

Specifications of the South African Hake 2014 Reference Case Assessment

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INTRODUCTION

This paper gives specifications of the 2014 South African hake Reference Case assessment. The data used as input to the Reference Case are listed in Appendix A, while the methodology is detailed in Appendix B. Figure 1 shows the distribution of the two hake species, and the boundary between west and south coasts adopted for analysis purposes.

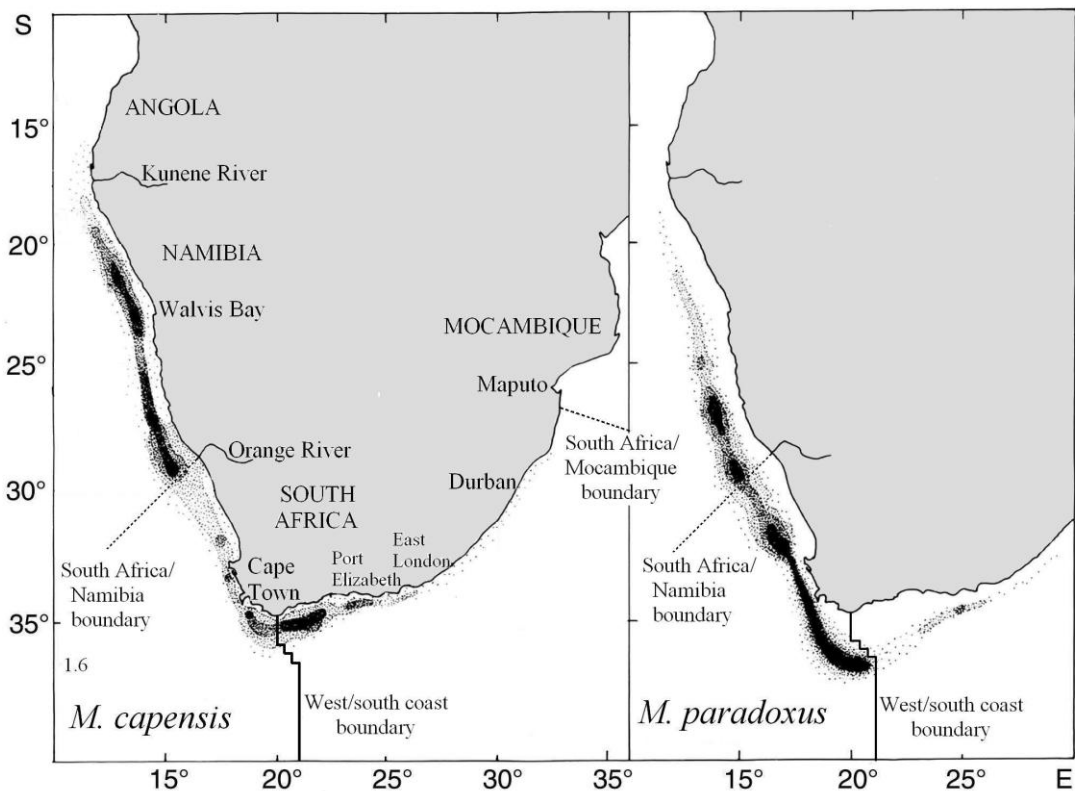


Figure 1: Management units and species distribution for southern African hake (adapted from Payne 1989).

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Appendix A - Data Tables

Table App.A.1a: Species-disaggregated catches (in thousand tons) by fleet of South African hake from the south and west coasts for the period 1917-1977. The Reference Case assessment assumes 1958 as the centre year of the shift from primarily *M. capensis* to primarily *M. paradoxus* in the offshore trawl catches.

	<i>M. paradoxus</i>		<i>M. capensis</i>		<i>M. paradoxus</i>		<i>M. capensis</i>	
	Offshore		Offshore		Offshore		Inshore	
	WC	WC	WC	SC	WC	SC	SC	
1917	-	1.000	1948	0.059	-	58.741	-	-
1918	-	1.100	1949	0.113	-	57.287	-	-
1919	-	1.900	1950	0.275	-	71.725	-	-
1920	-	0.000	1951	0.662	-	88.838	-	-
1921	-	1.300	1952	1.268	-	87.532	-	-
1922	-	1.000	1953	2.558	-	90.942	-	-
1923	-	2.500	1954	5.438	-	99.962	-	-
1924	-	1.500	1955	10.924	-	104.476	-	-
1925	-	1.900	1956	19.581	-	98.619	-	-
1926	-	1.400	1957	34.052	-	92.348	-	-
1927	-	0.800	1958	51.895	-	78.805	-	-
1928	-	2.600	1959	76.609	-	69.391	-	-
1929	-	3.800	1960	100.490	-	59.410	-	1.000
1930	-	4.400	1961	104.009	-	44.691	-	1.308
1931	-	2.800	1962	109.596	-	38.004	-	1.615
1932	-	14.300	1963	129.966	-	39.534	-	1.923
1933	-	11.100	1964	126.567	-	35.733	-	2.231
1934	-	13.800	1965	159.704	-	43.296	-	2.538
1935	-	15.000	1966	154.109	-	40.891	-	2.846
1936	-	17.700	1967	139.973	7.086	36.727	7.100	3.154
1937	-	20.200	1968	113.890	13.958	29.710	13.950	3.462
1938	-	21.100	1969	131.023	18.982	34.077	18.948	3.769
1939	-	20.000	1970	113.124	11.876	29.376	11.847	4.077
1940	-	28.600	1971	160.384	15.078	41.616	15.037	4.385
1941	-	30.600	1972	193.694	23.382	50.239	23.314	4.692
1942	0.001	34.499	1973	125.292	36.232	32.490	36.124	5.000
1943	0.001	37.899	1974	97.674	45.496	25.326	45.357	10.056
1944	0.002	34.098	1975	71.165	33.783	18.452	33.680	6.372
1945	0.004	29.196	1976	114.268	26.005	29.626	25.925	5.740
1946	0.011	40.389	1977	81.260	18.515	21.068	18.457	3.500
1947	0.021	41.379						

Table App.A.1b: Species-disaggregated catches (in thousand tons) by fleet of South African hake from the south and west coasts for the period 1978-present. The recent offshore trawl catches are from Glazer (2013), the recent inshore and handline catches are from Rob Cooper (pers. comm.) and the new longline catches from Sobahle Somhlaba (pers. comm.). For 2013, the catches are taken as the 2013 TAC with the same proportion by species and fleet as in 2012.

	<i>M. paradoxus</i>				<i>M. capensis</i>					
	Offshore		Longline		Offshore		Inshore	Longline		Handline
	WC	SC	WC	SC	WC	SC	SC	WC	SC	SC
1978	107.701	4.937	-	-	19.812	2.648	4.931	-	-	-
1979	101.890	3.575	-	-	31.633	3.345	6.093	-	-	-
1980	105.483	3.676	-	-	28.045	2.784	9.121	-	-	-
1981	95.330	1.767	-	-	25.601	3.719	9.400	-	-	-
1982	88.933	5.057	-	-	24.417	6.300	8.089	-	-	-
1983	74.173	7.034	0.126	-	20.260	5.482	7.672	0.104	-	-
1984	86.045	5.718	0.200	0.005	25.210	5.217	9.035	0.166	0.011	-
1985	98.283	12.694	0.638	0.091	26.788	7.322	9.203	0.529	0.201	0.065
1986	107.907	11.539	0.753	0.094	25.898	4.427	8.724	0.625	0.208	0.084
1987	96.162	10.536	1.952	0.110	21.363	5.148	8.607	1.619	0.243	0.096
1988	83.606	8.664	2.833	0.103	22.976	5.852	8.417	2.350	0.228	0.071
1989	85.298	9.039	0.158	0.010	21.961	9.873	10.038	0.132	0.022	0.137
1990	84.969	13.622	0.211	-	18.668	9.169	10.012	0.175	-	0.348
1991	89.371	15.955	-	0.932	17.079	6.119	8.206	-	2.068	1.270
1992	86.777	22.368	-	0.466	16.510	4.094	9.252	-	1.034	1.099
1993	105.114	12.472	-	-	12.951	1.789	8.870	-	-	0.278
1994	106.287	8.588	0.882	0.194	17.580	2.464	9.569	0.732	0.432	0.449
1995	102.877	5.395	0.523	0.202	18.020	1.755	10.630	0.434	0.448	0.756
1996	110.460	11.080	1.308	0.568	18.715	2.209	11.062	1.086	1.260	1.515
1997	103.035	13.651	1.410	0.582	14.119	2.185	8.834	1.170	1.290	1.404
1998	113.083	11.703	0.505	0.457	14.570	2.450	8.283	0.419	1.014	1.738
1999	89.147	13.435	1.532	1.288	14.614	1.912	8.595	1.272	2.856	2.749
2000	97.417	9.920	2.706	3.105	20.285	3.610	10.906	2.000	1.977	5.500
2001	101.990	11.016	2.045	0.370	15.606	5.141	11.836	1.750	1.347	7.300
2002	91.720	15.445	4.469	1.585	13.211	3.140	9.581	2.391	2.546	3.500
2003	95.143	21.107	3.305	1.252	10.233	3.926	9.883	2.526	3.078	3.000
2004	86.916	30.746	2.855	1.196	11.315	4.024	10.004	2.297	2.731	1.600
2005	87.540	25.051	3.091	0.472	7.727	4.195	7.881	2.773	3.270	0.700
2006	83.840	22.133	3.241	0.485	9.657	2.494	5.524	2.520	3.227	0.400
2007	96.332	15.825	2.512	3.021	12.537	1.420	6.350	2.522	2.522	0.400
2008	88.290	14.940	2.255	0.809	11.085	2.567	5.496	1.937	1.893	0.231
2009	69.716	13.269	2.410	1.069	10.783	2.431	5.639	2.828	2.520	0.265
2010	70.156	17.863	2.045	0.370	9.738	1.649	5.472	1.750	1.347	0.275
2011	76.744	20.447	3.261	0.905	15.505	1.543	6.013	2.705	2.009	0.185
2012	82.531	19.204	3.582	0.573	11.978	1.751	3.223	2.972	1.272	0.008
2013	101.350	23.583	4.399	0.704	14.709	2.151	3.958	3.650	1.562	0.010

Table App.A.1c: Male proportion of the longline catches for *M. paradoxus* and *M. capensis* (Somhlaba, pers. comm.). For years for which data are not available, the average over the whole period from 1999 to 2011 is used.

	<i>M. paradoxus</i>		<i>M. capensis</i>	
	WC	SC	WC	SC
1999	0.152	0.460	0.187	0.293
2000	0.357	0.460	0.098	0.293
2001	0.054	0.526	0.134	0.382
2002	0.263	0.460	0.139	0.365
2003	0.227	0.460	0.223	0.317
2004	0.125	0.460	0.108	0.266
2005	0.058	0.460	0.149	0.165
2006	0.046	0.288	0.103	0.272
2007	0.037	0.460	0.344	0.293
2008	0.223	0.346	0.293	0.279
2009	0.224	0.615	0.337	0.212
2010	0.054	0.526	0.134	0.382
2011	0.152	0.460	0.187	0.293

Table App.A.2: South and west coast historic (ICSEAF 1989) and GLM standardized CPUE data (Glazer, 2013) for *M. paradoxus* and *M. capensis*.

GLM CPUE (kg min ⁻¹)			GLM CPUE (kg min ⁻¹)				
Species combined			<i>M. paradoxus</i>		<i>M. capensis</i>		
Year	West Coast	South Coast	Year	West Coast	South Coast	West Coast	South Coast
1955	17.31	-	1978	4.20	1.09	0.74	1.68
1956	15.64	-	1979	4.16	1.08	1.21	1.84
1957	16.47	-	1980	3.86	1.59	1.08	2.07
1958	16.26	-	1981	3.85	1.03	1.11	2.01
1959	16.26	-	1982	3.79	1.31	0.95	1.99
1960	17.31	-	1983	4.08	1.43	1.28	2.46
1961	12.09	-	1984	4.17	1.55	1.35	2.84
1962	14.18	-	1985	4.88	2.24	1.49	3.45
1963	13.97	-	1986	4.25	2.18	1.22	2.87
1964	14.60	-	1987	3.50	1.94	1.07	2.63
1965	10.84	-	1988	3.46	1.60	0.96	2.79
1966	10.63	-	1989	3.74	1.57	1.03	3.00
1967	10.01	-	1990	4.02	2.34	0.87	3.59
1968	10.01	-	1991	4.37	2.32	1.08	3.24
1969	8.62	1.28	1992	3.81	2.72	1.32	2.96
1970	7.23	1.22	1993	3.89	2.25	1.24	2.13
1971	7.09	1.14	1994	4.16	1.81	1.58	2.88
1972	4.90	0.64	1995	3.63	1.39	1.63	2.71
1973	4.97	0.56	1996	3.94	1.99	1.89	2.50
1974	4.65	0.54	1997	3.52	2.28	1.64	1.82
1975	4.66	0.37	1998	3.93	1.93	1.87	2.09
1976	5.35	0.40	1999	3.19	2.20	1.72	2.00
1977	4.84	0.42	2000	2.84	1.67	1.61	2.22
			2001	2.30	1.65	1.10	1.64
			2002	2.23	1.41	1.06	1.84
			2003	2.64	1.93	0.90	2.07
			2004	2.20	1.52	0.81	1.65
			2005	2.19	1.34	0.58	1.48
			2006	2.38	1.46	0.62	1.03
			2007	2.92	1.53	0.60	0.98
			2008	3.23	1.65	0.81	1.56
			2009	3.23	2.05	1.42	2.89
			2010	3.54	2.25	1.22	2.22
			2011	3.30	2.62	1.47	2.79
			2012	2.92	2.17	1.24	1.62

Table App.A.3: Survey abundance estimates and associated standard errors in thousand tons for *M. paradoxus* for the depth range 0-500m for the South Coast and for the West Coast. Values in bold are for the surveys conducted by the *Africana* with the new gear (Fairweather, 2012).

Year	West coast				South coast			
	Summer		Winter		Spring (Sept)		Autumn (Apr/May)	
	Biomass	(s.e.)	Biomass	(s.e.)	Biomass	(s.e.)	Biomass	(s.e.)
1985	166.294	(35.299)	264.839	(52.949)	-	-	-	-
1986	196.111	(36.358)	172.477	(24.122)	13.758	(3.554)	-	-
1987	284.805	(53.101)	195.482	(44.415)	21.554	(4.605)	-	-
1988	158.758	(27.383)	233.041	(64.003)	-	-	30.316	(11.104)
1989	-	-	468.780	(124.830)	-	-	-	-
1990	282.174	(78.945)	226.862	(46.007)	-	-	-	-
1991	327.020	(82.180)	-	-	-	-	26.638	(10.460)
1992	226.687	(32.990)	-	-	-	-	24.304	(15.195)
1993	334.151	(50.234)	-	-	-	-	198.849	(98.452)
1994	330.270	(58.319)	-	-	-	-	111.469	(34.627)
1995	324.554	(80.357)	-	-	-	-	55.068	(22.380)
1996	430.908	(80.604)	-	-	-	-	85.546	(25.484)
1997	569.957	(108.200)	-	-	-	-	135.192	(51.031)
1998	-	-	-	-	-	-	-	-
1999	569.364	(114.536)	-	-	-	-	321.478	(113.557)
2000	-	-	-	-	-	-	-	-
2001	-	-	-	-	19.929	(9.956)	-	-
2002	267.487	(35.068)	-	-	-	-	-	-
2003	411.177	(69.431)	-	-	88.442	(36.051)	108.857	(37.528)
2004	259.527	(56.021)	-	-	63.900	(17.894)	48.898	(20.343)
2005	286.416	(39.849)	-	-	-	-	26.605	(7.952)
2006	315.310	(49.490)	-	-	72.415	(15.500)	34.799	(8.325)
2007	397.049	(71.564)	-	-	52.287	(19.231)	129.646	(60.661)
2008	246.542	(51.973)	-	-	24.816	(8.775)	39.505	(11.408)
2009	330.235	(28.526)	-	-	-	-	102.834	(28.670)
2010	589.533	(85.686)	-	-	-	-	169.560	(67.650)
2011	347.082	(92.540)	-	-	-	-	24.105	(7.089)
2012	377.515	(50.690)	-	-	-	-	-	-
2013	-	-	-	-	-	-	-	-

Table App.A.4: Survey abundance estimates and associated standard errors in thousand tons for *M. capensis* for the depth range 0-500m for the South Coast and for the West Coast (Fairweather, 2012). Values in bold are for the surveys conducted by the *Africana* with the new gear.

Year	West coast				South coast			
	Summer		Winter		Spring (Sept)		Autumn (Apr/May)	
	Biomass	(s.e.)	Biomass	(s.e.)	Biomass	(s.e.)	Biomass	(s.e.)
1985	125.028	(22.719)	181.487	(27.476)	-	-	-	-
1986	117.810	(23.636)	119.587	(18.489)	121.197	(16.625)	-	-
1987	75.693	(10.241)	87.391	(11.198)	159.088	(17.233)	-	-
1988	66.725	(10.765)	47.120	(9.568)	-	-	165.939	(21.871)
1989	-	-	323.833	(67.295)	-	-	-	-
1990	455.798	(135.237)	157.800	(23.561)	-	-	-	-
1991	77.357	(14.995)	-	-	-	-	274.298	(44.395)
1992	95.407	(11.744)	-	-	-	-	138.085	(15.357)
1993	92.598	(14.589)	-	-	-	-	158.340	(13.733)
1994	121.257	(35.951)	-	-	-	-	160.555	(23.701)
1995	199.142	(26.812)	-	-	-	-	236.025	(31.840)
1996	83.337	(9.285)	-	-	-	-	244.410	(25.107)
1997	257.293	(46.056)	-	-	-	-	183.087	(18.906)
1998	-	-	-	-	-	-	-	-
1999	196.992	(32.059)	-	-	-	-	191.203	(14.952)
2000	-	-	-	-	-	-	-	-
2001	-	-	-	-	133.793	(20.858)	-	-
2002	106.253	(15.813)	-	-	-	-	-	-
2003	75.960	(13.314)	-	-	82.928	(9.010)	128.450	(20.062)
2004	205.939	(33.216)	-	-	106.119	(15.596)	99.902	(12.027)
2005	70.983	(13.845)	-	-	-	-	76.932	(5.965)
2006	88.420	(22.851)	-	-	99.867	(9.803)	130.900	(14.816)
2007	82.040	(11.491)	-	-	74.615	(7.383)	70.940	(5.615)
2008	50.877	(5.355)	-	-	94.232	(11.456)	108.195	(9.978)
2009	175.289	(39.920)	-	-	-	-	124.004	(11.808)
2010	163.545	(34.444)	-	-	-	-	184.960	(37.720)
2011	89.392	(23.218)	-	-	-	-	117.222	(11.857)
2012	92.588	(11.926)	-	-	-	-	-	-
2013	-	-	-	-	-	-	-	-

Table App.A.5: Commercial length frequencies available for use in the Reference Case. *The longline data starred are available disaggregated by sex (Somhlaba and Leslie, 2014).

	Offshore trawl		Inshore trawl	Longline				
	Species combined		<i>M. capensis</i>	Spp combined	<i>M. paradoxus</i>	<i>M. capensis</i>	<i>M. paradoxus</i>	<i>M. capensis</i>
	WC	SC	SC		WC		SC	
1975	-	✓	-	-	-	-	-	-
1976	-	✓	-	-	-	-	-	-
1977	-	✓	-	-	-	-	-	-
1978	-	✓	-	-	-	-	-	-
1979	-	✓	-	-	-	-	-	-
1980	-	✓	-	-	-	-	-	-
1981	✓	✓	✓	-	-	-	-	-
1982	✓	✓	✓	-	-	-	-	-
1983	✓	✓	✓	-	-	-	-	-
1984	✓	✓	✓	-	-	-	-	-
1985	✓	✓	✓	-	-	-	-	-
1986	✓	✓	✓	-	-	-	-	-
1987	✓	✓	✓	-	-	-	-	-
1988	✓	✓	✓	-	-	-	-	-
1989	✓	✓	✓	-	-	-	-	-
1990	✓	✓	✓	-	-	-	-	-
1991	✓	✓	✓	-	-	-	-	-
1992	✓	✓	✓	-	-	-	-	-
1993	✓	✓	✓	-	-	-	-	-
1994	✓	✓	✓	✓	-	-	-	✓
1995	✓	✓	✓	✓	-	-	-	✓
1996	✓	✓	✓	✓	-	-	-	✓
1997	✓	-	-	✓	-	-	-	✓
1998	✓	-	✓	-	-	-	-	-
1999	✓	-	✓	-	-	-	-	-
2000	-	-	✓	-	✓*	✓*	-	✓*
2001	-	-	✓	-	✓*	✓*	✓*	✓*
2002	-	-	-	-	✓*	✓*	-	✓*
2003	-	-	-	-	✓*	✓*	-	✓*
2004	-	-	-	-	✓*	✓*	-	✓*
2005	✓	-	-	-	✓*	✓*	-	✓*
2006	✓	-	-	-	✓*	✓*	✓*	✓*
2007	✓	-	✓	-	✓*	✓*	-	-
2008	✓	-	✓	-	✓*	✓*	✓*	✓*
2009	✓	-	✓	-	✓*	✓*	✓*	✓*
2010	✓	-	✓	-	✓*	✓*	✓*	✓*
2011	✓	-	✓	-	-	-	-	-
2012	✓	-	✓	-	-	-	-	-

Table App.A.6: Survey length frequencies available for use in the Reference Case. *The data starred are available disaggregated by sex.

Year	West coast		South coast	
	Summer	Winter	Spring	Autumn
1985	✓	✓	-	-
1986	✓	✓	✓	-
1987	✓	✓	✓	-
1988	✓	✓	-	✓
1989	-	✓	-	-
1990	✓	✓	-	-
1991	✓	-	-	-
1992	✓	-	-	✓
1993	✓*	-	-	✓*
1994	✓*	-	-	✓*
1995	✓*	-	-	✓*
1996	✓*	-	-	✓*
1997	✓*	-	-	✓*
1998	-	-	-	-
1999	✓*	-	-	✓
2000	-	-	-	-
2001	-	-	✓	-
2002	✓	-	-	-
2003	✓	-	✓	✓
2004	✓	-	✓	✓
2005	✓	-	-	✓
2006	✓*	-	✓*	✓*
2007	✓*	-	✓*	✓*
2008	✓*	-	✓*	✓*
2009	✓*	-	-	✓*
2010	✓*	-	-	✓*
2011	✓*	-	-	✓*
2012	✓*	-	-	✓*
2013	-	-	-	-

Table App.A.7: Species- and sex-disaggregated age at length data available for use in the Reference Case by reader, with sample sizes shown.

Reader	<i>M. paradoxus</i>											Reader	
	Number of otolith's readings										Data actually used		
	UR	AD	LB	KG	JP	AP	DJ	PM	TA	KB			
West coast summer survey	1990	351										351	UR
	1991	349										349	UR
	1992				310	310	44					309	KG
	1993				311	313		49				311	KG
	1994				290	290		4				290	KG
	1995				303						303	303	KG
	1996	292										292	UR
	1997	333		334									
	1999	268	307	299								283	LB
	2004			503									
	2005			353								333	LB
	2006		465	468									
2007		557	554								533	LB	
2008		410	409								410	LB	
West coast winter survey	1988	471										470	UR
	1990	303										303	UR
South coast spring survey	1994	10										10	UR
	2004												
	2006		489	243								243	LB
	2007		116									116	AD
	2008		149									148	AD
South coast autumn survey	1991	109										109	UR
	1992				40	40	5					40	KG
	1993				95	95		23				95	KG
	1994				69	95		27				69	KG
	1995	95										95	UR
	1996	60										60	UR
	1997	85										84	UR
	1999		139	139					140	140	140	134	LB
	2004												
	2005		194	193								185	LB
	2006		444	358								352	LB
2007		215	214								208	LB	
2008		137									132	AD	
Offshore commercial	1992				521	521	46					521	KG
	1993				646	645		75				645	KG
	1994				330	330		38				330	KG
Longline commercial	1994				314	314		9				314	KG

Table App.A.8: Female maturity-at-length ogive (equation B.50) parameter estimates (from Singh *et al.* 2011).

	l_{50} (cm)	Δ (cm)
<i>M. paradoxus</i>	41.53	2.98
<i>M. capensis</i>	53.83	10.14

Table App.A.9: Length-weight relationship estimates (from Singh 2013).

	α (gm/cm ^{β})	β
<i>M. paradoxus:</i>		
Males	0.007750	2.977
Females	0.005700	3.071
<i>M. capensis:</i>		
Males	0.006750	3.044
Females	0.005950	3.075

APPENDIX B: South African hake 2014 Reference Case assessment model specifications

The model used is a gender-disaggregated Statistical Catch-at-Age (SCAA), which is fitted directly to age-length keys (ALKs) and length frequencies. The model also assesses the two species as two independent stocks and is fitted to species-disaggregated data as well as species-combined data. A distinction is made between the west and the south coasts (see Figure 1), with hake movement surrogated using the “areas-as-fleets” approach. “Fleet” below therefore refer to a combination of gear type (offshore trawl, inshore trawl, longline and handline) and area (west and south coasts). The general specifications and equations of the overall model are set out below, together with some key choices in the implementation of the methodology. Details of the contributions to the log-likelihood function from the different data considered are also given. Quasi-Newton minimisation is used to minimise the total negative log-likelihood function (implemented using AD Model BuilderTM, Otter Research, Ltd. (Fournier *et al.* 2011)).

App.B.1 Population Dynamics

App.B1.1 Numbers-at-age

The resource dynamics of the two populations (*M. capensis* and *M. paradoxus*) of the South African hake are modelled by the following set of equations.

Note: for ease of reading, the ‘species’ subscript *s* has been omitted below where equations are identical for the two species.

$$N_{y+1,0}^g = R_{y+1}^g \quad (\text{B.1})$$

$$N_{y+1,a+1}^g = \left(N_{ya}^g e^{-M_a^g/2} - \sum_f C_{fya}^g \right) e^{-M_a^g/2} \quad \text{for } 0 \leq a \leq m - 2 \quad (\text{B.2})$$

$$N_{y+1,m}^g = \left(N_{y,m-1}^g e^{-M_{m-1}^g/2} - \sum_f C_{f,y,m-1}^g \right) e^{-M_{m-1}^g/2} + \left(N_{ym}^g e^{-M_m^g/2} - \sum_f C_{fym}^g \right) e^{-M_m^g/2} \quad (\text{B.3})$$

where

N_{ya}^g is the number of fish of gender *g* and age *a* at the start of year y^1 ;

R_y^g is the recruitment (number of 0-year-old fish) of fish of gender *g* at the start of year *y*;

m is the maximum age considered (taken to be a plus-group);

M_a^g denotes the natural mortality rate on fish of gender *g* and age *a*; and

C_{fya}^g is the number of hake of gender *g* and age *a* caught in year *y* by fleet *f*.

App.B.1.2 Recruitment

The number of recruits (i.e. new zero-year old fish) at the start of year *y* is assumed to be related to the corresponding female spawning stock size (i.e., the biomass of mature female fish). The underlying assumptions are that female spawning output can limit subsequent recruitment, but that there are always sufficient males to provide adequate fertilisation. The recruitment and corresponding female spawning stock size are related by means of the Beverton-Holt (Beverton and Holt 1957) or a modified (generalised) form of the Ricker stock-recruitment relationship. These

¹ In the interests of less cumbersome notation, subscripts have been separated by commas only when this is necessary for clarity.

forms are parameterized in terms of the “steepness” of the stock-recruitment relationship, h , the pre-exploitation equilibrium female spawning biomass, $K_y^{\circ,sp}$, and the pre-exploitation recruitment, R_0 and assuming a 50:50 sex-split at recruitment:

$$R_y^g = \frac{4hR_0 B_y^{\circ,sp}}{K_y^{\circ,sp}(1-h) + (5h-1)B_y^{\circ,sp}} e^{(\zeta_y - \sigma_R^2/2)} \quad (B.4a)$$

for the Beverton-Holt stock-recruitment relationship and

$$R_y^g = \alpha B_y^{\circ,sp} \exp\left(-\beta(B_y^{\circ,sp})^\gamma\right) e^{(\zeta_y - \sigma_R^2/2)} \quad (B.4b)$$

with

$$\alpha = R_0 \exp\left(\beta(K_y^{\circ,sp})^\gamma\right) \quad \text{and} \quad \beta = \frac{\ln(5h)}{(K_y^{\circ,sp})^\gamma (1-5^{-\gamma})}$$

for the modified Ricker relationship (for the true Ricker, $\gamma=1$) where

ζ_y reflects fluctuation about the expected recruitment in year y ;

σ_R is the standard deviation of the log-residuals, which is input ($\sigma_R = 0.45$ and is taken to decrease linearly from this value to 0.1 over the last five years to statistically stabilise estimates of recent recruitment).

$B_y^{\circ,sp}$ is the female spawning biomass at the start of year y , computed as:

$$B_y^{\circ,sp} = \sum_{a=1}^m f_a^{\circ} w_a^{\circ} N_{ya}^{\circ} \quad (B.5)$$

where

w_a^g is the begin-year mass of fish of gender g and age a ;

f_a^g is the proportion of fish of gender g and age a that are mature (converted from maturity-at-length, see equation App.B.47); and

$$R_0 = K_y^{\circ,sp} / \left[\sum_{a=1}^{m-1} f_a^{\circ} w_a^{\circ} e^{-\sum_{a=0}^{a-1} M_a^g} + f_m^{\circ} w_m^{\circ} \frac{e^{-\sum_{a=0}^{m-1} M_a^g}}{1 - e^{-M_m^g}} \right] \quad (B.6)$$

For the Beverton-Holt form, h is bounded above by 0.98 to preclude high recruitment at extremely low spawning biomass, whereas for the modified Ricker form, h is bounded above by 1.5 to preclude extreme compensatory behaviour. The Reference Case uses the modified Ricker form to model recruitment.

App.B.1.3 Total catch and catches-at-age

The fleet-disaggregated catch by mass, in year y is given by:

$$C_{fy} = \sum_g \sum_{a=0}^m \tilde{w}_{fy,a+1/2}^g C_{fya}^g = \sum_g \sum_{a=0}^m \tilde{w}_{fy,a+1/2}^g N_{ya}^g e^{-M_a^g/2} F_{fy} S_{fya}^g \quad (B.7)$$

where

C_{fya}^g is the catch-at-age, i.e. the number of fish of gender g and age a , caught in year y by fleet f ;

F_{fy} is the fishing mortality of a fully selected age class, for fleet f in year y (independent of g);

$$S_{fya}^g = \sum_l S_{fyl}^g P_{a+1/2,l}^g \quad (B.8)$$

S_{fya}^g is the commercial selectivity of gender g at age a for fleet f and year y ;

S_{fyt}^g is the commercial selectivity of gender g at length l for year y , and fleet f ;

$$\tilde{w}_{fy,a+1/2}^g = \frac{\sum_l S_{fyt}^g w_l^g P_{a+1/2,l}^g}{\sum_l S_{fyt}^g P_{a+1/2,l}^g} \quad (\text{B.9})$$

$\tilde{w}_{fy,a+1/2}^g$ is the selectivity-weighted mid-year weight-at-age a of gender g for fleet f and year y ;

w_l^g is the weight of fish of gender g and length l ;

$P_{a+1/2,l}^g$ is the mid-year proportion of fish of age a and gender g that fall in the length group l (i.e., $\sum_l P_{a+1/2,l}^g = 1$ for all ages a).

The matrix P is calculated under the assumption that length-at-age is log-normally distributed about a mean given by the von Bertalanffy equation, i.e.:

$$\ln l_a \sim N \left[\ln(l_\infty (1 - e^{-k(a-t_0)})); \left(\frac{\theta_a}{l_\infty (1 - e^{-k(a-t_0)})} \right)^2 \right] \quad (\text{B.10})$$

where θ_a is the standard deviation of length-at-age a , which is estimated directly in the model fitting for age 0, and for ages 1 and above a linear relationship applies:

$$\theta_a = \begin{cases} B_0 & \text{for } a = 0 \\ (\beta a + \alpha) & \text{for } 1 \leq a \leq m \end{cases}$$

with species and gender-specific B_0 , α and β estimated in the model fitting procedure. A penalty is added to ensure that θ_a is increasing with age, i.e. $\beta > 0$.

App.B.1.4 Exploitable and survey biomasses

The model estimate of the mid-year exploitable ("available") component of biomass for each species and fleet is calculated by converting the numbers-at-age into mid-year mass-at-age and applying natural and fishing mortality for half the year:

$$B_{fy}^{ex} = \sum_g \sum_{a=0}^m \tilde{w}_{fy,a+1/2}^g S_{fya}^g N_{ya}^g e^{-M_a^g/2} \left(1 - \sum_f S_{fya}^g F_{fy} / 2 \right) \quad (\text{B.11})$$

The model estimate of the survey biomass at the start of the year (summer) is given by:

$$B_y^{surv} = \sum_g \sum_{a=0}^{m_s} \tilde{w}_a^{g,sum} S_a^{g,sum} N_{ya}^g \quad (\text{B.12})$$

and in mid-year (winter):

$$B_y^{surv} = \sum_g \sum_{a=0}^m \tilde{w}_{a+1/2}^{g,win} S_a^{g,win} N_{ya}^g e^{-M_a^g/2} \left(1 - \sum_f S_{fya}^g F_{fy} / 2 \right) \quad (\text{B.13})$$

where

$S_a^{g,sum/win}$ is the survey selectivity of gender g for age a , converted from survey selectivity-at-length in the same manner as for the commercial selectivity (eqn B.8);

$\tilde{w}_a^{g,i}$ is the survey selectivity-weighted weight-at-age a of gender g for survey i , computed in the same manner as for the commercial selectivity-weight-at-age (equation App.II.9) and taking account of the begin-year ($\tilde{w}_{y,a}^{g,sum}$ from $P_{a,l}^g$) or mid-year ($\tilde{w}_{y,a+1/2}^{g,win}$ from $P_{a+1/2,l}^g$) nature of the surveys.

Note that both the spring and autumn surveys are taken to correspond to winter (mid-year), and that as with the commercial catch the areas-as-fleets approach underlies the use of selectivity to reflect differences between the west and south coast surveys.

It is assumed that the resource is at the deterministic equilibrium that corresponds to an absence of harvesting at the start of the initial year considered, i.e., $B_1^{g,sp} = K^{g,sp}$, and the year $y=1$ corresponds to 1917 when catches commence.

App.B.2 MSY and related quantities

The equilibrium catch for a fully selected fishing proportion F^* is calculated as:

$$C(F^*) = \sum_g \sum_a \tilde{w}_{a+1/2}^g S_a^g F^* N_a^g(F^*) e^{-((M_a^g + S_a^g F^*)/2)} \quad (\text{B.14})$$

where

S_a^g is the average selectivity across all fleets, for the most recent five years;

$$S_a^g = \frac{\sum_{y=2005}^{2009} \sum_f S_{fya}^g F_{fy}}{\max \left(\sum_{y=2005}^{2009} \sum_f S_{fya}^g F_{fy} \right)} \quad (\text{B.15})$$

where the maximum is taken over genders and ages; and with

$$N_a^g(F^*) = \begin{cases} R_1(F^*) & \text{for } a = 1 \\ N_{a-1}^g(F^*) e^{-M_{a-1}^g (1 - S_{a-1}^g F^*)} & \text{for } 1 < a < m \\ N_{m-1}^g(F^*) e^{-M_{m-1}^g (1 - S_{m-1}^g F^*)} & \text{for } a = m \\ \frac{N_{m-1}^g(F^*) e^{-M_{m-1}^g (1 - S_{m-1}^g F^*)}}{(1 - e^{-M_m^g (1 - S_m^g F^*)})} & \end{cases} \quad (\text{B.16})$$

where

$$R_1(F^*) = \frac{\alpha B_{MSY}^{g,sp}(F^*)}{\beta + B_{MSY}^{g,sp}(F^*)} \quad (\text{B.17})$$

for a Beverton-Holt stock–recruitment relationship.

The maximum of $C(F^*)$ is then found by searching over F^* to give F_{MSY}^* , with the associated female spawning biomass given by:

$$B_{MSY}^{g,sp} = \sum_a f_a^{g,sp} w_a^{g,sp} N_a^g(F_{MSY}^*) \quad (\text{B.18})$$

App.B.3 The likelihood function

The model is fit to CPUE and survey biomass indices, commercial and survey length frequencies, survey age-length keys, as well as to the stock-recruitment curve to estimate model parameters. Contributions by each of these to the negative of the log-likelihood ($-\lambda nL$) are as follows².

App.B.3.1 CPUE relative biomass data

The likelihood is calculated by assuming that the observed biomass index (here CPUE) is log-normally distributed about its expected value:

$$I_y^i = \hat{I}_y^i e^{\varepsilon_y^i} \quad \text{or} \quad \varepsilon_y^i = \lambda n(I_y^i) - \lambda n(\hat{I}_y^i) \quad (\text{B.19})$$

where

I_y^i is the biomass index for year y and series i (which corresponds to a specified species and fleet);

² Strictly it is a penalised log-likelihood which is maximised in the fitting process, as some contributions that would correspond to priors in a Bayesian estimation process are added.

$\hat{I}_y^i = \hat{q}^i \hat{B}_{fy}^{ex}$ is the corresponding model estimate, where \hat{B}_{fy}^{ex} is the model estimate of exploitable resource biomass, given by equation B.11;

\hat{q}^i is the constant of proportionality for biomass series i ; and

ε_y^i from $N(0, (\sigma_y^i)^2)$.

In cases where the CPUE series are based upon species-aggregated catches (as available pre-1978), the corresponding model estimate is derived by assuming two types of fishing zones: z1) an “*M. capensis* only zone”, corresponding to shallow-water and z2) a “mixed zone” (see diagrammatic representation in Figure B.1).

The total catch of hake of both species (*BS*) by fleet f in year y ($C_{BS, fy}$) can be written as:

$$C_{BS, fy} = C_{C, fy}^{z1} + C_{C, fy}^{z2} + C_{P, fy} \quad (B.20)$$

where

$C_{C, fy}^{z1}$ is the *M. capensis* catch by fleet f in year y in the *M. capensis* only zone (z1);

$C_{C, fy}^{z2}$ is the *M. capensis* catch by fleet f in year y in the mixed zone (z2); and

$C_{P, fy}$ is the *M. paradoxus* catch by fleet f in year y in the mixed zone.

Catch rate is assumed to be proportional to exploitable biomass. Furthermore, let γ be the proportion of the *M. capensis* exploitable biomass in the mixed zone ($\gamma = B_{C, fy}^{ex, z2} / B_{C, fy}^{ex}$) (assumed to be constant throughout the period for simplicity) and ψ_{fy} be the proportion of the effort of fleet f in the mixed zone in year y ($\psi_{fy} = E_{fy}^{z2} / E_{fy}$), so that:

$$C_{C, fy}^{z1} = q_C^{i, z1} B_{C, fy}^{ex, z1} E_{fy}^{z1} = q_C^{i, z1} (1 - \gamma) B_{C, fy}^{ex} (1 - \psi_{fy}) E_{fy} \quad (B.21)$$

$$C_{C, fy}^{z2} = q_C^{i, z2} B_{C, fy}^{ex, z2} E_{fy}^{z2} = q_C^{i, z2} \gamma B_{C, fy}^{ex} \psi_{fy} E_{fy} \quad \text{and} \quad (B.22)$$

$$C_{P, fy} = q_P^i B_{P, fy}^{ex} E_{fy}^{z2} = q_P^i B_{P, fy}^{ex} \psi_{fy} E_{fy} \quad (B.23)$$

where

$E_{fy} = E_{fy}^{z1} + E_{fy}^{z2}$ is the total effort of fleet f , corresponding to combined-species CPUE series i which consists of the effort in the *M. capensis* only zone (E_{fy}^{z1}) and the effort in the mixed zone (E_{fy}^{z2});

$q_C^{i, zj}$ is the catchability for *M. capensis* (*C*) for biomass series i , and zone zj ; and

q_P^i is the catchability for *M. paradoxus* (*P*) for biomass series i .

It follows that:

$$C_{C, fy} = B_{C, fy}^{ex} E_{fy} \left[q_C^{i, z1} (1 - \gamma) (1 - \psi_{fy}) + q_C^{i, z2} \gamma \psi_{fy} \right] \quad (B.24)$$

$$C_{P, fy} = B_{P, fy}^{ex} E_{fy} q_P^i \psi_{fy} \quad (B.25)$$

From solving equations B.24 and B.25:

$$s_{fy} = \frac{q_C^{i, z1} (1 - \gamma)}{\left\{ \frac{C_{C, fy} B_{P, fy}^{ex} q_P^i}{B_{C, fy}^{ex} C_{P, fy}} - q_C^{i, z2} \gamma + q_C^{i, z1} (1 - \gamma) \right\}} \quad (B.26)$$

and:

$$\hat{I}_y^i = \frac{C_{fy}}{E_{fy}} = \frac{C_{fy} B_{P, fy}^{ex} q_P^i \psi_{fy}}{C_{P, fy}} \quad (B.27)$$

Zone 1 (z1): <i>M. capensis</i> only	Zone 2 (z2): Mixed zone
<i>M. capensis</i> : biomass (B_C^{z1}), catch(C_C^{z1})	<i>M. capensis</i> : biomass (B_C^{z2}), catch(C_C^{z2}) <i>M. paradoxus</i> : biomass (B_p), catch(C_p)
Effort in zone 1 (E^{z1})	Effort in zone 2 (E^{z2})

Figure B.1: Diagrammatic representation of the two conceptual fishing zones.

Two species-aggregated CPUE indices are available: the ICSEAF West Coast and the ICSEAF South Coast series. For consistency, q 's for each species (and zone) are forced to be in the same proportion:

$$q_s^{SC} = r q_s^{WC} \quad (B.28)$$

To correct for possible negative bias in estimates of variance (σ_y^i) and to avoid according unrealistically high precision (and so giving inappropriately high weight) to the CPUE data, lower bounds on the standard deviations of the residuals for the logarithm of the CPUE series have been enforced: for the historic ICSEAF CPUE series (separate West Coast and South Coast series) the lower bound is set to 0.25, and to 0.15 for the recent GLM-standardised CPUE series, i.e.: $\sigma^{ICSEAF} \geq 0.25$ and $\sigma^{GLM} \geq 0.15$.

The contribution of the CPUE data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\lambda \ln L^{CPUE} = \sum_i \sum_y \left[\lambda \ln(\sigma_y^i) + (\varepsilon_y^i)^2 / 2(\sigma_y^i)^2 \right] \quad (B.29)$$

where

σ_y^i is the standard deviation of the residuals for the logarithms of index i in year y .

Homoscedasticity of residuals for CPUE series is customarily assumed³, so that $\sigma_y^i = \sigma^i$ is estimated in the fitting procedure by its maximum likelihood value:

$$\hat{\sigma}^i = \sqrt{1/n_i \sum_y (\lambda \ln(I_y^i) - \lambda \ln(\hat{I}_y^i))^2} \quad (B.30)$$

where n_i is the number of data points for biomass index i .

In the case of the species-disaggregated CPUE series, the catchability coefficient q^i for biomass index i is estimated by its maximum likelihood value, which in the more general case of heteroscedastic residuals is given by:

$$\ln \hat{q}^i = \frac{\sum_y (\ln I_y^i - \ln \hat{B}_{fy}^{ex}) / (\sigma_y^i)^2}{\sum_y 1 / (\sigma_y^i)^2} \quad (B.31)$$

In the case of the species-combined CPUE, $q_C^{WC,z1}$, $q_C^{WC,z2}$, q_P^{WC} , r and γ are estimated directly in the fitting procedure.

³ There are insufficient data in any series to enable this to be tested with meaningful power.

App.B.3.2 Survey biomass data

Data from the research surveys are treated as relative biomass indices in a similar manner to the species-disaggregated CPUE series above, with survey selectivity function $S_a^{g, sum/win}$ replacing the commercial selectivity S_{fya}^g (see equations B.12 and B.13 above, which also take account of the begin- or mid-year nature of the survey).

An estimate of sampling variance is available for most surveys and the associated σ_y^i is generally taken to be given by the corresponding survey CV. However, these estimates likely fail to include all sources of variability, and unrealistically high precision (low variance and hence high weight) could hence be accorded to these indices. The contribution of the survey data to the negative log-likelihood is of the same form as that of the CPUE biomass data (see equation B.29). The procedure adopted takes into account an additional variance $(\sigma_A)^2$ which is treated as another estimable parameter in the minimisation process, i.e:

$$-\lambda n L^{Survey} = \sum_i \sum_y \left[\lambda n \left(\sqrt{(\sigma_y^i)^2 + (\sigma_A)^2} \right) + (\varepsilon_y^i)^2 / 2 \left((\sigma_y^i)^2 + (\sigma_A)^2 \right) \right] \quad (B.32)$$

This procedure is carried out enforcing the constraint that $(\sigma_A)^2 > 0$, i.e. the overall variance cannot be less than its externally input component.

In June 2003, the trawl gear on the *Africana* was changed and a different value for the multiplicative bias factor q is taken to apply to the surveys conducted with the new gear. Calibration experiments have been conducted between the *Africana* with the old gear (hereafter referred to as the "old *Africana*") and the *Nansen*, and between the *Africana* with the new gear ("new *Africana*") and the *Nansen*, in order to provide a basis to relate the multiplicative biases of the *Africana* with the two types of gear (q_{old} and q_{new}). A recent calibration analysis based on "Model 1" (see Table 1, "Model 1" of Smith *et al.*, 2013) provided the following estimates:

$$\begin{aligned} (q^{new}/q^{old})^{capensis} &= 0.652 && \text{with SE}=0.073 \text{ and} \\ (q^{new}/q^{old})^{paradoxus} &= 0.883 && \text{with SE}=0.082. \end{aligned}$$

The following contribution is therefore added as a penalty (or a log prior in a Bayesian context) to the negative log-likelihood in the assessment:

$$-\lambda n L^{q-ch} = (\lambda n q_{new} - \lambda n q_{old} - \Delta \lambda n q)^2 / 2 \sigma_{\Delta n q}^2 \quad (B.33)$$

A different length-specific selectivity is estimated for the "old *Africana*" and the "new *Africana*".

The survey's catchability coefficients q (for the survey with the old *Africana* gear) are constrained to values below 1 (i.e. it is assumed that the nets do not herd the hake):

$$pen^q = \sum_i (q_{old}^i - 1)^2 / 0.02^2 \quad \text{if} \quad q_{old}^i > 1 \quad (B.34)$$

App.B.3.3 Commercial proportions at length

Commercial proportions at length from the offshore trawl fleet cannot be disaggregated by species and gender as the data collected did not distinguish these. The model is therefore fit to the proportions at length as determined for both species and gender combined. The catches made by the inshore trawl fleet are assumed to consist of *M. capensis* only, and species and sex information is available over the 2000-2010 period for the longline fleet.

The catches at length are computed as:

$$C_{fjl} = \sum_s \sum_g \sum_{a=0}^m N_{sya}^g F_{sfl} S_{sfl}^g P_{s,a+1/2,l}^g e^{-M_{sa}^g/2} \left(1 - \sum_f S_{sfl}^g F_{fjl} / 2 \right) \quad (B.35)$$

where the summation over species and gender is taken only where appropriate.

The predicted proportions at length:

$$\hat{p}_{yl}^i = C_{fyl} / \sum_l C_{fyl} \quad (\text{B.36})$$

The contribution of the proportion at length data to the negative of the log-likelihood function when assuming an “adjusted” lognormal error distribution (Punt and Kennedy, 1997) is given by:

$$-\lambda n L^{\text{length}} = 0.1 \sum_y \sum_l \left[\lambda n \left(\sigma_{len}^i / \sqrt{p_{yl}^i} \right) + p_{yl}^i \left(\lambda n p_{yl}^i - \lambda n \hat{p}_{yl}^i \right)^2 / 2 \left(\sigma_{len}^i \right)^2 \right] \quad (\text{B.37})$$

where

the superscript ‘*i*’ refers to a particular series of proportions at length data which reflect a specified fleet, species and sex (or combination thereof); and

σ_{len}^i is the standard deviation associated with the proportion at length data, which is estimated in the fitting procedure by:

$$\hat{\sigma}_{len}^i = \sqrt{\sum_y \sum_l p_{yl}^i \left(\ln p_{yl}^i - \ln \hat{p}_{yl}^i \right)^2 / \sum_y \sum_l 1} \quad (\text{B.38})$$

The initial 0.1 multiplicative factor in equation B.37 is a somewhat arbitrary downweighting to allow for correlation between proportions in adjacent length groups. The coarse basis for this adjustment is the ratio of effective number of age-classes present to the number of length groups in the minimisation, under the argument that independence in variability is likely to be more closely related to the former.

Commercial proportions at length are incorporated in the likelihood function using equation B.37, for which the summation over length *l* is taken from length l_{minus} (considered as a minus group) to l_{plus} (a plus group). The length for the minus- and plus-groups are fleet specific and are chosen so that typically a few percent, but no more, of the fish sampled fall into these two groups.

App.B.3.4 Survey proportions at length

The survey proportions at length are incorporated into the negative of the log-likelihood in an analogous manner to the commercial catches-at-age, assuming an adjusted log-normal error distribution (equation B.37). In this case however, data are disaggregated by species, and for some surveys further disaggregated by gender:

$$p_{syl}^{g,surv} = \frac{C_{syl}^{g,surv}}{\sum_g \sum_l C_{syl}^{g,surv}} \quad \text{is the observed proportion of fish of species } s, \text{ gender } g \text{ and length } l \text{ from survey } surv \text{ in}$$

year *y*; and

$\hat{p}_{syl}^{g,surv}$ is the expected proportion of fish of species *s*, gender *g* and length *l* in year *y* in the survey *surv*, given by:

$$\hat{p}_{syl}^{g,surv} = \frac{\sum_a S_{sl}^{g,sum} P_{sal}^g N_{sya}^g}{\sum_g \sum_l \sum_a S_{sl}^{g,sum} P_{sal}^g N_{sya}^g} \quad (\text{B.39})$$

for begin-year (summer) surveys, or

$$\hat{p}_{syl}^{g,surv} = \frac{\sum_a S_{sl}^{g,win} P_{s,a+1/2,l}^g N_{sya}^g e^{-M_{sa}^g/2} \left(1 - \sum_f S_{sfa}^g F_{sfy} / 2 \right)}{\sum_g \sum_l \sum_a S_{sl}^{g,win} P_{s,a+1/2,l}^g N_{sya}^g e^{-M_{sa}^g/2} \left(1 - \sum_f S_{sfa}^g F_{sfy} / 2 \right)} \quad (\text{B.40})$$

for mid-year (autumn, winter or spring) surveys.

8.II.3.5 Age-length keys

Under the assumption that fish are sampled randomly with respect to age within each length-class, the contribution to the negative log-likelihood for the ALK data (ignoring constants) is:

$$-\ln L^{ALK} = -w \sum_i \sum_l \sum_a \left[A_{i,l,a}^{obs} \ln(\hat{A}_{i,l,a}) - A_{i,l,a}^{obs} \ln(A_{i,l,a}^{obs}) \right] \quad (B.41)$$

where

w is a downweighting factor to allow for overdispersion in these data compared to the expectation for a multinomial distribution with independent data; this downweighting factor is somewhat arbitrarily set to 0.01 to avoid these data overriding trend information in the indices of biomass;

$A_{i,a,l}^{obs}$ is the observed number of fish of size class l that fall in age a , for ALK i (a specific combination of survey, year, species and gender);

$\hat{A}_{i,a,l}$ is the model estimate of $A_{i,a,l}^{obs}$, computed as:

$$\hat{A}_{i,a,l} = W_{i,l} \frac{C_{i,a,l} \tilde{A}_{a,l}}{\sum_{a'} C_{i,a',l} \tilde{A}_{a',l}} \quad (B.42)$$

where

$W_{i,l}$ is the number of fish in length class l that were aged for ALK i ,

$\tilde{A}_{a,l} = \sum_a P(a'|a) A_{a,l}$ is the ALK for age a and length l after accounting for age-reading error,

with $P(a'|a)$, the age-reading error matrix, representing the probability of an animal of true age a being aged to be that age or some other age a' .

Age-reading error matrices have been computed for each reader and for each species as reported in Appendix A.

When multiple readers age the same fish, these data are considered to be independent information in the model fitting.

App.B.3.6 Stock-recruitment function residuals

The stock-recruitment residuals are assumed to be log-normally distributed. Thus, the contribution of the recruitment residuals to the negative of the log-likelihood function is given by:

$$-\lambda \ln L^{SR} = \sum_s \left[\sum_{y=y1}^{y2} \zeta_{sy}^2 / 2\sigma_R^2 + \left(\sum_{y=y1}^{y2} \zeta_{sy} \right)^2 / 0.01^2 \right] \quad (B.43)$$

where

ζ_{sy} is the recruitment residual for species s , and year y , which is assumed to be log-normally distributed with standard deviation σ_R and which is estimated for year $y1$ to $y2$ (see equation B.4) (estimating the stock-recruitment residuals is made possible by the availability of catch-at-age data, which give some indication of the age-structure of the population); and

σ_R is the standard deviation of the log-residuals, which is input.

The stock-recruitment residuals are estimated for years 1985 to 2013, with recruitment for other years being set deterministically (i.e. exactly as given by the estimated stock-recruitment curve) as there is insufficient catch-at-age information to allow reliable residual estimation for earlier years. A limit on the recent recruitment fluctuations is set by having the σ_R (which measures the extent of variability in recruitment – see equation – App.B.43) decreasing linearly from 0.45 in 2004 to 0.1 in 2009, thereby effectively forcing recruitment over the last years to lie closer to the stock-recruitment relationship curve.

The second term on the right hand side is introduced to force the average of the residuals estimated over the period from y_1 to y_2 to be close to zero, to assist stabilise the estimation.

App.B.4 Model parameters

App.B.4.1 Estimable parameters

The primary parameters estimated are the species-specific female virgin spawning biomass ($K_s^{\varphi,sp}$) and “steepness” (h_s) and γ (for the modified Ricker curve used in the Reference Case, see equation B.4b) of the stock-recruitment relationship. The standard deviations σ^i for the CPUE series residuals (the species-combined as well as the GLM-standardised series) as well as the additional variance (σ_A^i)² for each species are treated as estimable parameters in the minimisation process. Similarly, in the case of the species-combined CPUE, $q_C^{WC,z1}$, $q_C^{WC,z2}$, q_P^{WC} , r and γ are directly estimated in the fitting procedure.

The species- and gender-specific von Bertalanffy growth curve parameters (l_∞ , κ and t_0) are estimated directly in the model fitting process, as well as the B_0 , α and β , values used to compute the standard deviation of the length-at-age a .

The following parameters are also estimated in the model fits undertaken (if not specifically indicated as fixed):

App.B.4.1.1 Natural mortality:

Natural mortality (M_{sa}^g) is assumed to be age-specific and is calculated using the following functional form (the selection of the specific form here is based on convenience and is somewhat arbitrary):

$$M_{sa}^{\varphi} = \begin{cases} M_{s2}^{\varphi} & \text{for } a \leq 1 \\ \alpha_s^M + \frac{\beta_s^M}{a+1} & \text{for } 2 \leq a \leq 5 \\ M_{s5}^{\varphi} & \text{for } a > 5 \end{cases} \quad (\text{B.44})$$

and

$$M_{sa}^{males} = \nu^s M_{sa}^{females} \quad (\text{B.45})$$

M_{s0} and M_{s1} are set equal to M_{s2} ($= \alpha_s^M + \beta_s^M / 3$) as there are no data (hake of ages younger than 2 are rare in catch and survey data) which would allow independent estimation of M_{s0} and M_{s1} .

When M values are estimated in the fit, a penalty is added to the total $-\ln L$ so that $M_{s2} \geq M_{s5}$:

$$pen^M = \sum_s (M_{s5} - M_{s2})^2 / 0.01^2 \quad \text{if } M_{s2} < M_{s5} \quad (\text{B.46})$$

For the Reference Case, the following values are fixed: $M_{s2}^{\varphi} = 0.75$ and $M_{s5}^{\varphi} = 0.375$ for both species and genders.

App.B.4.1.2 Stock-recruitment residuals:

Stock-recruitment residuals ζ_{sy} are estimable parameters in the model fitting process. They are estimated separately for each species from 1985 to the present, and set to zero pre-1985 because there are no catch-at-length data for that period to provide the information necessary to inform estimation.

Table B.1 summarises the estimable parameters, excluding the selectivity parameters.

App.B.4.1.3 Survey fishing selectivity-at-length:

The survey selectivities are estimated directly for seven pre-determined lengths for *M. paradoxus* and *M. capensis*. When the model was fitted to proportion-at-age rather than proportion-at-length, survey selectivities were estimated

directly for each age (i.e. seven age classes). The lengths at which selectivity is estimated directly are survey specific (at constant intervals between the minus and plus groups) and are given in Table B.2. Between these lengths, selectivity is assumed to change linearly. The slope (trend) from lengths $l_{minus}+1$ to l_{minus} is assumed to continue exponentially to lower lengths down to length 1, and similarly the slope from lengths $l_{plus}-1$ to l_{plus} for *M. paradoxus* and *M. capensis* to continue for greater lengths.

For the South Coast spring and autumn surveys, gender-specific selectivities are estimated for *M. paradoxus*. Furthermore, the female selectivities are scaled down by a parameter estimated for each of these two surveys to allow for the male predominance in the survey catch. This is done for *M. paradoxus* on the South Coast only, as the catch-at-length data for *M. paradoxus* West Coast surveys and *M. capensis* on both coasts do not show substantial gender differences

A penalty is added to the total $-\ln L$ to smooth the selectivities to smooth the selectivities by penalising deviations from straight line dependence (the choice of a weighting of 3 was made empirically to balance this term having sufficient but not undue influence):

$$pen^{survS} = \sum_i \sum_{L=L_i+1}^{L_i-1} 3(S_{L-1}^i - 2S_L^i + S_{L+1}^i)^2 \quad (B.47)$$

where i is a combination of survey, species and gender.

App.B.4.1.4 Commercial fishing selectivity-at-length:

The fishing selectivity-at-length for each species and fleet, S_{sfl} , is estimated in terms of a double normal curve given by:

$$S_{sfl} = \exp\left(-\frac{(l-l_{max})^2}{2\sigma_{Left}^2}\right) \quad \text{for } l \leq l_{max}$$

$$S_{sfl} = \exp\left(-\frac{(l-l_{max})^2}{2\sigma_{Right}^2}\right) \quad \text{for } l > l_{max} \quad (B.48)$$

where σ_{Left} , σ_{Right} and l_{max} are estimable parameters.

Periods of fixed and changing selectivity have been assumed for the offshore trawl fleet to take account of the change in the selectivity at low ages over time in the commercial catches, likely due to the phasing out of the (illegal) use of net liners to enhance catch rates.

Two selectivity periods are also assumed for the longline fleet.

On the South Coast, for *M. paradoxus*, the female offshore trawl selectivity (only the trawl fleet is assumed to catch *M. paradoxus* on the South Coast) is scaled down by a factor taken as the average of those estimated for the South Coast spring and autumn surveys. Although there is no gender information for the commercial catches, the South Coast spring and autumn surveys catch a much higher proportion of male *M. paradoxus* than female (ratios of about 7:1 and 3.5:1 for spring and autumn respectively). This is assumed to reflect a difference in distribution of the two genders which would therefore affect the commercial fleet similarly.

Details of the fishing selectivities (including the number of parameters estimated) that are used in the assessment are shown in Table B.3.

App.B.4.2 Input parameters and other choice for application to hake

App.B.4.2.1 Age-at-maturity:

The proportion of fish of species s , gender g and length l that are mature is assumed to follow a logistic curve with the parameter values given in Table App.A.8:

$$f_{sl}^g = \left(1 + e^{\frac{l-l_{50}^{s,g}}{\Delta^{s,g}}}\right)^{-1} \quad (B.50)$$

• Maturity-at-length is then converted to maturity-at-age as follows:

$$f_{sa}^g = \sum_l f_{sl}^g P_{a,l}^g \tag{B.50}$$

App.B.4.2.2 Weight-at-length:

The weight-at-length for each species and gender is calculated from the mass-at-length function, with values of the parameters for this function listed in Table App.A.9.

App.B.4.2.3 Minus- and plus-groups

Because of a combination of gear selectivity and mortality, a relatively small number of fish in the smallest and largest length classes are caught. In consequence, there can be relatively larger errors (in terms of variance) associated with these data. To reduce this effect, the assessment is conducted with minus- and plus-groups obtained by summing the data over the lengths below and above l_{minus} and l_{plus} respectively. The minus- and plus-group used are given in Table B.4. Furthermore, the proportions at length data (both commercial and survey) are summed into 2cm length classes for the model fitting.

Table B.1: Parameters estimated in the model fitting procedure, excluding selectivity parameters.

	No of parameters	Parameters estimated	Bounds enforced
K^{\varnothing}	2	$\ln(K_{cap}^{\varnothing})$ and $\ln(K_{para}^{\varnothing})$	(3.5; 9.0)
h	2	h_{cap} and h_{para}	(0.2; 0.98) for BH, (0.2; 1.5) for modified Ricker
Additional variance	2	$\sigma_{A,cap}$ and $\sigma_{A,para}$	(0; 0.5)
Recruitment residuals	58	$\zeta_{cap,1985-2013}$ and $\zeta_{para,1985-2013}$	(-5; 5)
σ_{CPUE}	6	1 for each series	ICSEAF: (0.25; 1), GLM (0.15; 1)
ICSEAF CPUE	5	$q_C^{WC,z1}$, $q_C^{WC,z2}$, q_P^{WC} , r and γ_R	q and r : (0,10), and γ_R (0; 1)
θ_a	12	For each species and gender: θ_0 , θ_1 and θ_{14}	θ_0 : (1; 100), θ_1 and θ_{14} : (1; 100)
Growth	12	For each species and gender: L_5 , κ and t_0	L_5 : (30; 60) κ : (0.00005; 0.2) and t_0 : (-10; 0)

Table B.2: Lengths (in cm) at which survey selectivity is estimated directly.

<i>M. paradoxus</i>	West coast summer	13	18	23	28	32	37	42	47
	West coast winter	13	18	24	29	35	40	46	51
	South coast spring	21	26	30	35	39	44	48	53
	South coast autumn	21	26	31	36	42	47	52	65
<i>M. capensis</i>	West coast summer	13	20	26	33	39	46	52	59
	West coast winter	13	17	21	30	40	47	54	61
	South coast spring	13	19	28	38	46	54	63	71
	South coast autumn	13	19	28	36	44	52	61	69

Table B.3: Details for the commercial selectivity-at-length for each fleet (trawl unless otherwise indicated) and species combination for the Reference Case, as well as indications of the nature of the data which are available.

	<i>M. paradoxus</i>		<i>M. capensis</i>		Data available
	No of est. parameters	Comments	No of est. parameters	Comments	
1. West coast offshore					
1917-1976	0	average between 77-84 and 93-2013 period	0	average between 77-84 and 93-2013 period	species combined
1977-1984	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	0	as 93-13 but σ_{Left} same difference as for paradoxus btw 77-84 and 93-13 periods	
1985-1992	0	linear change between 1984 and 1993 selectivity	0	linear change between 1984 and 1993 selectivity	
1993-2013	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	0	Based on inshore selectivity: $I_{max}=I_{max}(inshore)+5$, $\sigma_{Left}=\sigma_{Left}(inshore)$ and $\sigma_{Right}=3*\sigma_{Right}(inshore)$	
2. South coast offshore					
1917-1976	0	double logistic (σ_{Left} , σ_{Right} and I_{max})	0	average between 77-84 and 93-2013 period	species combined
1977-1984	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	0	as 93-13 but σ_{Left} same difference as for paradoxus btw 77-84 and 93-13 periods	
1985-1992	0	linear change between 1984 and 1993 selectivity	0	linear change between 1984 and 1993 selectivity	
1993-2013	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	0	Based on inshore selectivity: $I_{max}=I_{max}(inshore)+5$, $\sigma_{Left}=\sigma_{Left}(inshore)$	
	0	female downscaling factor (av. of SC spring and autumn surveys's factors)			
3. South coast inshore	-	-	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	<i>M. capensis</i>
4. West coast longline					
1984-1999	1	$I_{max}^{84-99}=I_{max}^{00-05}-\delta_1^{WCpara}$ (δ_1^{WCpara} estimated, same for males and females)	1	$I_{max}^{84-99}=I_{max}^{00-05}-\delta_1^{WCcap}$ (δ_1^{WCcap} estimated, same for males and females)	species and gender combined
2000-2005	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	
	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	species and gender disaggregated
2006-2013	1	$I_{max}^{06-13}=I_{max}^{00-05}-\delta_2^{WCpara}$ (δ_2^{WCpara} estimated, same for males and females)	1	$I_{max}^{06-13}=I_{max}^{00-05}-\delta_2^{WCcap}$ (δ_2^{WCcap} estimated, same for males and females)	
	0		0		
5. South coast longline					
1984-1999	1	$I_{max}^{84-99}=I_{max}^{00-05}-\delta_1^{SCpara}$ (δ_1^{SCpara} estimated, same for males and females)	1	$I_{max}^{84-99}=I_{max}^{00-05}-\delta_1^{SCcap}$ (δ_1^{SCcap} estimated, same for males and females)	species and gender combined
2000-2005	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	
	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	3	double logistic (σ_{Left} , σ_{Right} and I_{max})	species and gender disaggregated
2006-2013	1	$I_{max}^{06-13}=I_{max}^{84-99}-\delta_2^{SCpara}$ (δ_2^{SCpara} estimated, same for males and females)	1	$I_{max}^{06-13}=I_{max}^{84-99}-\delta_2^{SCcap}$ (δ_2^{SCcap} estimated, same for males and females)	
	0		0		
6. South coast handline	-	-	0	parameters taken as average of SC longline (female, 00-05) and inshore parameters	-
Total	28		19		

Table B.4: Minus- and plus-groups taken for the surveys and commercial proportion at length data.

SURVEY DATA		Sex-aggregated		Males		Females	
		Minus	Plus	Minus	Plus	Minus	Plus
West coast summer	<i>M. paradoxus</i>	13	47	20	47	20	47
	<i>M. capensis</i>	13	59	20	59	20	59
West coast winter	<i>M. paradoxus</i>	13	51	-	-	-	-
	<i>M. capensis</i>	13	61	-	-	-	-
South coast spring	<i>M. paradoxus</i>	21	53	20	53	20	53
	<i>M. capensis</i>	13	71	20	71	20	71
South coast autumn	<i>M. paradoxus</i>	21	65	20	59	20	59
	<i>M. capensis</i>	13	69	20	69	20	69

COMMERCIAL DATA		Sex-aggregated		Males		Females	
Fleet	Species	Minus	Plus	Minus	Plus	Minus	Plus
West coast offshore trawl	species combined	23	65	-	-	-	-
South coast offshore trawl	species combined	27	75	-	-	-	-
South coast inshore trawl	<i>M. capensis</i>	27	65	-	-	-	-
West coast longline	species combined	45	91	-	-	-	-
	<i>M. paradoxus</i>	-	-	41	85	41	95
	<i>M. capensis</i>	-	-	41	85	41	95
South coast longline	<i>M. paradoxus</i>	-	-	41	85	41	80
	<i>M. capensis</i>	45	91	41	95	41	80