S}}{w_{j,1}^S} = 1.40$, $\frac{\overline{w_{j,3}^S}}{w_{j,1}^S} = 1.69$, $\frac{\overline{w_{j,4}^S}}{w_{j,1}^S} = 1.88$, and $\frac{\overline{w_{j,5+}^S}}{w_{j,1}^S} = 2.00$ for the single stock hypothesis. For the two stock hypothesis, $\frac{\overline{w_{1,2}^S}}{w_{1,1}^S} = 1.39$, $\frac{\overline{w_{1,3}^S}}{w_{1,1}^S} = 1.65$, $\frac{\overline{w_{1,4}^S}}{w_{1,1}^S} = 1.80$, $\frac{\overline{w_{1,5+}^S}}{w_{1,1}^S} = 1.89$, $\frac{\overline{w_{2,2}^S}}{w_{2,1}^S} = 1.39$, $\frac{\overline{w_{2,3}^S}}{w_{2,1}^S} = 1.68$, $\frac{\overline{w_{2,4}^S}}{w_{2,1}^S} = 1.89$, and $\frac{\overline{w_{2,5+}^S}}{w_{2,1}^S} = 2.02$.

Estimable Parameters and Prior Distributions for the Base Case Hypotheses

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve. For the single stock hypothesis, the variance about the stock recruitment curve is assumed to differ between peak and non-peak years, i.e.the prior pdfs for the recruitment residuals are given by:

$$\varepsilon_{j,y}^S \sim N\left(0,\left(\sigma_r^S\right)^2\right) \quad , \qquad y = y_1,\ldots,1999,2005,\ldots,y_{n-1}$$

$$\varepsilon_{j,y}^S \sim N\left(0,\left(\sigma_{r,peak}^S\right)^2\right) \quad , \qquad y = 2000,\ldots,2004$$

while for the two stock hypothesis, the variance about the stock recruitment curves is assumed to differ between stocks, but not over years, i.e.

$$\varepsilon_{j,y}^S \sim N\left(0,\left(\sigma_{j,r}^S\right)^2\right) \quad , \quad y = y_1,\ldots,y_{n-1}$$

$k_{ac}^S \sim N\left(0.714, 0.077^2\right)$, see Appendix B

The remaining estimable parameters are defined as having the following near non-informative prior distributions:

$move_y \sim U(0,1)$, $y = y_1,\ldots,y_n$, for the two stock hypothesis only

$$k_{\text{cov}}^S \sim U(0.3,1)$$

$$k_{\text{cov}\,E}^S \sim U(0,1)$$

$$\left(\lambda_{j,N/r}^S\right)^2 \sim U(0,10)$$

Initial results indicated that survey selectivity-at-length could be reasonably well reflected by these constant levels:

$S_{j,l}^{survey} = S_j^{survey1} \sim U(0.6,1.1)$[13], $l = 1,\ldots,12$

$S_{j,l}^{survey} = S_j^{survey5} \sim U(0.9,1.1)$, $l = 34,\ldots,40$

While the priors for the commercial selectivity-at-length parameters are:

$$\chi_j \sim U(0,1)$$

[13] By design, surveys aim to achieve equal selectivity over all ages. Age 1 sardine distributed inshore may be under caught in comparison to other ages. On the other hand older, faster fish may be more able to avoid day-time trawls and thus be under represented in any day-time (about ½) trawl samples. It is, however, most likely that selectivity of ages 3 to 5+ is flat (Coetzee pers. comm.).

$\bar{l}_{1,j} \sim U(5cm, 15cm)$

$\bar{l}_{2,j} - \bar{l}_{1,j} \sim U(0cm, 15cm)$

$\left(\sigma_{1,j}^{sel}\right)^2 \sim U(2,7)$

$\left(\sigma_{2,j}^{sel}\right)^2 \sim U(0,2)$

For the single stock hypothesis: $\left(\sigma_r^S\right)^2 \sim U(0.16, 10)$ and $\left(\sigma_{r,peak}^S\right)^2 \sim U(0.16, 10)$

While for the two stock hypothesis: $\left(\sigma_{j,r}^S\right)^2 \sim U(0.16, 10)$, corresponding to a range of $\sigma_{j,r}^S$ from 0.4 to about 3.2

$N_{j=1,1983,a}^S \sim U(0, 50)$ billion $a = 0,1,2$

$N_{j=2,1983,a}^S \sim U(0, 50)$ billion $a = 0$

$N_{j=1,1983,a}^S = N_{j=1,1983,a-1}^S e^{-\left(Finit_j + \bar{M}_a^S\right)}$ $a = 3,4$

$N_{j=2,1983,a}^S = N_{j=2,1983,a-1}^S e^{-\left(Finit_j + \bar{M}_a^S\right)}$ $a = 1,2,3,4$

$N_{j,1983,5+}^S = N_{j,1983,4}^S \dfrac{e^{-\left(Finit_j + \bar{M}_{ad}^S\right)}}{1 - e^{-Finit_j - \bar{M}_{ad}^S}}$, with $Finit_j \sim U(0,1)$

$\vartheta_{j,a} \sim U(0.01, 3)$, for $a = 1, \ldots, 5+$ These parameters are assumed to be the same for the "west" and "south" stocks.

$L_{j,\infty} \sim U(10,30)$

$\kappa_j \times L_{j,\infty} \sim U(0,10)$

$t_{0,j} \sim U(-4, 4)$ These parameters are assumed to be the same for the "west" and "south" stocks

Further Outputs

Recruitment serial correlation:

$$s_{j,cor}^S = \frac{\sum_{y=y1}^{yn-2} \varepsilon_{j,y} \varepsilon_{j,y+1}}{\sqrt{\left(\sum_{y=y1}^{yn-2} \varepsilon_{j,y}^2\right)\left(\sum_{y=y1}^{yn-2} \varepsilon_{j,y+1}^2\right)}} \tag{A.27}$$

and the standardised recruitment residual value for 2011:

$$\eta_{j,2010}^S = \frac{\varepsilon_{j,2010}^S}{\sigma_{j,r}^S} \tag{A.28}$$

are also required as input into the OM.

Appendix B: Glossary of parameters used in this document

Annual numbers and biomass:

$N^S_{j,y,a}$ - model predicted number (in billions) of sardine of age a at the beginning of November in year y of stock j

$B^S_{j,y}$ - model predicted biomass (in thousand tonnes) of adult sardine of stock j at the beginning of November in year y, associated with the November survey

$SSB^S_{j,y}$ - model predicted spawning stock biomass (in thousand tonnes) of stock j at the beginning of November in year y

$w^S_{j,y,a}$ - mean mass (in grams) of sardine of age a of stock j sampled during the November survey of year y

$w^{S1+}_{j,y}$ is the total (1+) mean mass (in grams) of sardine of stock j sampled during the November survey of year y

$\overline{\dfrac{w^S_{j,a}}{w^S_{j,1}}}$ is the average ratio of mean mass (in grams) of sardine of stock j aged a to age 1 obtained from the growth curve

$N^S_{j,y,r}$ - model predicted number (in billions) of juvenile sardine of stock j at the time of the recruit survey in year y

t^S_y - time lapsed (in months) between 1 May and the start of the recruit survey in year y

$move_y$ - proportion of west stock recruits which migrate to the east stock at the beginning of November of year y

Natural mortality:

$M^S_{a,y}$ - rate of natural mortality (in year^{-1}) of sardine of age a in year y

$\overline{M}^S_{ju}$ - median juvenile rate of natural mortality (in year^{-1})

$\overline{M}^S_{ad}$ - median adult rate of natural mortality (in year^{-1})

ε^{ad}_y - annual residuals about adult natural mortality

η^{ad}_y - normally distributed error used in calculating ε^{ad}_y

σ_{ad} - standard deviation in the annual residuals about adult natural mortality

σ_j - standard deviation in the annual residuals about juvenile natural mortality

p - annual autocorrelation coefficient in annual residuals about adult natural mortality

Commercial selectivity

$S_{j,y,l}$ - commercial selectivity at length l during year y of stock j

χ_j - denotes the height of the near-normal curve for stock j

l_{mid} - the midpoint (in cm) of length class l

$l_{\max} = 23.5cm$ - one length class above the maximum

$\bar{l}_{1,j}$ - the mean of the near-normal distribution for stock j

$\bar{l}_{2,j}$ - the median of the near-lognormal distribution for stock j

$\left(\sigma_{1,j}^{sel}\right)^2$ - the variance parameter of the near-normal distribution for stock j

$\left(\sigma_{2,j}^{sel}\right)^2$ - the variance parameter of the near-lognormal distribution for stock j

$S_{j,y,q,a}$ - commercial selectivity at age a during quarter q of year y of stock j

Catch:

$C_{j,y,a,q}^{S}$ - model predicted umber (in billions) of sardine of age a of stock j caught during quarter q of year y

$C_{j,y,m,l}^{RLF}$ - number of fish in length class l landed in month m of year y of stock j (the 'raised length frequency')

$lcut_{y,m}$ - cut off length for recruits in month m of year y

$C_{j,y,q,a}^{bycatch}$ - the number of fish of age $a \geq 1$ from the anchovy-directed fishery in quarter q of year y

$F_{j,y,q}$ - fished proportion in quarter q of year y for a fully selected age class a of stock j, by the directed and redeye bycatch fisheries

$\tilde{C}_{j,y,0bs}^{S}$ - number (in billions) of juvenile sardine of stock j caught between 1 May and the day before the start of the recruit survey

Proportions at age:

$p_{j,y,a}^{S}$ - model predicted proportion-at-age a of stock j in the November survey of year y

$S_{j,a}^{survey}$ - survey selectivity at age a in the November survey for stock j

$p_{j,y,q,a}^{com,S}$ - model predicted proportion-at-age a of stock j in the directed and redeye bycatch commercial catch of quarter q of year y

Recruitment:

h_j^S - "steepness" of the stock-recruitment relationship for stock j

K_j^S - carrying capacity for stock j

K_{peak}^S - carrying capacity during peak years (only for single stock hypothesis)

a_j^S - maximum recruitment of stock j in the hockey stick model;

b^S_j - spawner biomass for stock j below which the expectation for recruitment is reduced below the maximum

c^S - constant recruitment (distribution median) during the "peak" years of 2000 to 2004 (only for single stock hypothesis)

$\varepsilon^S_{j,y}$ - annual lognormal deviation of sardine recruitment.

$\sigma^S_{j,r}$ - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve of stock j

$\sigma^S_{r,peak}$ - standard deviation in the residuals (lognormal deviation) about the stock recruitment curve during peak years in the single stock hypothesis

Proportions at length and growth curve:

$p^S_{j,y,l}$ - model predicted proportion-at-length l of stock j associated with the November survey in year y

$A^{sur}_{j,a,l}$ - proportion of sardine of age a of stock j that fall in the length group l in November

$p^{coml,S}_{j,y,q,l}$ - model predicted proportion-at-length l of stock j in the directed and redeye bycatch commercial catch of quarter q of year y

$A^{com}_{j,q,a,l}$ - proportion of sardine of age a of stock j that fall in the length group l in quarter q

$L_{j,\infty}$ - maximum length of sardine of stock j

κ_j - annual growth rate of sardine of stock j

$t_{0,j}$ - age at which the length of sardine of stock j is zero

$\vartheta_{j,a}$ - standard deviation about the mean length for age a of sardine of stock j

Likelihoods:

$-\ln L^{Nov}$ - contribution to the negative log likelihood from the model fit to the November 1+ biomass data

$-\ln L^{rec}$ - contribution to the negative log likelihood from the model fit to the May recruit data

$-\ln L^{sur\ propl\ \min}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data for the minus length class only

$-\ln L^{sur\ propl}$ - contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data for the minus length class only

$-\ln L^{com\ propl}$ - contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data for the remaining length classes

$\hat{B}^S_{j,y}$ - acoustic survey estimate (in thousands of tonnes) of adult sardine biomass of stock j from the November survey in year y

$\sigma^S_{j,y,Nov}$ - survey sampling CV associated with $\hat{B}^S_{j,y}$ that reflects survey inter-transect variance

$k^S_{j,N}$ - constant of proportionality (multiplicative bias) associated with the November survey of stock j

k_{ac}^{S} - multiplicative bias associated with the acoustic survey

$\hat{N}_{j,y,r}^{S}$ - acoustic survey estimate (in billions) of sardine recruitment numbers of stock j from the recruit survey in year y

$\sigma_{j,y,rec}^{S}$ - survey sampling CV associated with $\hat{N}_{j,y,r}^{S}$ that reflects survey inter-transect variance

$k_{j,r}^{S}$ - constant of proportionality (multiplicative bias) associated with the recruit survey of stock j

k_{cov}^{S} - multiplicative bias associated with the coverage of the recruits by the recruit survey in comparison to the 1+ biomass by the November survey

$k_{\mathrm{cov}\,E}^{S}$ - multiplicative bias associated with the coverage of the east stock recruits by the recruit survey in comparison to the west stock recruits during the same survey

ϕ_{ac}^{S} - the CV associated with factors which cause bias in the acoustic survey estimates and which vary inter-annually;

$(\lambda_{j,N/r}^{S})^{2}$ - additional variance (over and above $\sigma_{y,Nov/rec}^{S}$ and ϕ_{ac}^{S}) associated with the November/recruit surveys of stock j;

$\hat{p}_{j,y,l}^{S}$ - observed proportion (by number) of sardine from stock j in length group l in the November survey of year y;

$w_{propl\,\mathrm{min}}^{sur}$ - weighting applied to the survey proportion at length data for the minus length class;

w_{propl}^{sur} - weighting applied to the remaining survey proportion at length data;

$\sigma_{j}^{S,surl\,\mathrm{min}}$ - variance-related parameter for the log-transformed survey proportion-at-length data for the minus length class;

$\sigma_{j}^{S,surl}$ - variance-related parameter for the log-transformed survey proportion-at-length data;

$\hat{p}_{j,y,q,l}^{S,coml}$ - observed proportion (by number) of the directed and redeye bycatch commercial catch in length group l of during quarter q of year y;

w_{propl}^{com} - weighting applied to the commercial proportion at length data

$\sigma_{j}^{S,coml}$ - variance-related parameter for the log-transformed commercial proportion-at-length data

Other:

F_{init} - rate of fishing mortality assumed in the initial year

$s_{j,cor}^{S}$ - recruitment serial correlation for stock j

$\eta_{j,2009}^{S}$ - standardised recruitment residual value for 2009 for stock j

$\overline{w}^{S}_{j,a}$ - mean mass (in grams) of sardine of age a from stock j sampled during each November survey, averaged over all years

$w^{catch}_{j,y,a}$ - mean mass (in grams) in the catch of sardine of age a from stock j in year y (from de Moor *et al*. 2012a).