

# Penguin model projections: 20 years, autocorrelation, and immigration

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Three requests for further results which were raised at a previous Pelagic Scientific Working Group meeting are addressed:

1. Plots of projected numbers of penguins show 20 year projections, corresponding to the full period for which the sardine operating model is projected (Figure 1).
2. Autocorrelation (AR1 process) has been included in the calculation of projected random variations about the penguin mortality–sardine biomass relationship. This has almost no effect on the projected median values, but the variance increases slightly (Figure 2).
3. Immigration has been introduced into the first 11 years of the projections, matching the annual immigration estimated to have taken place between 1989 and 1999. With the “optimistic” future sardine biomass distribution, the posterior median number of projected penguins increases rapidly and then levels off when immigration stops (Figure 3a). With the less optimistic future sardine distribution, there is little change to the median and lower 10%-ile projections, but the upper end of the probability envelope shown is substantially higher (Figure 3b).

The calculations for the first and third figures are straightforward and need no further explanation.

The autocorrelation coefficient for the penguin mortality–sardine biomass relationship,  $\rho = 0.259$ , is the value of  $\rho$  which minimizes

$$\sum_y (\varepsilon_{y+1} - \rho\varepsilon_y)^2$$

where  $\varepsilon_y$  are the residuals of the biomass–mortality relationship:

$$\varepsilon_y = M_y - (M_{\min} + f_S(B_{S,y}))$$

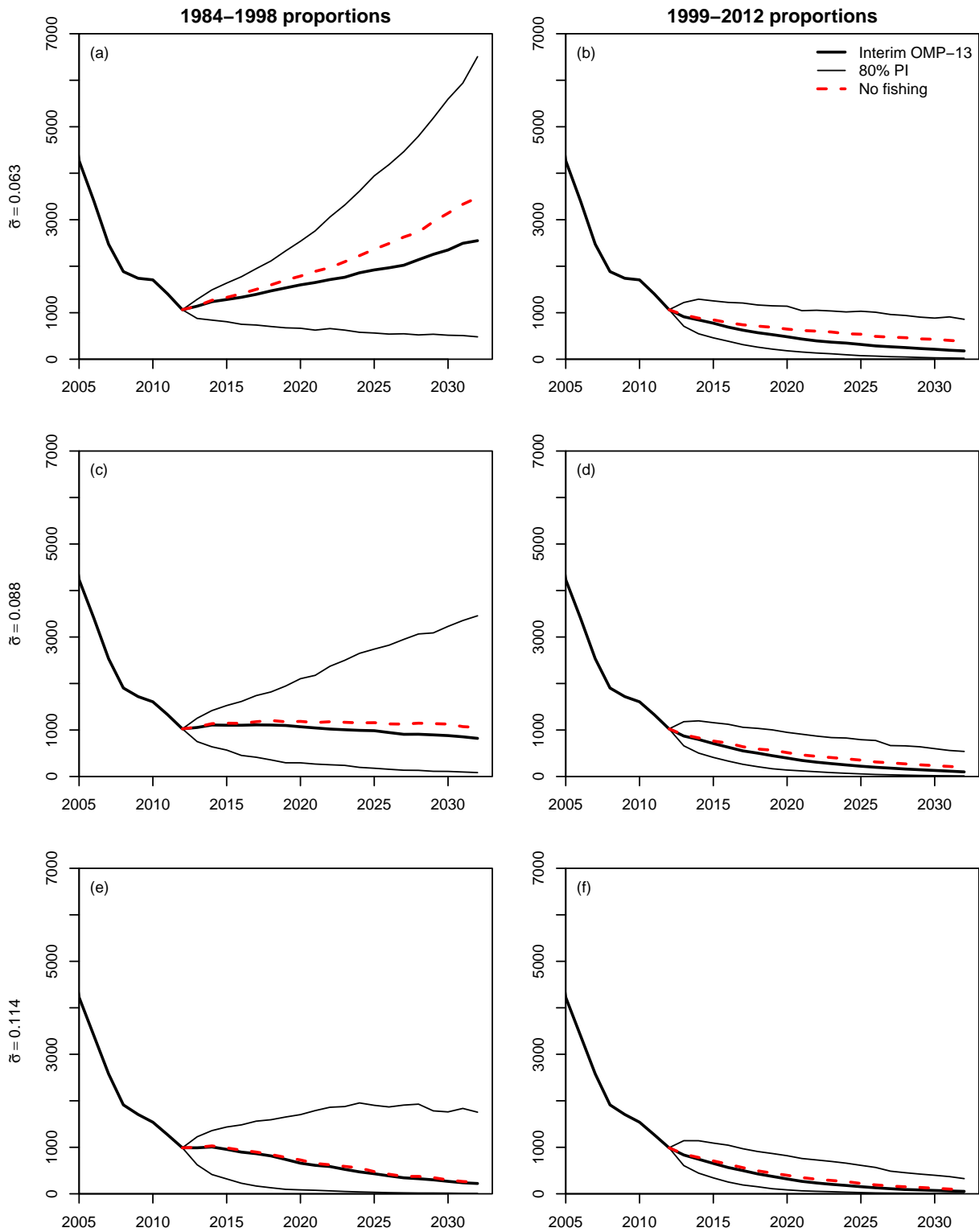
The simulated random effects for the projections are then calculated as:

$$\varepsilon_{y+1} = \rho\varepsilon_y + \sqrt{1 - \rho^2} \eta_y$$

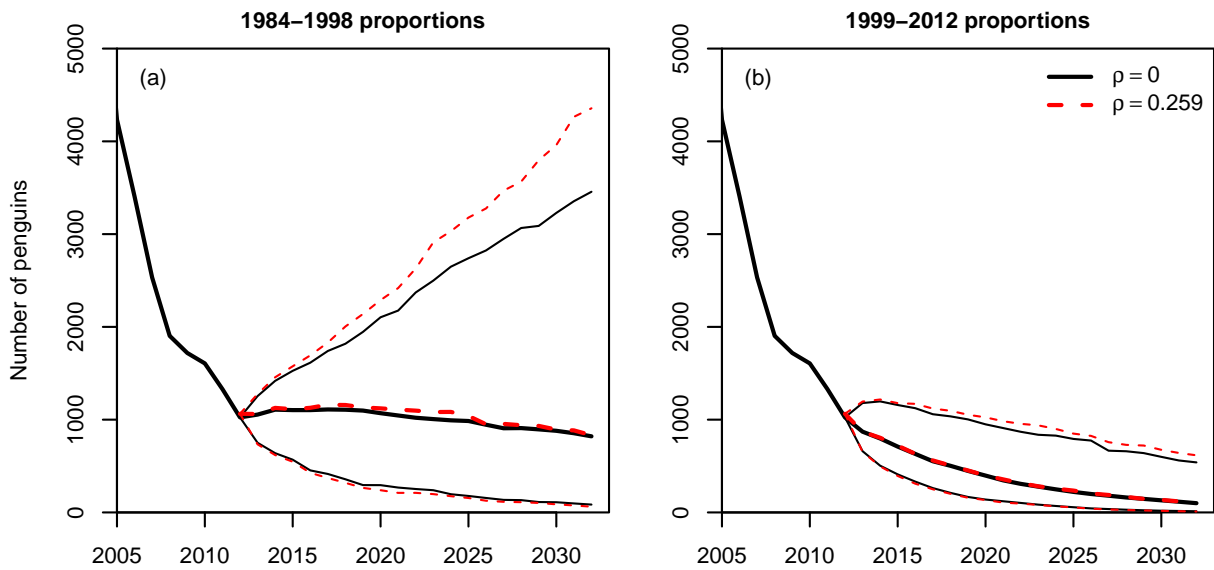
where  $\eta_y \sim N(0, 1)$ .

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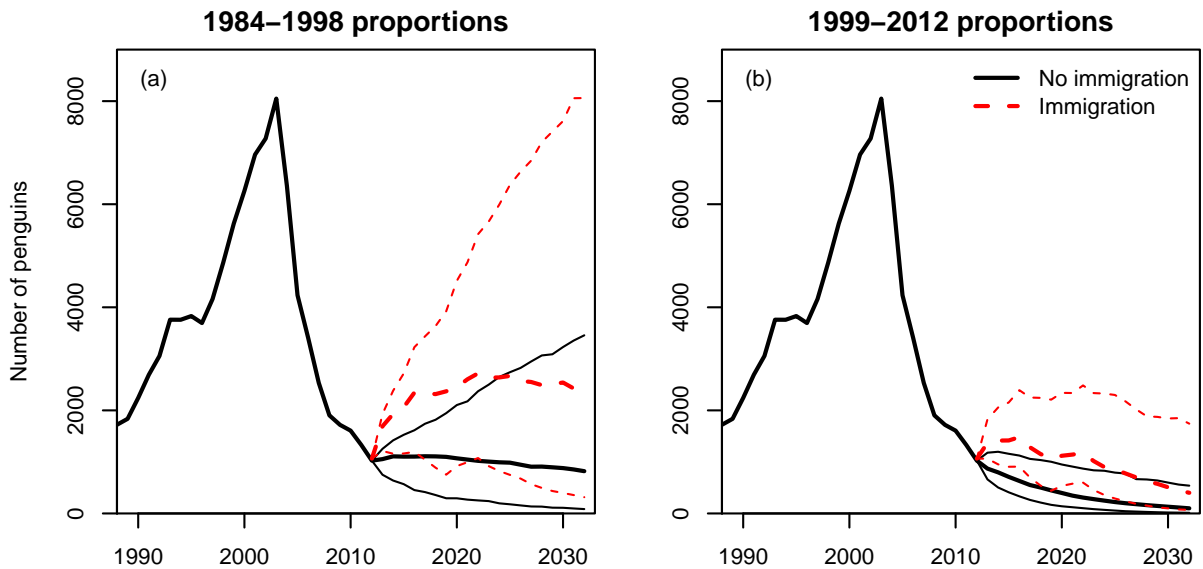
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**Figure 1:** Penguin population projections extended to 2032 for various values of  $\tilde{\sigma}$  and different assumptions concerning the future distribution of sardine biomass. The projections show posterior medians and 80% probability envelopes.



**Figure 2:** Penguin population projections for  $\tilde{\sigma} = 0.088$  (solid lines) compared with similar projections incorporating autocorrelation in the mortality random effects (dashed lines). The thick lines show posterior medians and the thin lines indicate the 80% probability envelopes.



**Figure 3:** Penguin population posterior median projections for  $\tilde{\sigma} = 0.088$  (solid lines) compared with similar projections incorporation immigration in the first 11 years (dashed lines). The thin lines indicate the 80% probability envelopes.