## Further updated SCRL assessment results for alternative selectivity functions

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This document extends previous assessments in two ways:

- It changes the model so that B<sub>exp</sub> mid-year takes into account of half of the "selectivity weighted fishing mortality losses" see Table 1 for the changes this makes to results.
- It investigates alternate selectivity functions –see Tables 2 and 3 and Figures 1-4.

#### Methods - the new selectivity function

$$S_{y,l}^{m/f,A} = \frac{e^{-\mu^{m/f,A}.l}}{1 + e^{(-\delta^{m/f,A}(l-l_*^{m/f,A}))}}$$
(1)

Thus there are three estimable parameters for each sex and each area ( $\mu$ ,  $\delta$  and  $l^*$ ).

For Area A1E and A1W – selectivity is assumed to remain constant over time.

For Area A2+3 selectivity is allowed to vary over time for the period for which there are catch-at-length data (1995-2010).

Thus for y=1995, 2010:

$$\begin{split} l^m_* &\to l^m_* + \varepsilon^m_{l^*,y} & \varepsilon^m_{l^*,y} \sim N(0,\sigma^2_{l^*,m}) \\ l^f_* &\to l^f_* + \varepsilon^f_{l^*,y} & \varepsilon^f_{l^*,y} \sim N(0,\sigma^2_{l^*,f}) \\ \mu^m &\to \mu^m + \varepsilon^m_{\mu,y} & \varepsilon^m_{\mu,y} \sim N(0,\sigma^2_{\mu,m}) \\ \mu^f &\to \mu^f + \varepsilon^f_{\mu,y} & \varepsilon^f_{\mu,y} \sim N(0,\sigma^2_{\mu,f}) \\ \delta^m &\to \delta^m + \varepsilon^m_{\delta,y} & \varepsilon^m_{\delta,y} \sim N(0,\sigma^2_{\delta,m}) \\ \delta^f &\to \delta^f + \varepsilon^f_{\delta,y} & \varepsilon^f_{\delta,y} \sim N(0,\sigma^2_{\delta,f}) \end{split}$$

Extra terms are added to the negative log likelihood to limit the extent to which the  $\varepsilon$  estimates differ from zero:

$$-lnL \rightarrow -lnL + \frac{1}{2} \sum_{y, \frac{m}{f}} \left(\frac{\varepsilon_{l*, y}^{m}}{\sigma_{l*, m}}\right)^{2} + \left(\frac{\varepsilon_{l*, y}^{f}}{\sigma_{l*, f}}\right)^{2} + \left(\frac{\varepsilon_{\mu, y}^{m}}{\sigma_{\mu, m}}\right)^{2} + \left(\frac{\varepsilon_{\delta, y}^{f}}{\sigma_{\delta, m}}\right)^{2} + \left(\frac{\varepsilon_{\delta, y}^{f}}{\sigma_{\delta, m}}\right)^{2} + \left(\frac{\varepsilon_{\delta, y}^{f}}{\sigma_{\delta, m}}\right)^{2}$$
(2)

An issue to be taken into account is that for equation (1), if for example  $l_*^m$  decreases, this means that selectivity is increasing on younger lobsters; however given that the model fitting procedure assumes that:

$$\hat{CPUE}_{y} = q \sum_{l} w_{l} (1 - S_{l,a} F_{y} / 2) N_{l,a} e^{-M/2}$$
(3)

this situation seems implausible as an enhanced CPUE would result even if there was not any increase in abundance.

Presumably enhanced catches of younger animals are achieved by spatially redistributing effort on a scale finer than captured by the GLM standardisation of the CPUE. A standard method to adjust for this, while maintaining a constant catchability coefficient q, is to renormalise the selectivity function in some way:

$$S_{y,l}^{m/f,A} \to S_{y,l}^{*,m/f,A} = S_{y,l}^{m/f,A} / X_{y}^{m/f,A}$$
(4)

where here as a simple initial approach we have chosen:

$$X_{y}^{m/f,A} = \sum_{l_{1}^{m/f,A}}^{l_{2}^{m/f,A}} \frac{S_{y,l}^{m/f,A}}{l_{2}^{m/f,A} - l_{1}^{m/f,A} + 1}$$
(5)

i.e., normalising selectivity by its average over a certain length range, so that now if  $\delta_y^{m/f,A}$  decreases, the  $S_{y,l}^{*,m/f,A}$  will decrease for large *l* to compensate for the effort spread to locations where younger animals are found associated with the increase for smaller *l*.

The values of  $l_1^{m/f,A}$  and  $l_2^{m/f,A}$  have been fixed at the following values to ensure that the ranges associated with these *I* values cover the greater part of these distributions. [Note that for the moment, these values remain the same as were used for previous OM1 type assessments.]

m/f	area	$l_1^{m/f,A}$	$l_2^{m/f,A}$
m	1E	65mm	90mm
f	1E	65mm	90mm
m	1W	65mm	90mm
f	1W	65mm	90mm
m	2+3	55mm	90mm
f	2+3	55mm	90mm

Note that female selectivity is then scaled finally by a scalar  $\gamma^A$  for each area.

A number of variants were explored where the amount of variation permitted in the time-varying selectivity parameters was varied.

Model	$\sigma_{l^{*,m}}$	$\sigma_{l^{*,f}}$	$\sigma_{\mu,m}$	$\sigma_{\mu,f}$	$\sigma_{\delta,m}$	$\sigma_{\delta,f}$
N1	0	0	0	0	0	0
N2	9	9	0.03	0.03	0.3	0.3
N3a	3	3	0	0	0	0
N3b	0	0	0.01	0.01	0	0
N3c	0	0	0	0	0.1	0.1
N4	1	1	0.003	0.003	0.03	0.03
N5	3	3	0.01	0.01	0.1	0.1

#### Results

The changes to the calculation of  $B_{exp}$  result in a more optimistic appraisal of resource status, with an increase in spawning biomass both in absolute terms and as a fraction of its pristine level (from 35 to 40% - see Table 1).

Table 2 shows that with the addition of time-variability to the various parameters of the selectivity function of equation (1), the greatest improvement to the fit is achieved by admitting variability in the  $\mu$  parameter which controls the extent of doming in the function at larger lengths (case N3a). Thus further results on the form of Figures are restricted to that case and to N5 and N2 which sequentially allow greater variability in all the parameters.

The three options all fit the CPUE effectively equally well. The real test, however, is whether they reduce the extent of systematic patterning (non-randomness) in the standardised residuals which is evident in these results for the Var2 option (see Figure 4a). Some indications of improvement in this regard are evident in these results for cases N2 (Figure 4c) and N5 (Figure 4e), though patterning still remains evident.

If pressed for a preference at this stage, our choice would be N5 over N2, as the latter probably admits more variability in the selectivity parameters than the available data can justify. Note that N5 reflects a slightly better current resource status than N2 (spawning biomass depletion of 0.36 rather than 0.33 – see Table 2.

Table 1: Estimated model parameters and -lnL values for VAR2 as previously reported, and VAR2 updated to account for half fishing mortality having taken place by mid-year.

	VAR2	VAR2
		updated
		With Bexp mid
		year
	Var2.tpl	Var2.tpl
# parameters	134	134
W <sub>len</sub>	1.0	1.0
$\sigma_{sel}$	7.5	7.5
$\sigma_R$	0.8	0.8
$\sigma_{\lambda}$	1.0	1.0
-InL Total	-387.53	-389.28
-InI CPUE	-116.83	-124.21
-Inl CPUE A1E	-17.29	-17.23
-InI CPUE A1W	-48.49	-50.87
-InI CPUE A2+3	-51.04	-56.10
-In SCI CAL	-287.16	-285.16
-In SCI CAL A1E	-11.26	-10.04
-In SCI CAL A1W	-96.20	-95.20
-In SCI CAL A2+3	-179.70	-179.92
CPUE Α1Ε σ	0.365	0.365
CPUE A1W σ	0.146	0.135
CPUE A2+3 σ	0.135	0.116
SCI CAL A1E σ	0.138	0.140
SCI CAL A1W σ	0.091	0.092
SCI CAL A2+3 σ	0.068	0.068
К	2598	2469
$\lambda^{A1E}$	0.174	0.179
$\lambda^{A1W}$	0.295	0.292
$\lambda^{A2+3}$	0.531	0.529
g75	3.404	3.422
карра	0.112	0.114
Δgm	0.880	0.885
Δg1E	-2.800	-2.829
Δg1W	-0.491	-0.566
$B_{\rm sp}(2011) (B_{\rm sp}(2011)/K_{\rm sp})$	897 (0.35)	978 (0.40)
B <sub>exp</sub> (2011) (B <sub>exp</sub> (2011)/K <sub>exp</sub> ) A1E	56 (0.16)	33 (0.13)
$B_{exp}(2011) (B_{exp}(2011)/K_{exp}) A1W$	692 (0.42)	340 (0.41)
$B_{exp}(2011) (B_{exp}(2011)/K_{exp}) A2+3$	1666 (0.32)	591 (0.29)

Table 2: Estimated model parameters and –*InL* values for the current BC and four variants.

	VAR2 Old Selectivity With TVS	N1 (no TVS)	N2 ("totally" free TVS)	N3a (free µ only medium)	N3b (free /* only medium)	N3c (free δ only medium)	N4 (all free low)	N5 (all free medium)
	Var2.tpl	Scl.tpl						
# parameters	134	219	219	219	219	219	219	219
-InL Total	-389.28	-334.51	-514.82	-392.25	-353.42	-343.08	-356.32	-425.64
-InL Total*	-398.30	-334.41	-542.40	-410.71	-364.35	-347.63	-373.12	-470.26
-InI CPUE	-124.21	-125.10	-114.91	-124.06	-127.95	-123.45	-124.70	-121.11
-InI CPUE A1E	-17.23	-17.93	-17.59	-18.37	-17.95	-17.90	-17.96	-18.02
-InI CPUE A1W	-50.87	-52.17	-52.38	-52.33	-52.57	-52.29	-52.25	-52.45
-Inl CPUE A2+3	-56.10	-55.00	-44.84	-53.33	-57.43	-53.26	-54.49	-50.64
-In SCI CAL	-285.16	-219.17	-432.62	-292.28	-247.16	-232.74	-256.99	-355.03
-In SCI CAL A1E	-10.04	-9.22	-9.16	-8.55	-9.50	-8.81	-9.10	-9.13
-In SCI CAL A1W	-95.20	-96.86	-96.42	-97.73	-96.44	-96.32	-96.72	-96.92
-In SCI CAL A2+3	-179.92	-113.09	-327.03	-186.00	-141.23	-127.61	-151.17	-248.97
κ	2469	3074	3435	3152	2989	3289	3176	3258
$\lambda^{A1E}$	0.179	0.179	0.172	0.175	0.180	0.175	0.176	0.172
$\lambda^{A1W}$	0.292	0.299	0.274	0.283	0.299	0.291	0.292	0.280
$\lambda^{A2+3}$	0.529	0.522	0.554	0.542	0.521	0.534	0.532	0.548
$B_{\rm sp}(2011) (B_{\rm sp}(2011)/K_{\rm sp})$	978 (0.40)	1207 (0.39)	1123 (0.33)	1172 (0.37)	1157 (0.39)	1259 (0.38)	1219 (0.38)	1160 (0.36)
B <sub>exp</sub> (2011) (B <sub>exp</sub> (2011)/K <sub>exp</sub> ) A1E	33 (0.13)	45 (0.16)	45 (0.16)	59 (0.19)	44 (0.15)	45 (0.16)	46 (0.16)	50 (0.17)
<i>B</i> <sub>exp</sub> (2011) ( <i>B</i> <sub>exp</sub> (2011)/ <i>K</i> <sub>exp</sub> ) A1W	340 (0.41)	414 (0.51)	403 (0.51)	424 (0.53)	423 (0.52)	412 (0.51)	415 (0.51)	424 (0.53)
$B_{exp}(2011) (B_{exp}(2011)/K_{exp}) A2+3$	591 (0.29)	824 (0.36)	592 (0.33)	719 (0.34)	772 (0.53)	882 (0.37)	833 (0.36)	699 (0.33)

\*without TVS penalty

Table 3: Estimated selectivity parameters.

	N1	N2	N3a	N3b	N3c	N4	N5
	(no TVS)	("totally"	(free μ	(free /*	(free $\delta$	(all free	(all free
		free TVS)	only	only	only	low)	medium)
			medium)	medium)	medium)		
$\mu^{m,A1E}$	0.031	0.025	0.044	0.027	0.029	0.031	0.033
$\mu^{f,A1E}$	0.032	0.031	0.036	0.034	0.031	0.032	0.036
$\mu^{m,A1W}$	0.023	0.044	0.031	0.022	0.044	0.031	0.038
$\mu^{f,A1W}$	0.021	0.014	0.030	0.016	0.017	0.020	0.020
$\mu^{m,A2+3}$	0.020	0.017	0.023	0.020	0.017	0.019	0.021
$\mu^{f,A2+3}$	0.000	0.026	0.001	0.000	0.000	0.000	0.004
$l *^{m,A1E}$	69.95	69.73	70.24	69.79	69.93	69.97	70.00
$l *^{f,A1E}$	72.16	72.29	72.46	72.26	72.15	72.20	72.48
$l *^{m,A1W}$	64.97	62.03	64.07	64.98	68.63	65.52	64.37
$l *^{f,A1W}$	65.93	65.71	66.10	65.78	65.82	65.90	65.85
$l *^{m,A2+3}$	70.28	70.16	70.46	70.29	70.12	70.23	70.38
l * f, A2+3	61.49	60.38	61.28	61.61	61.69	61.72	61.11
$\delta^{m,A1E}$	0.34	0.34	0.35	0.34	0.34	0.34	0.34
$\delta^{f,A1E}$	0.28	0.28	0.29	0.28	0.28	0.28	0.29
$\delta^{m,A1W}$	0.35	0.47	0.38	0.35	0.25	0.35	0.36
$\delta^{f,A1W}$	0.41	0.41	0.42	0.41	0.41	0.41	0.41
$\delta^{m,A2+3}$	0.29	0.28	0.29	0.29	0.28	0.28	0.29
$\delta^{f,A2+3}$	0.43	0.53	0.47	0.42	0.43	0.46	0.48
$\gamma^{A1E}$	0.50	0.50	0.50	0.50	0.50	0.50	0.50
$\gamma^{A1W}$	0.50	0.50	0.50	0.50	0.50	0.50	0.50
$\gamma^{A2+3}$	0.50	0.50	0.50	0.50	0.50	0.50	0.50







### Figure 2a: N2 (all "totally" free) A2+3 selectivity functions.



### Figure 2b; N3a (mu only free) A2+3 selectivity functions.



### Figure 2c; N5 (all free medium) A2+3 selectivity functions.



Figure 3a: N2 (all "totally" free) fits to A2+3 CAL data.





Figure 3b: N3a (mu only free) fits to A2+3 CAL data.





### Figure 3c: N5 (all free medium) fits to A2+3 CAL data.



Figure 4a: Var2 A2+3 CAL residuals.



Figure 4b: N1 (no TVS) A2+3 CAL residuals.





Figure 4c: N2 (all "totally" free) A2+3 CAL residuals.



Figure 4d: N3a (mu only free) A2+3 CAL residuals.



Figure 4e: N5 (all free medium) A2+3 CAL residuals.