The Age-structured Production Modelling approach for assessment of the Rock Lobster Resources at the Tristan da Cunha group of islands

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The stock assessment approach for all four islands of the Tristan da Cunha group is to use an age-structured production model (ASPM) to fit to catch, longline standardised CPUE and catch-at-length (CAL) data. The models consider only catches from 1990, i.e. models are initiated in 1990. The method for setting up the initial population age structure in 1990 is given below. Note that these assessments do not take into account any effect of the OLIVA oil/soya spill even which took place in March 2011, as the data for these assessments were collected prior to this event.

1. The population model

The resource dynamics are modeled by the equations:

$$N^{m}_{y+1,0} = R_{y+1} \quad (1)$$

$$N^{f}_{y+1,0} = R_{y+1} \quad (2)$$

$$N^{m}_{y+1,a+1} = \sum_{l} \,[\vec{N}^{m}_{y,a,l} e^{-M^{m}/2} - \vec{C}^{m}_{y,a,l} - D^{m}_{y,a,l}] e^{-M_a/2} \quad (3)$$

$$N^{f}_{y+1,a+1} = \sum_{l} \,[\vec{N}^{f}_{y,a,l} e^{-M^{f}/2} - \vec{C}^{f}_{y,a,l} - D^{f}_{y,a,l}] e^{-M_a/2} \quad (4)$$

$$N^{m}_{y+1,p} = \sum_{l} \,[\vec{N}^{m}_{y,p-1,l} e^{-M_a/2} - \vec{C}^{m}_{y,p-1,l} - D^{m}_{y,p-1,l}] e^{-M_a/2} + \sum_{l} [\vec{N}^{m}_{y,p,l} e^{-M_a/2} - \vec{C}^{m}_{y,p,l} - D^{m}_{y,p,;}] e^{-M_a/2} \quad (5)$$

$$N^{f}_{y+1,p} = \sum_{l} \,[\vec{N}^{f}_{y,p-1,l} e^{-M_a/2} - \vec{C}^{f}_{y,p-1,l} - D^{f}_{y,p-1,l}] e^{-M_a/2} + \sum_{l} [\vec{N}^{f}_{y,p,l} e^{-M_a/2} - \vec{C}^{f}_{y,p,l} - D^{f}_{y,p,l}] e^{-M_a/2} \quad (6)$$

where

$N^{m/f}_{y,a}$ is the number of male or female (*m/f*) lobsters of age *a* at the start of year *y* ,

$\vec{N}^{m/f}_{y,a,l}$ is the number of male or female (*m/f*) lobsters of age *a* of length *l* at the start of year *y* (see equation 15),

M_a denotes the natural mortality rate for male and female lobsters aged *a* years (and here identical for male and female lobsters). Note that for the Reference Case Operating Model this value is fixed at 0.10 for ages 0 to 9, and increased to a value of 1.5 for ages *a*=10+. Alternate values of *M* for lobsters aged *a*=10+ are explored in robustness tests.

$\vec{C}^{m/f}_{y,a,l}$ is the catch of male or female (*m/f*) lobsters of age *a* of length *l* in year *y* ,

$D^{m/f}_{y,a,l}$ is the number of male or female (*m/f*) lobsters of age *a* of length *l* in year *y* that die due to discard mortality, and

p is the maximum age considered (taken to be a plus-group, and set equal to 20 here).

The number of recruits of age 0, of each sex, at the start of year *y* is related to the spawner stock size by a stock-recruitment relationship:

$$R_y = \frac{\alpha B_y^{sp}}{\beta + (B_y^{sp})^{\gamma}} e^{\varsigma_y - \sigma_R^2/2} \tag{7}$$

where

α, β and γ are spawner biomass-recruitment parameters (γ=1 for a Beverton-Holt relationship),

ς_y reflects fluctuation about the expected (mean) recruitment for year *y* (here we estimate stock-recruit residuals for the period 1992-2010) where $\varsigma_y \sim N(0, \sigma_R^2)$ with σ_R=0.4 and

B_y^{sp} is the spawner biomass at the start of year *y*, given by:

$$B_y^{sp} = \sum_{a=1}^{p} \sum_{l=1}^{180} [f_l Q_{a,l}^f w_l^f N_{y,a}^f] \tag{8a}$$

$$B_y^{sp} = \sum_{a=1}^{p} N_{y,a}^f \sum_{l=1}^{180} [f_l Q_{a,l}^f \, w_l^f] \tag{8b}$$

$$B_y^{sp} = \sum_{a=1}^{p} [N_{y,a}^f X_a \,] \tag{8c}$$

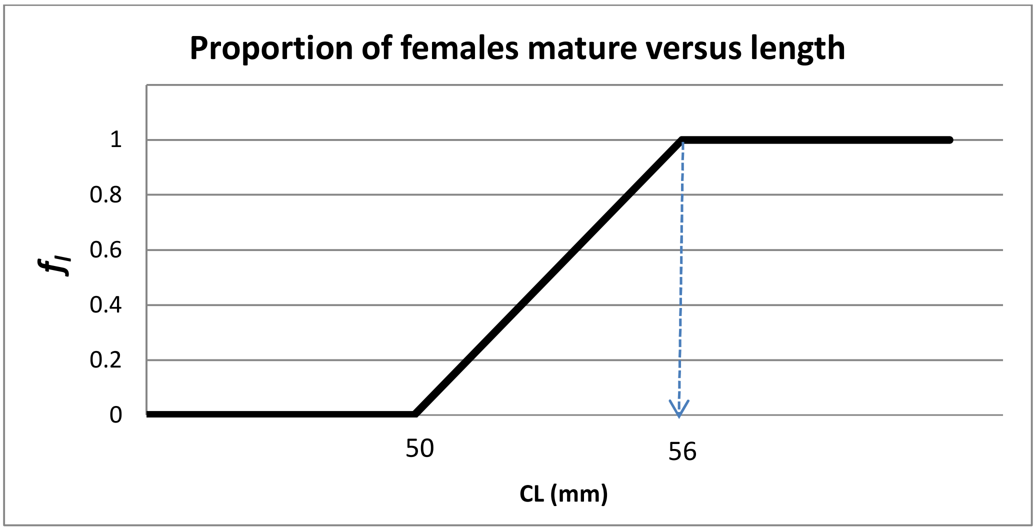
where

$$X_a = \sum_{l=1}^{180} [f_l \quad Q_{a,l}^f w_l^f] \tag{8d}$$

where w_l^f is the mass of female lobsters at length l, and f_l is the proportion of lobster of length l that are mature.

Maturity at length

Pollock (1991) produced plots of the proportion of female lobsters mature for different carapace lengths at Inaccessible and Nightingale islands. Here we assume that the results for Inaccessible island are likely to be similar to those at Tristan. Using Pollock's values, the function below is assumed to apply for Tristan lobsters:



In order to work with estimable parameters that are more meaningful biologically, the stock-recruit relationship is re-parameterised in terms of the pre-exploitation equilibrium female spawning biomass, K^{sp}, and the "steepness" of the stock-recruit relationship (recruitment at $B^{sp} = 0.2K^{sp}$ as a fraction of recruitment at $B^{sp} = K^{sp}$):

$$\alpha = \frac{4hR_1}{5h-1} \tag{9}$$

and

$$\beta = \frac{(K^{sp}(1-h))}{5h-1} \tag{10}$$

where

$$R_1 = K^{sp} / \left[\sum_{a=1}^{p-1} X_a e^{-\sum_{a'=0}^{a-1} M_{a'}} + X_p \frac{e^{-\sum_{a'=0}^{p-1} M_{a'}}}{1-e^{-M_p}} \right] \tag{11}$$

The total catch by mass in year *y* is given by:

$$C_y = \sum_{m/f} \sum_{a} \sum_{l \geq \min} w_l^{m/f} \vec{C}_{y,a,l}^{m/f} \tag{12}$$

where

$$\vec{C}_{y,a,l}^{m} = \vec{N}_{y,a,l}^{m} S_{y,l}^{m,comm} F_y \tag{13}$$

$$\vec{C}_{y,a,l}^{f} = \vec{N}_{y,a,l}^{f} S_{y,l}^{f,comm} F_y \tag{14}$$

where $w_l^{m/f}$ denotes the mass of a *m/f* lobster of length l, and where

$S_{y,l}^{m/f,gear}$ is the length-specific selectivity for male/female lobsters in year y for a given gear type (either *comm*ercial or *sur*vey),

F_y is the fishing proportion in year *y* for lobsters, and which is constrained to be ≤ 0.90, and where

$$F_y = \frac{C_y^{obs}}{\sum_a \sum_{l \geq min}[w_l^m \vec{N}_{y,a,l}^m S_{y,l}^{m,comm} e^{-\frac{M_a}{2}}][w_l^f \vec{N}_{y,a,l}^f S_{y,l}^{f,comm} e^{-\frac{M_a}{2}}]}$$

min is the minimum legal carapace length in mm, and

$$\vec{N}_{y,a,l}^{m/f} = N_{y,a}^{m/f} Q_{a,l}^{m/f} \tag{15}$$

where $Q_{a,l}^{m/f}$ is the proportion of fish of age *a* that fall in the length group *l* for the sex and area concerned (thus $\sum_l Q_{a,l}^{m/f} = 1$ for all ages *a*).

The matrix *Q* is calculated under the assumption that length-at-age is normally distributed about a mean given by the von Bertalanffy equation, i.e.:

$$l_a \sim N^*\left[l_\infty^{m/f}\left(1-e^{-\kappa(a-t_0)}\right);\theta_a^2\right] \tag{16}$$

where

*N** is the normal distribution truncated at ± 3 standard deviations, and

θ_a is the standard deviation of length-at-age *a*, which is modeled to be proportional to the expected length-at-age *a*, i.e.:

$$\theta_a = \beta^* l_\infty^{m/f}\left(1-e^{-\kappa(a-t_0)}\right) \tag{17}$$

with β^* a fixed parameter of the model, and set here to 0.20.

1.1 Initial conditions

For the first year (1990) considered in the model, the stock is assumed to be at a fraction (θ) of its pre-exploitation spawning biomass, i.e.:

$$B_{1990}^{sp} = \theta \cdot K^{sp} \tag{18}$$

with the starting age structure for the first year given by:

$$N_{1990,0}^{m,f} = \theta^* R \tag{19}$$

where *R* is the recruitment corresponding to the *K* (the mean unexploited population size). The numbers at age for the starting population size in 1990 are then calculated as follows:

$$N_{1990,a}^{m/f} = N_{1990,a-1}^{m/f} e^{-(M_a+\varphi)} \quad \text{for } 1 \leq a \leq m \tag{20}$$

and

$$N_{1990,m}^{m,f} = N_{1990,m}^{m,f}/[1-e^{-(M_m+\varphi)}] \quad \text{for } a=m \tag{21}$$

where φ is the average fishing proportion over the years immediately preceding 1990.

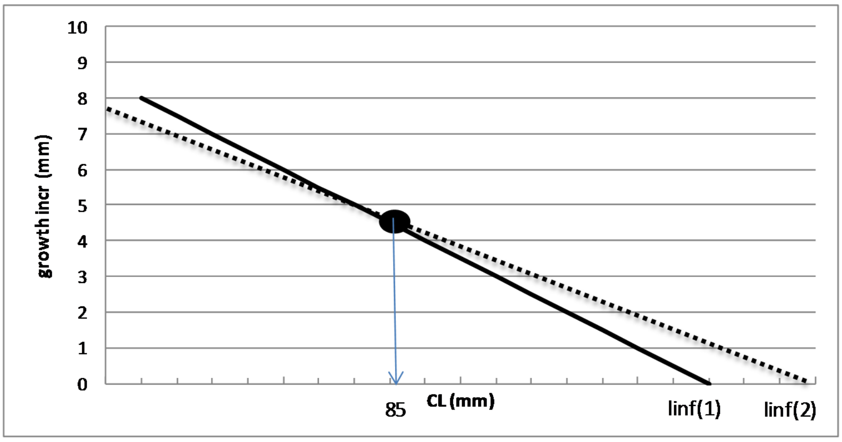
The value of φ is fixed at 0.01, and θ is an estimable parameter.

By adding a penalty to the likelihood, the value of fishing proportion in 2009, F_{1990}, can be set equal to any required level.

1.2 The von Bertalanffy Growth Function

Johnston and Butterworth (2011) reports the von Bertalanffy growth parameters (see equation 16) for each island and sex as found in the literature. Note that the data for males were from Tristan and Nightingale only, and for females data from Tristan only. Johnston and Butterworth (2011) provides suggested von Bertalanffy parameters for each island and sex, based on discussions with James Glass (pers. commn). The original growth parameter estimates were obtained from studies by Pollock and Roscoe (1977) and Pollock (1981). The tagging data centered around lobsters of carapace length 85mm. Original model fits using the von Bertalanffy parameters as reported in Johnston (2011) did not produce satisfactory fits to catch-at-length data. It was found that by changing the l_∞ value slightly one could greatly improve model fits. The authors thus decided to follow the following method for setting the von Bertalnaffy growth parameters:

- Since most of the tagging data centered around carapace length 85mm, it would be assumed that the length increment for this length (i.e. 85mm CL) would remain fixed at the value reported in the literature.
- The l_∞ value would be allowed to be increased or decreased in order to produce better fits to the CPUE and CAL data (this was done by fixing the l_∞ values at different values and inspecting the resultant model fits)
- The κ value would be re-calculated for the new l_∞ value, assuming a "pivot" through the growth increment line at 85mm, thus as the l_∞ value changes, so does the κ value, but the growth increment at 85mm is not altered.



In the figure above, the solid line shows fit to the data as reported in the literature with l_∞ (1) (linf(1) in plot above) being the estimate produced. The dotted line shows how the authors modified this line by increasing the l_∞ value in this case, but retaining the growth increment at the pivot CL of 85mm.

For a new l_∞ value, $l_\infty(2)$, a new kappa value, $\kappa(2)$ (the slope parameter) is calculated as follows:

$$\kappa(2) = \frac{l_\infty(1)-85}{l_\infty(2)-85}.\kappa(1) \qquad (22)$$

The table below reports the values used in the final assessments.

	Inaccessible and Tristan	Nightingale and Gough – Pollock growth	Nightingale and Gough – James Glass growth*
$l_\infty^m(1)$	132.4	156.5	147
$l_\infty^m(2)$	125	150	147
$l_\infty^f(1)$	99.8	99.8	99
$l_\infty^f(2)$	90	90	99
$\kappa^m(1)$	0.11	0.066	0.116
$\kappa^f(1)$	0.06	0.06	0.06
t_0^m	0	0	0
t_0^f	-15	-15	-15

*For the assessments using the James Glass growth, the "pivot method" was not implemented as it was found not to improve model fits.

1.3 Discard Mortality

The number of lobsters that die due to discard mortality is calculated as follows:

$$D_{y,a,l}^m = d(\vec{N}_{y,a,l}^m S_{y,l}^{m,comm} F_y) \qquad (23)$$

$$D_{y,a,l}^f = d(\vec{N}_{y,a,l}^f S_{y,l}^{f,comm} F_y) \qquad (24)$$

where $D_{y,a,l}^{m/f}$ is calculated for $l < \min$, and *d* is the value of discard mortality which is set equal to 0.1 here.

1.4 Biomass estimates

The model estimate of mid-year exploitable biomass for the commercial catch is given by:

$$B_y = B_y^m + B_y^f \qquad (25)$$

where

$$B_y^f = \sum_a \sum_{l \geq \min} [S_{y,l}^{f,comm} w_l^f \vec{N}_{y,a,l}^f e^{-M_a/2}][1-(F_y S_{y,l}^{f,comm}/2)] \qquad (26)$$

$$B_y^m = \sum_a \sum_{l \geq \min} [S_{y,l}^{m,comm} w_l^m \vec{N}_{y,a,l}^m e^{-M_a/2}][1-(F_y S_{y,l}^{m,comm}/2)] \qquad (27)$$

and where

B_y is the total (male plus female) model estimate of mid-year exploitable biomass for year *y*.

The model estimate of begin-year biomass for the biomass survey is given by:

$$B_y^{surv,Leg1} = B_y^{m,surv,Leg1} + B_y^{f,surv,Leg1} \tag{28}$$

where

$$B_y^{f,surv,Leg1} = \sum_a \sum_l [S_{y,l}^{f,surv} w_l^f \vec{N}_{y,a,l}^f] \tag{29a}$$

$$B_y^{m,surv,Leg1} = \sum_a \sum_l [S_{y,l}^{m,surv} w_l^m \vec{N}_{y,a,l}^m] \tag{29b}$$

and where

$B_y^{surv,Leg1}$ is the total (male plus female) model estimate of begin-year survey biomass for year *y*.

The Leg2 biomass survey is assumed to occur at the end of the season once all the fishing and natural mortality for that season has occurred, thus

$$B_y^{f,surv,Leg2} = \sum_a \sum_l w_l^f [S_{y,l}^{f,surv} \vec{N}_{y,a,l}^f e^{-M_a/2} - \vec{C}_{y,a,l}^f - \vec{D}_{y,a,l}^f] e^{-M_a/2} \tag{30a}$$

$$B_y^{m,surv,Leg2} = \sum_a \sum_l w_l^m [S_{y,l}^{m,surv} \vec{N}_{y,a,l}^m e^{-M_a/2} - \vec{C}_{y,a,l}^m - \vec{D}_{y,a,l}^m] e^{-M_a/2} \tag{30b}$$

The total Leg2 biomass survey is thus:

$$B_y^{surv,Leg2} = B_y^{m,surv,Leg2} + B_y^{f,surv,Leg2} \tag{31}$$

1.5 Commercial catch-at-length proportions

$$\hat{p}_{y,l}^{comm,m} = \frac{\sum_a \vec{c}_{y,a,l}^m}{\sum_a \vec{c}_{y,a,l}^m + \vec{c}_{y,a,l}^f} \tag{32}$$

$$\hat{p}_{y,l}^{comm,f} = \frac{\sum_a \vec{c}_{y,a,l}^f}{\sum_a \vec{c}_{y,a,l}^m + \vec{c}_{y,a,l}^f} \tag{33}$$

where $\hat{p}_{y,l}^{comm,m/f}$ is the estimated proportion of commercial catch of *m/f* lobsters in length class *l* in year *y.*

1.6 Biomass survey catch-at-length proportions

These are calculated separately for Leg1 (begin-season) and Leg2 (end-season). For Leg we have:

$$\hat{p}_{y,l}^{Leg1,m} = \frac{\sum_a \vec{N}_{y,a,l}^{m} S_l^{m,surv}}{\sum_a [\vec{N}_{y,a,l}^{m} S_l^{m,surv} + \vec{N}_{y,a,l}^{f} S_l^{f,surv}]} \tag{34a}$$

$$\hat{p}_{y,l}^{Leg1,f} = \frac{\sum_a \vec{N}_{y,a,l}^{f} S_l^{f,surv}}{\sum_a [\vec{N}_{y,a,l}^{m} S_l^{m,surv} + \vec{N}_{y,a,l}^{f} S_l^{f,surv}]} \tag{34b}$$

where $\hat{p}_{y,l}^{Leg1,m/f}$ is the estimated proportion of biomass survey lobsters in Leg1 of *m/f* lobsters in length class l in year *y.*

For Leg2 we have:

$$\hat{p}_{y,l}^{Leg2,m} = \frac{\sum_a [\vec{N}_{y,a,l}^{m} S_l^{m,surv} e^{-M_a/2} - \vec{C}_{y,a,l}^{m} - D_{y,a,l}^{m}] e^{-M_a/2}}{\sum_a [\vec{N}_{y,a,l}^{m} S_l^{m,surv} e^{-M_a/2} - \vec{C}_{y,a,l}^{m} - D_{y,a,l}^{m}] e^{-M_a/2} + [\vec{N}_{y,a,l}^{f} S_l^{f,surv} e^{-M_a/2} - \vec{C}_{y,a,l}^{f} - D_{y,a,l}^{f}] e^{-M_a/2}]} \tag{35a}$$

$$\hat{p}_{y,l}^{Leg2,f} = \frac{\sum_a [\vec{N}_{y,a,l}^{f} S_l^{f,surv} e^{-M_a/2} - \vec{C}_{y,a,l}^{f} - D_{y,a,l}^{f}] e^{-M_a/2}}{\sum_a [\vec{N}_{y,a,l}^{m} S_l^{m,surv} e^{-M_a/2} - \vec{C}_{y,a,l}^{m} - D_{y,a,l}^{m}] e^{-M_a/2} + [\vec{N}_{y,a,l}^{f} S_l^{f,surv} e^{-M_a/2} - \vec{C}_{y,a,l}^{f} - D_{y,a,l}^{f}] e^{-M_a/2}]} \tag{35b}$$

where $\hat{p}_{y,l}^{Leg2,m/f}$ is the estimated proportion of biomass survey lobsters in Leg2 of *m/f* lobsters in length class l in year *y.*

1.7 Commercial selectivity-at-length function

The selectivity function (which depends on length) is assumed to change over time. For each island, three fixed periods of selectivity are modelled – these vary slightly between islands and were selected after studying the residual trends from the model fits to the catch-at-length data. The time-variation is effected by estimating **three different μ values** (for m and f separately) for the selectivity function for each of the different time periods. The time periods assumed for the fixed selectivity intervals in these assessments are:

These periods were selected based on examination of residual plots of the catch-at-length data.

1990-2000
2001-2005
2006+

Male and female selectivities are estimated separately as follows:

$$S_{y,l}^{m,comm} = \frac{e^{-\mu_y^m l}}{1+e^{-\delta^m (l-l_*^m)}} \tag{36}$$

$$S_{y,l}^{f,comm} = P\frac{e^{-\mu_y^f l}}{1+e^{-\delta^f (l-l_*^f)}} \tag{37}$$

The estimable parameters are thus:

- $l_*^{m/f}$,
- $\delta^{m/f}$,
- $\mu^{m/f}$ [with three values for each of the three selectivity periods shown in table above]
- *P* (the female scaling parameter)

The selectivity functions for males are scaled so that the maximum selectivity value is 1.0, and the female selectivity function is scaled by the multiplicative parameter *P* so that the maximum selectivity value for females is equal to *P*.

1.8 Survey selectivity-at-length function

The selectivity functions for the gear used in the biomass surveys are assumed to be time invariant.

Male and female selectivities are estimated separately as follows:

$$S_{l}^{m,surv} = \frac{e^{-\mu^m l}}{1+e^{-\delta^m (l-l_*^m)}} \tag{38}$$

$$S_{l}^{f,surv} = P\frac{e^{-\mu^f l}}{1+e^{-\delta^f (l-l_*^f)}} \tag{39}$$

The estimable parameters for the survey selectivity function are thus:

- $l_*^{m/f}$,
- $\delta^{m/f}$,
- $\mu^{m/f}$ and
- *P* (the female scaling parameter)

It was found that much improved fits to both the commercial and survey CAL male data could be obtained if the selectivity of male lobsters for lengths 110mm+ was reduced by 25%. Thus for all OMs the following applies:

$S_{y,l}^{m,comm} \rightarrow S_{y,l}^{m,comm} * 0.75$ and,

$S_{y,l}^{m,surv} \rightarrow S_{y,l}^{m,surv} * 0.75$ for $l \geq 110mm$

Where the values on the right hand sides on these equation are calculated from equations 38 to 39.

2. The likelihood function

The model is fitted to CPUE, survey abundance, commercial catch-at-length (male and female separately) data and survey catch-at-length (male and female separately) data to estimate model parameters. Contributions by each of these to the negative log-likelihood (-ln*L*), and the various additional penalties added are as follows. For the outer islands, the model is also fitted to discard ratio data.

2.1 Relative abundance data (CPUE) from commercial catch

The likelihood is calculated assuming that the observed abundance index is log-normally distributed about its expected (median) value:

$$CPUE_y = q\, B_y e^{\varepsilon_y} \text{ or } \varepsilon_y = \ln(CPUE_y) - \ln(q\, B_y) \tag{40}$$

where

$CPUE_y$ is the CPUE abundance index for year *y*,

B_y is the model estimate of mid-year exploitable biomass for year *y* in given by equation 25,

q is the constant of proportionality (catchability coefficient), and

ε_y from $N(0,(\sigma\ )^2)$.

The contribution of the abundance data to the negative of the log-likelihood function (after removal of constants) is given by:

$$-\ln L = \sum_y \left[(\varepsilon_y)^2 / 2(\sigma\ )^2 + \ln(\sigma\ )\right] \tag{41}$$

where

σ is the residual standard deviation estimated in the fitting procedure by its maximum likelihood value:

$$\hat{\sigma}\ = \sqrt{1/n \sum_y \left(\ln CPUE_y - \ln \hat{q}\ \hat{B}_y\right)^2} \tag{42}$$

where

n is the number of data points in the CPUE series, and

q is the catchability coefficient, estimated by its maximum likelihood value:

$$\ln \hat{q}\ = 1/n \sum_y \left(\ln CPUE_y - \ln \hat{B}_y\right) \tag{43}$$

2.2 Relative abundance data from the biomass survey

Data from Leg1 (around September in each season) and Leg2 (around February each season) are fitted independently. The likelihood is calculated assuming that the observed abundance index is log-normally distributed about its expected (median) value:

$$SUR_y^{Leg} = q^{Leg} B_y^{sur} e^{\varepsilon_y}, \qquad \text{i.e.} \qquad \varepsilon_y = \ln(SUR_y^{Leg}) - \ln(q^{Leg} B_y^{sur}) \tag{44}$$

where

$SURV_y^{Leg}$ is the survey biomass abundance index for either Leg1 or Leg2 in year *y*,

B_y^{sur} is the model estimate of mid-year exploitable survey biomass for year *y* given by equation 28,

q^{Leg} is the constant of proportionality (catchability coefficient) for either Leg1 or Leg2, and

ε_y from $N(0,(\sigma\ )^2)$.

The contribution of the biomass survey abundance data to the negative of the log-likelihood function (after removal of constants) is given by:

$$-\ln L = \sum_y \left[(\varepsilon_y)^2 / (2((CV_y^{Leg})^2 + \sigma_{add}^2)) + 0.5.\ln((CV_y^{Leg})^2 + \sigma_{add}^2) \right] \quad (45)$$

where

CV_y^{Leg} is the survey sampling CV of the biomass survey in year *y* for either Leg1 or Leg2,

q^{Leg} is the catchability coefficient for either Leg1 or Leg2, estimated by its maximum likelihood value:

$$\ln\hat{q}^{Leg} = 1/n\sum_y \left(\ln SUR_y^{Leg} - \ln\hat{B}_y^{sur}\right), \text{ and} \quad (46)$$

σ_{add} is an estimable parameter which reflects variance additional to the estimated survey sampling variance.

2.3 Commercial catches-at-length

The following term is added to the negative log-likelihood:

$$-\ell n L^{\text{length}} = w_{len}\sum_y\sum_l\sum_{m/f} \left[\ln\left(\sigma_{len} / \sqrt{p_{y,l}^{comm,m/f}}\right) + p_{y,l}^{comm,m/f}\left(\ln p_{y,l}^{comm,m/f} - \ln\hat{p}_{y,l}^{comm,m/f}\right)^2 / 2\left(\sigma_{len}\right)^2 \right] \quad (47)$$

where

$p_{y,l}^{comm,m/f}$ is the observed proportion of *m/f* lobsters (by number) in length group *l* in the commercial catch in year *y*, and

σ_{len} is the standard deviation associated with the length-at-age data, which is estimated in the fitting procedure by:

$$\hat{\sigma}_{len} = \sqrt{\sum_{m/f}\ \sum_y\sum_l p_{y,l}^{comm,m/f}\left(\ln p_{y,l}^{comm,m/f} - \ln\hat{p}_{y,l}^{comm,m/f}\right)^2 / \sum_{m/f}\ \sum_y\sum_l 1} \quad (48)$$

Equation (47) makes the assumption that proportion-at-length data are log-normally distributed about their model-predicted values. The associated variance is taken to be inversely proportional to $p_{y,l}^{comm,m/f}$ to downweight contributions from observed small proportions which will correspond to small predicted sample sizes. The value of w_{len} is set equal to 0.1.

2.4 Biomass survey catches-at-length

The following term is added to the negative log-likelihood:

$$-\ell n L^{\text{length}} = w_{len} \sum_{y} \sum_{l} \sum_{m/f} \left[\ln\left(\sigma_{len} / \sqrt{p_{y,l}^{Leg,m/f}}\right) + p_{y,l}^{Leg,m/f} \left(\ln p_{y,l}^{Leg,m/f} - \ln \hat{p}_{y,l}^{Leg,m/f} \right)^2 / 2\left(\sigma_{len}\right)^2 \right] \tag{49}$$

where

$p_{y,l}^{Leg,m/f}$ is the observed proportion of *m/f* lobsters (by number) in length group *l* in the biomass survey in year *y* during Leg1 or Leg2, and

σ_{len} is the standard deviation associated with the length-at-age data, which is estimated in the fitting procedure by:

$$\hat{\sigma}_{len} = \sqrt{\sum_{m/f} \sum_{y} \sum_{l} p_{y,l}^{Leg,m/f} \left(\ln p_{y,l}^{Leg,m/f} - \ln \hat{p}_{y,l}^{Leg,m/f} \right)^2 / \sum_{m/f} \sum_{y} \sum_{l} 1} \tag{50}$$

Equation (49) makes the assumption that proportion-at-length data are log-normally distributed about their model-predicted values. The associated variance is taken to be inversely proportional to $p_{y,l}^{surv,m/f}$ to downweight contributions from observed small proportions which will correspond to small predicted sample sizes. The value of w_{len} is set equal to 0.1.

2.3 **Discard % (for Nightingale, Inaccessible and Gough)**

The longline catch and effort databases provide information on the weight of discarded lobsters. In this document the discard % is expressed as % weight of discards relative to the weight of the total catch (under and over legal size) hauled. This information is incorporated into the likelihood function when fitting the assessment models to the data by including the following term:

$$-lnL = -lnL + \sum_{y} \left(\ln \mathrm{D}_{\mathrm{y}}^{\mathrm{obs}} - \ln \widehat{\mathrm{D}_{\mathrm{y}}} \right)^{2} / 2\mathrm{CV}^{2} \tag{51}$$

where

D_y^{obs} is the observed discard percentage for year *y*, and

$\widehat{D}_y$ is the model estimated value of discard percentage for year y, where

$$D_y^* = \sum_{a=0}^{P} \sum_{l=1}^{min-1} \left(w_l^m D_{y,a,l}^m + w_l^f D_{y,a,l}^f \right), \tag{52}$$

and

$$\widehat{D}_y = \left[D_y^* / (D_y^* + C_y) \right] * 100 \tag{53}$$

The CV is set at a value of 0.6 for Inaccessible and Nightingale, and 0.8 for Gough.

2.4 Stock-recruitment function residuals

The assumption that these residuals are log-normally distributed (and could be serially correlated) defines a corresponding joint prior distribution. This can be equivalently regarded as a penalty function added to the log-likelihood, which for fixed serial correlation ρ is given by:

$$-\ln L = -\ln L + \sum_{y=y1}^{y2}\left[\frac{\varepsilon_y - \rho\varepsilon_{y-1}}{\sqrt{1-\rho^2}}\right]^2 / 2\sigma_R^2 \tag{54}$$

where

$\varsigma_y = \rho\tau_{y-1} + \sqrt{1-\rho^2}\varepsilon_y$ is the recruitment residual for year *y* (see equation 1),

$\varepsilon_y \sim N(0,\sigma_R^2)$,

σ_R is the standard deviation of the log-residuals, which is input,

ρ is their auto-correlation coefficient, and

y1=1992 and *y2*=2010 here.

Note that here , ρ is set equal to zero, i.e. the recruitment residuals are assumed uncorrelated, and σ_R is set equal to 0.4. Recruitment residuals are estimated for years 1992 to 2010 only.

The following term is added to constrain the size of these terms (i.e. to fit to genuine difference rather than to noise) and to force the average of the residuals to equal zero:

$$-\ln L = \ln L + W[\sum_{1992}^{2010}\frac{\varepsilon_y}{\sigma_R}]^2 \tag{55}$$

where the weighting factor *W* is set high to ensure that the sum above ends as zero. This is to ensure that when projecting, the stock-recruitment curve used more closely reflects the past patterns of recruitment and its variability.

Future recruitment: The model estimates residuals for 1992-2010. For 2011+ recruitment is set equal to its expected values given the fitted stock-recruit relationship. The relationship itself is $R_y = \frac{\alpha B_y^{sp}}{\beta + B_y^{sp}} e^{\varepsilon_y - \sigma_R^2/2}$ where $\varepsilon_y \sim N(0,\sigma_R^2)$ and $\sigma_R = 0.4$. This means that the expected recruitment $E[R_y] = \frac{\alpha B_y^{sp}}{\beta + B_y^{sp}}$ (56)

The residuals for years 1990 and 1991 are set equal to zero.

3. Further Model parameters

Age-at-maturity: The proportion of lobsters of age *a* that are mature is approximated by $f_a = 1$ for *a* > 5 years (i.e. $f_a = 0$ for *a* = 0, …,5).

Minimum age: Age 0.

Maximum age: *p* = 20, and is taken as a plus-group.

Minimum length: 1mm.

Maximum length: 180mm, which is taken as a plus-group.

Mass-at-age: The mass $w_a^{m/f}$ of a *m/f* lobster at age *a* is given by:

$$w_a^{m/f} = \alpha^{m/f}\left[\hat{l}_\infty^{m/f}\left(1-e^{-\hat{\kappa}^{m/f}\left(a-\hat{t}_0^{m/f}\right)}\right)/10\right]^{\beta^{m/f}} \tag{57}$$

where the values assumed for the observed length-weight are:

$$\alpha^m = 0.4789$$
$$\alpha^f = 0.5907$$
$$\beta^m = 3.024$$
$$\beta^f = 2.9449$$

This provides weight of in units of kgs.

4. The Bayesian approach

The Bayesian method entails updating prior distributions for model parameters according to the respective likelihoods of the associated population model fits to the CPUE and catch-at-length, to provide posterior distribution for these parameters and other model quantities.

The catchability coefficients (*q*) and the standard deviations associated with the commercial CPUE and catch-at-length data (σ and σ_{len}) are estimated in the fitting procedure by their maximum likelihood values, rather than integrating over these three parameters as well. This is considered adequately accurate given reasonably large sample sizes.

Modes of posteriors, obtained by finding the maximum of the product of the likelihood and the priors, are then estimated rather than performing a full Bayesian integration, due to the time intensiveness of the latter.

4.1 Priors

The following prior distributions are assumed:

h	N(0.95, SD^2) with SD=0.2, where the normal distribution is truncated at h = 1.
$l_*^{m/f}$:	U[1, 180] mm
$\mu^{m/f}$	U[0, 1]
$\delta^{m/f}$	U[0, 1]
P	U[0,5]
θ	U[0,1]
ς_y	U[-5,5]
σ_{add}	U[0,1]

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