Assessment of the South African anchovy resource using data from 1984 – 2014: base case results at the posterior mode

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Abstract

The operating model (OM) for the South African anchovy resource has been updated from that used to develop OMP-14 given three more years of data. The model has been altered from previous assessments to now fit directly to length frequency data, removing the earlier need for estimates of proportions of anchovy-at-age 1 during the annual November hydroacoustic survey. A Beverton Holt stock recruitment relationship is used for the base case. Time-invariant natural mortality is assumed to be 1.2year^{-1} for both juvenile and adult natural mortality as before. The resource abundance is estimated to be above the historical (1984-2013) average, with a total biomass of 4.2 million tons in November 2014. Recruitment reflects three major peaks over the past 20 years, although the lowest points in these fluctuations were still large, being similar to the maximum recruitment prior to 2000.

Introduction

The operating model of the South African anchovy resource has been updated from the last assessment (de Moor and Butterworth 2012) to take account of data collected between 2012 and 2014. There have been substantial changes in the model formulation, in particular to be able to fit directly to length-frequency data from the November survey and from commercial catches. The time series of estimates of proportions of 1 year old anchovy in the November survey (de Moor et al. 2013) which was used previously is now no longer required.

This document presents the updated base case operating model with results at the posterior mode only. A subsequent separate document will show the full posterior distributions and compare these results to those for a number of robustness tests.

Available Data

de Moor et al. (2015) detail all the data used in this assessment. Key changes from the data used by de Moor and Butterworth (2012), and how they are utilised in the model, include the following.

i) The incorporation of three more year's survey data from November 2012 to 2014.

ii) The model fits to November survey length-structured data, instead of estimates of proportions-at-age 1 in the November survey.

iii) The model fits to quarterly commercial length-structured data, instead of assuming catch-at-age (calculated using monthly and annually varying cut-off lengths) is observed without error.

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Population Dynamics Model

The operating model used for the South African anchovy resource is detailed in Appendix A. All parameters used in this document are listed with definitions as well as parameter values, prior distributions or associated equations in Table A.1.

Key changes in the population dynamics model from de Moor and Butterworth (2012) include the following.

i) The model is still age-structured at its core, but has been extended using estimated length-at-age distributions (equations A.4 and A.20) to be able to fit directly to length- rather than age-structured data.

ii) Quarterly catches-at-age are estimated within the model (equations A.11 and A.15). Catches of ages older than 1 are thus allowed, while for de Moor and Butterworth (2012) the catch was split between ages 0 and 1 only, using monthly and annually varying cut-off lengths.

iii) A commercial selectivity curve is thus now also required, and changes in commercial selectivity between quarters is allowed in the estimation process (equation A.9).

iv) The assumption is made that the November survey estimate of biomass is an estimate of total (0+) biomass, i.e. all anchovy of lengths ≥2cm (equation A.7), rather than only 1+ biomass.

v) A trawl survey selectivity-at-length is used, to reflect the lower selectivity on anchovy <7cm in the trawls used to capture survey length-frequency data. Given the survey design, uniform trawl selectivity is assumed for all lengths ≥7cm.

vi) Instead of assuming all 1+ anchovy to be mature, spawner biomass is calculated from 1+ anchovy after taking a maturity-at-length relationship (Melo, 1990) into account (equation A.8).

vii) Weight-at-length, rather than weight-at-age, is now used, being more appropriate for this revised formulation. In addition, the weight-at-length formula used in the assessment at the time of the November survey, and the monthly-varying weight-at-length formula used to re-adjust the monthly observed commercial catch length-frequency to a length-frequency consistent with the observed tonnage landed, are both new relationships (de Moor and Butterworth 2015).

Larger anchovy are generally landed earlier in the year than smaller anchovy, resulting in changes in the proportion-at-length distribution between the quarters of the year. This is primarily due to the targeting of larger anchovy early in the year before recruits become available to the fishery. This is taken into account in the model in a variety of ways. Modelling catch to be taken once a quarter allows account for quarterly changes in the length distribution of the population. This naturally has a greater effect on the fast growing juveniles. Secondly, as some fishing vessels turn their attention to target recruits mid-way through the year, the model allows for a change in fishing selectivity by quarter. This change in selectivity reflects a change in targeting (e.g. area) rather than a gear effect. One further advantage of modelling catch quarterly is that it allows for changes in the timing of the peak of anchovy catches[1] over the years.

de Moor and Butterworth (2015) estimated new weight-at-length relationships for anchovy based separately on survey and commercial data. Although de Moor and Butterworth (2015) found that these relationships could change from year-to-year, this assessment does not allow for such changes. This is because assumptions would need to be made regarding the relationship applied in past and future years for which no data exist to calculate the associated annual weight-at-length relationships. Such assumptions are premature while research continues to attempt to find environmental co-variates which explain these changes. In addition, the annually-varying relationships were shown to not differ to biologically meaningful extents from the time-invariant relationships (de Moor and Butterworth 2015). Thus, in the meantime, a time-invariant relationship is used in this assessment, although different relationships are applied to the anchovy associated with the November survey and to the monthly commercial data.

<u>Stock recruitment relationship</u>

The following alternative stock recruitment relationships have been considered (Table 1):

A_{BH} – Beverton Holt stock-recruitment curve, with uniform priors on steepness and carrying capacity (the base case)

A_{2BH} – two Beverton Holt stock-recruitment curves, with uniform priors on steepness and carrying capacity, one estimated using data from 1984 to 1999 and the other from 2000 to 2010

A_R – Ricker stock-recruitment curve, with uniform priors on steepness and carrying capacity

A_{HS} – hockey stick stock-recruitment curve, with uniform priors on the log of the maximum recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity

A_{2HS} – two hockey stick stock-recruitment curves, with uniform priors on the log of the maximum recruitment and on the ratio of the spawning biomass at the inflection point to carrying capacity, one estimated using data from 1984 to 1999 and the other from 2000 to 2010.

In cases where a second curve is estimated from 2000 to 2012, the variance about the stock recruitment curve over this time period, $\left(\sigma_{r,2000+}^{A}\right)^2$, is estimated separately from that for the earlier time period, $\left(\sigma_{r}^{A}\right)^2$.

<u>Retrospective runs</u>

A_{BH} is run using data from 1984 to 1999, to 2003, to 2006, and to 2011 to compare the base case model estimates to those which would have resulted from data corresponding to the years used as input to the OMs used for testing OMP-02, OMP-04, OMP-08 and OMP-14. Note that the data used in A_{BH} and the retrospective runs do NOT compare directly with those used for the former OMs due to methodological updates over time, corrections to historic time series of data and the replacement of proportion-at-age 1 inputs with length-structured data.

[1] Following inspection of the raw data, de Moor and Butterworth (2012) assumed there was a shift in the timing of the annual pulse of age-0 anchovy catch between 1998 and 1999.

Results

Stock recruitment relationship

Table 2 lists the various contributions to the negative log posterior pdf at the posterior mode for the alternative stock-recruitment relationships considered. AIC_c is used to coarsely[2] compare amongst alternative stock-recruitment relationships, suggesting that the preferred stock-recruitment relationship is the Hockey Stick, with the Beverton Holt and Ricker being hardly distinguishable for second choice. Models with different stock-recruitment relationships before and after the turn of the century were not favoured by AIC_c, even though they result in a better fits to the data. This is due to the additional number of estimable parameters required for these models. Both A_{2HS} and A_{2BH} estimate a higher recruitment for the same spawner biomass after 2000 than before (Figure 2). A_{BH} is thus chosen as the base case operating model to use during the development of the next OMP, with robustness being tested to A_R and A_{HS} (Figures 1 and 2). This curve reflects a more productive resource than was estimated at the joint posterior mode by de Moor and Butterworth (2012).

Base case (A_{BH}) results at posterior mode

The estimated parameter values and key outputs for A_{BH} are listed in Table 3. The fit to the November total biomass is very good (Figure 3). The joint posterior mode estimate of $k_N^A = 0.67$ indicates that the survey estimate of abundance is an over-estimate of total biomass, compared to the under-estimate of 1+ biomass indicated by the previous assessment (de Moor and Butterworth, 2012 had a joint posterior mode of $k_N^A = 1.16$). This is due firstly to the change in the assumption of the November survey being associated with total rather than 1+ biomass, together with the inclusion of a maturity-at-length ogive in the calculation of spawner biomass. de Moor and Butterworth (2012) assumed the time series of abundance estimates from the November hydroacoustic survey and DEPM reflected the same biomass. The model predicted SSB time series is higher than that estimated by de Moor and Butterworth (2012), but still reasonably within the range of DEPM estimates of abundance (Figure 4). There is some slight trend in the residuals from the model fit to the May survey estimates of recruitment (Figure 5). The model projected posterior mode estimates of May recruitment in 2007, 2008 and 2010 fall outside the 95% CIs for the survey results (although within the 95% CI which reflects both the survey inter-transect and additional variance) as a result of the model also being required to fit to November survey estimates of total biomass which generally have smaller CVs.

The model fits the November survey estimates of proportions-at-length obtained from trawl samples well (Figures 7 and 8), allowing for a lower net selectivity on anchovy of small lengths (Figure 6). Initial model testing indicated that some commercial selectivity parameters could be assumed to be the same over quarters (see Table A.1). The model estimated commercial selectivity-at-length curves that reflect near-constant selectivity between 7 and 13cm over November to January, with a steep decrease in selectivity for lengths less than 7cm (Figure 9). The selectivity-at-length

[2] Strictly AICc is for use in comparing between alternative frequentist models; the comparison here is made at the joint posterior mode.

estimated between February and April reflects the combination of the recruits of the year not yet being available to the fishery and the subsequent targeting of larger anchovy (Figure 9). The model estimated selectivity-at-length between May and October reflects the targeting of recruiting anchovy (Figures 9 and 10). In general, the model fits to the commercial proportions-at-length are reasonable (Figures 10 and 11).

The model predicted catch-at-age is shown in Figure 12, indicating the majority of catch (by number) is estimated to be of age 0 and 1, although small amounts of age 2+ anchovy are estimated to have been landed.

Figure 13 shows the model estimated von Bertalanffy growth curve and Figure 14 shows the distributions about this curve, with a greater variability estimated for age 0 compared to older ages (Table 3). It is interesting to note that the growth curve estimated from proportion-at-length data from 1984 to 2014 has a steeper increase and thus greater length-at-ages 1 and 2 compared to that estimated directly from ageing data from the November surveys in 1990, 1992 to 1995 (that ageing was conducted by M. Kerstan, Deon Durholtz pers. comm.).

The historical annual harvest rates are plotted in Figure 15 and the annual losses of anchovy to predation are listed in Table 4 showing catch over the past two decades to be no more than a low fraction (seldom exceeding 5%) of anchovy lost to natural mortality.

Retrospective analysis

There is little difference in the historic November total biomass trajectory and key model parameter estimates for the retrospective runs (Figure 16, Table 5). These results indicate that the more productive stock-recruitment relationship estimated here for A_{BH} compared to that estimated by de Moor and Butterworth (2012), is primarily due to the change in methodology and change from using age- to length-structured data, rather than to the three further years of data.

Discussion

This document has detailed the updated assessment of the South African anchovy resource, including a number of key changes in model formulation and data used to tune the model. The base case hypothesis assumes a Beverton Holt stock recruitment curve and time-invariant natural mortality, and is able to fit the new length-structured data reasonable well. Estimation of catch-at-age within the model results in the majority of catch being estimated to be of ages 0 and 1, in line with previous assumptions about anchovy landings. The total resource biomass in November 2014 is estimated to be substantially above the historical (1984-2013) average of 3.4 million tons, and is now estimated at 4.2 million tons for A_{BH}. Recruitment over the past 20 years reflects three major peaks, although the low points of these fluctuations were still large, being similar to the maximum recruitment observed prior to 2000. The harvest proportion over the past 19 years has only exceeded 0.15 once, in 2012 when the 305 000t of anchovy was landed, but this peak proportion remained below 0.25 (Figure 15).

References

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Table 1. The alternative stock-recruitment relationships considered. The parameters are defined in Appendix A, Table A.1, with, $X=\sum_{a=1}^{3}\overline{w}_a^A e^{-M_j^A-(a-1)\overline{M}_{ad}^A}+\overline{w}_{4^+}^A e^{-M_j^A-3\overline{M}_{ad}^A}\Big/\left(1-e^{-\overline{M}_{ad}^A}\right)$, where $\overline{w}_a^A$ is the average of $w_{y,a}^A=\sum_{l=2}^{16}A_{a,l}^{sur}w_{y,l}^A$, where $A_{a,l}^{sur}$ and $w_{y,l}^A$ are as defined in Appendix A.

Test	Stock recruitment relationship	$f\left(SSB_y^A\right)=$	Parameters
A_{BH}	Beverton Holt	$\frac{\alpha^A SSB_y^A}{\beta^A+SSB_y^A}$	$h^A\sim U(0.2,1)$ $K^A/1000\sim U(0,10)$ $\alpha^A=\frac{4h^A}{5h^A-1}\frac{K^A}{X}$ $\beta^A=\frac{K^A\left(1-h^A\right)}{5h^A-1}$
A_{2BH}	Beverton Holt (2 curves)	$\frac{\alpha_1^A SSB_y^A}{\beta_1^A+SSB_y^A}$ if $y<2000$ $\frac{\alpha_2^A SSB_y^A}{\beta_2^A+SSB_y^A}$ if $y\geq 2000$	$h_t^A\sim U(0.2,1)$ $K_t^A/1000\sim U(0,10)$ $\alpha_t^A=\frac{4h_t^A}{5h_t^A-1}\frac{K_t^A}{X}$ $\beta_t^A=\frac{K_t^A\left(1-h_t^A\right)}{5h_t^A-1}$ $t=1,2$
A_R	Ricker	$\alpha^A SSB_y^A e^{-\beta^A SSB_y^A}$	$h^A\sim U(0.2,1.5)$ $K^A/1000\sim U(0,10)$ $\alpha^A=\frac{1}{X}\left(\frac{h^A}{0.2}\right)^{1/0.8}$ $\beta^A=\frac{\ln\left(h^A/0.2\right)}{0.8K^A}$
A_{ModR}	Modified Ricker	$\alpha^A SSB_y^A e^{-\beta^A\left(SSB_y^A\right)^{c^A}}$	$h^A\sim U(0.2,1.5)$ $K^A/1000\sim U(0,10)$ $\alpha^A=\frac{1}{X}\left(\frac{h^A}{0.2}\right)^{\frac{1}{1-0.2^c}}$ $\beta^A=\frac{\ln\left(h^A/0.2\right)}{\left(K^A\right)^c\left[1-0.2^c\right]}$ $c^A\sim U(0,1)$
A_{HS}	Hockey stick	$\begin{cases} a^A & \text{if } SSB_y^A\geq b^A \\ a^A\frac{SSB_y^A}{b^A} & \text{if } SSB_y^A<b^A\end{cases}$	$\ln\left(a^A\right)\sim U(0,7.2)$[3] $b^A/K^A\sim U(0,1)$ $K^A=a^A e^{-0.5\sigma_r^A}X$[4]
A_{2HS}	Hockey stick (2 curves)	$\begin{cases} a_1^A & \text{if } SSB_y^A\geq b_1^A \\ a_1^A\frac{SSB_y^A}{b_1^A} & \text{if } SSB_y^A<b_1^A\end{cases}$ if $y<2000$ $\begin{cases} a_2^A & \text{if } SSB_y^A\geq b_2^A \\ a_2^A\frac{SSB_y^A}{b_2^A} & \text{if } SSB_y^A<b_2^A\end{cases}$ if $y\geq 2000$	$\ln\left(a_t^A\right)\sim U(0,7.2)$[3] $b_t^A/K_t^A\sim U(0,1)$ $K_t^A=a_t^A e^{-0.5\sigma_{r,t}^A}X$[4] $t=1,2$

[3] Given the lack of *a priori* information on the scale of a^A, a log-scale was used, with a maximum corresponding to about 10 million tons.
[4] For consistency, *K* relates throughout to corresponding means.

Table 2. The contributions to the negative log posterior pdf at the joint posterior mode, together with the values of various quantities at that mode, for alternative stock recruitment relationships.

	A_{BH}	A_{2BH}	A_R	A_{ModR}	A_{HS}	A_{2HS}
-ln(Posterior)	-610.1	-611.9*	-609.7	-609.8*	-608.8*	-612.2*
$-\ln L^{Nov}$	-14.5	-13.2	-14.4	-14.4	-15.3	-13.4
$-\ln L^{Egg}$	6.6	6.4	6.6	6.6	6.5	6.4
$-\ln L^{rec}$	14.8	14.4	14.7	14.7	15.9	15.0
$-\ln L^{sur\ propl}$	-389.3	-389.7	-389.2	-389.1	-390.0	-390.3
$-\ln L^{com\ propl}$	-264.7	-264.6	-264.7	-264.7	-264.7	-264.7
-ln(Priors)	36.0	33.8	36.2	36.2	37.9	33.9
# parameters	53	55	53	54	53	55
Sample size (i.e. data points)	5267	5267	5267	5267	5267	5267
AIC	-1188	-1183	-1188	-1186	-1189	-1184
AIC_c	-1187	-1182	-1187	-1185	-1188	-1183
h^A	0.49	0.47	0.47	0.47		
K^A	4818	4021	4668	4695	2280	1649
a^A				0.93	650	483
b^A					1001	925
h_2^A		0.63				
K_2^A		5278				3371
a_2^A						859
b_2^A						1126

* Convergence to the mode is not confirmed as a positive definite Hessian was not obtained.

Table 3. Key parameter values estimated at the joint posterior mode together with key model outputs. All parameters are defined in Table A.1. Fixed values are given in **bold**. Numbers are reported in billions and biomass in thousands of tons.

	A_{BH}	A_R	A_{HS}		A_{BH}	A_R	A_{HS}
-ln(Posterior)	-610	-610	-609	$S^{survey}_{l<7cm}$	0.24	0.24	0.24
$-\ln L^{Nov}$	-15	-14	-15	$l50_1$	6.3	6.3	6.3
$-\ln L^{Egg}$	7	7	6	$l50_2$	7.9	7.9	7.9
$-\ln L^{rec}$	15	15	16	$l50_3 = l50_4$	6.6	6.6	6.6
$-\ln L^{sur\ propl}$	-389	-389	-390	ψ_1	-4.8	-4.8	5.1
$-\ln L^{com\ propl}$	-265	-265	-265	$\psi_2 = \psi_3 = \psi_4$	-1.8	-1.8	-1.8
-ln(Prior rec residuals)	31	31	33	$\delta_1 = \delta_2$	-0.38	-0.38	-0.38
-ln(Prior growth parameters)	-4	-4	-4	$\delta_3 = \delta_4$	-0.75	-0.75	-0.75
-ln(Prior selectivity parameters)	-2	-2	-2	L_∞	11.1	11.1	11.1
-ln(Prior initial numbers)	11	11	11	t_0	0.12	0.13	0.13
-ln(Prior M residuals)				κ	2.6	2.6	2.6
$\overline{M}^A_j$	**1.2**	**1.2**	**1.2**	ϑ_0	2.0	2.0	2.0
$\overline{M}^A_{ad}$	**1.2**	**1.2**	**1.2**	ϑ_1	1.2	1.2	1.2
$N^A_{1983,0}$	51	51	53	ϑ_{2+}	0.96	0.95	0.95
$N^A_{1983,1}$	142	142	142	B^A_{2014}	4204	4236	4030
$N^A_{1983,2}$	349	349	349	$\overline{B}^A_{Nov}$ [5]	2009	2014	1978
$N^A_{1983,3}$	105	105	105	η^A_{2013}	-1.17	-0.23	0.14
$N^A_{1983,4^+}$	45	45	45	s^A_{cor}	0.13	0.10	0.23
k^A_N	0.67	0.67	0.69				
k^A_r	0.55	0.55	0.56				
k^A_r / k^A_N	0.82	0.82	0.82				
k^A_g	**1.00**	**1.00**	**1.00**				
$\left(\lambda^A_N\right)^2$	**0.00**	**0.00**	**0.00**				
$\left(\lambda^A_r\right)^2$	0.12	0.12	0.13				
a^A	1299	0.58	650				
b^A	1723	0.0002	1001				
K^A	4818	4668	2280				
h^A	0.49	0.47					
σ^A_r	0.68	0.68	0.72				

[5] This is the average over 1984 to 1999. The past three OMPs were developed using Risk defined as "the probability that adult anchovy biomass falls below 10% of the average adult anchovy biomass between November 1984 and November 1999 at least once during the projection period of 20 years".

Table 4. The annual estimated anchovy loss to predation (in '000t), P_y^A in Appendix C, compared to the annual anchovy catch (in '000t), and the annual total proportion fished, F_y^A in Appendix C. Note that these are calculated under the simplified assumption that catch is taken as a pulse mid-way through the year and thus are approximate to a certain extent.

		A_{BH}			A_{Mad}			A_{Mj}		
Year	Catch	Loss to *M*	Catch / Loss to *M*	Annual proportion fished	Loss to *M*	Catch / Loss to *M*	Annual proportion fished	Loss to *M*	Catch / Loss to *M*	Annual proportion fished
1984	265	6246	0.04	0.07	6773	0.04	0.08	6322	0.04	0.07
1985	280	3046	0.09	0.11	1707	0.16	0.11	5980	0.05	0.09
1986	300	4684	0.06	0.07	3395	0.09	0.08	1835	0.16	0.10
1987	600	4939	0.12	0.13	5082	0.12	0.13	5433	0.11	0.12
1988	570	4115	0.14	0.14	4389	0.13	0.13	5219	0.11	0.12
1989	297	2096	0.14	0.16	2280	0.13	0.15	6142	0.05	0.13
1990	152	1719	0.09	0.15	2267	0.07	0.16	2358	0.06	0.14
1991	151	4254	0.04	0.05	2069	0.07	0.07	941	0.16	0.07
1992	349	4802	0.07	0.12	2282	0.15	0.13	4336	0.08	0.12
1993	236	3153	0.07	0.11	4075	0.06	0.12	5755	0.04	0.09
1994	156	1628	0.10	0.14	1118	0.14	0.15	4084	0.04	0.13
1995	177	1585	0.11	0.18	2306	0.08	0.19	2599	0.07	0.15
1996	42	1116	0.04	0.06	1271	0.03	0.06	2645	0.02	0.06
1997	60	2084	0.03	0.04	1168	0.05	0.04	752	0.08	0.05
1998	108	2804	0.04	0.06	2667	0.04	0.05	4408	0.02	0.05
1999	179	4157	0.04	0.06	4120	0.04	0.06	3398	0.05	0.06
2000	268	8914	0.03	0.04	11128	0.02	0.04	5204	0.05	0.05
2001	285	13229	0.02	0.03	17100	0.02	0.03	9333	0.03	0.03
2002	216	11084	0.02	0.03	16575	0.01	0.03	15104	0.01	0.03
2003	256	8934	0.03	0.05	12661	0.02	0.06	10358	0.02	0.04
2004	192	6260	0.03	0.05	8239	0.02	0.05	10102	0.02	0.04
2005	282	6280	0.04	0.05	2875	0.10	0.06	4191	0.07	0.05
2006	136	4919	0.03	0.05	4821	0.03	0.04	7401	0.02	0.04
2007	251	5812	0.04	0.06	8555	0.03	0.05	7469	0.03	0.05
2008	259	7899	0.03	0.05	11530	0.02	0.05	7291	0.04	0.05
2009	181	8028	0.02	0.04	11097	0.02	0.03	9042	0.02	0.03
2010	220	5882	0.04	0.06	9485	0.02	0.05	11632	0.02	0.05
2011	120	3327	0.04	0.06	4632	0.03	0.06	7615	0.02	0.05
2012	305	5957	0.05	0.07	3656	0.08	0.09	1672	0.18	0.10
2013	77	8139	0.01	0.01	7543	0.01	0.01	7336	0.01	0.01
2014	243	7065	0.03	0.06	5977	0.04	0.06	8811	0.03	0.06

Table 5. Key parameter values estimated at the joint posterior mode for A_{BH} and the retrospective runs assuming a Beverton Holt stock recruitment relationship (with parameters α^A, β^A). A_{2003}, A_{2006} and A_{2011} assume data available up to 2003, 2006 and 2011 only. Comparisons are also shown to the values at the joint posterior mode from former operating models used to develop OMP-04, OMP-08 and OMP-14 – the former two of these were developed using operating models assuming a Hockey Stick stock recruitment relationship (with parameters a^A, b^A). Note that direct comparison between the average 1984-1999 model predicted November biomass, $\overline{B}^A_{Nov}$, from A_{BH} and the retrospective runs to the previous operating models is not possible as the former assumes that the November survey covers total biomass while the latter assumed the November survey covered only 1+ biomass. Numbers are reported in billions and biomass in thousands of tons.

	A_{HS}	A_{2011}	A_{2006}	A_{2003}	Previous operating models		
					OMP-04	OMP-08	OMP-14
$\overline{M}^A_j$	**1.2**	**1.2**	**1.2**	**1.2**	**0.9**	**0.9**	**1.2**
$\overline{M}^A_{ad}$	**1.2**	**1.2**	**1.2**	**1.2**	**0.9**	**0.9**	**1.2**
k^A_N	0.67	0.66	0.66	0.66	1.22	1.23	1.16
k^A_r	0.55	0.56	0.56	0.56	0.93	1.03	0.90
α^A / a^A	1299	1296	1296	1296	228	213	1078
β^A / b^A	1723	1856	1856	1856	461	368	2846
K^A	4818	4651	4651	4651	2492	2925	2705
h^A	0.49	0.47	0.47	0.47	1.00	1.00	0.33
σ^A_r	0.68	0.69	0.69	0.69	0.88	0.86	0.68
$\overline{B}^A_{Nov}$ [6]	2009	2020	2020	2020	1169	1103	1157
s^A_{cor}	0.13	0.19	0.16	0.19	0.47	0.43	0.10

[6] See footnote 7.

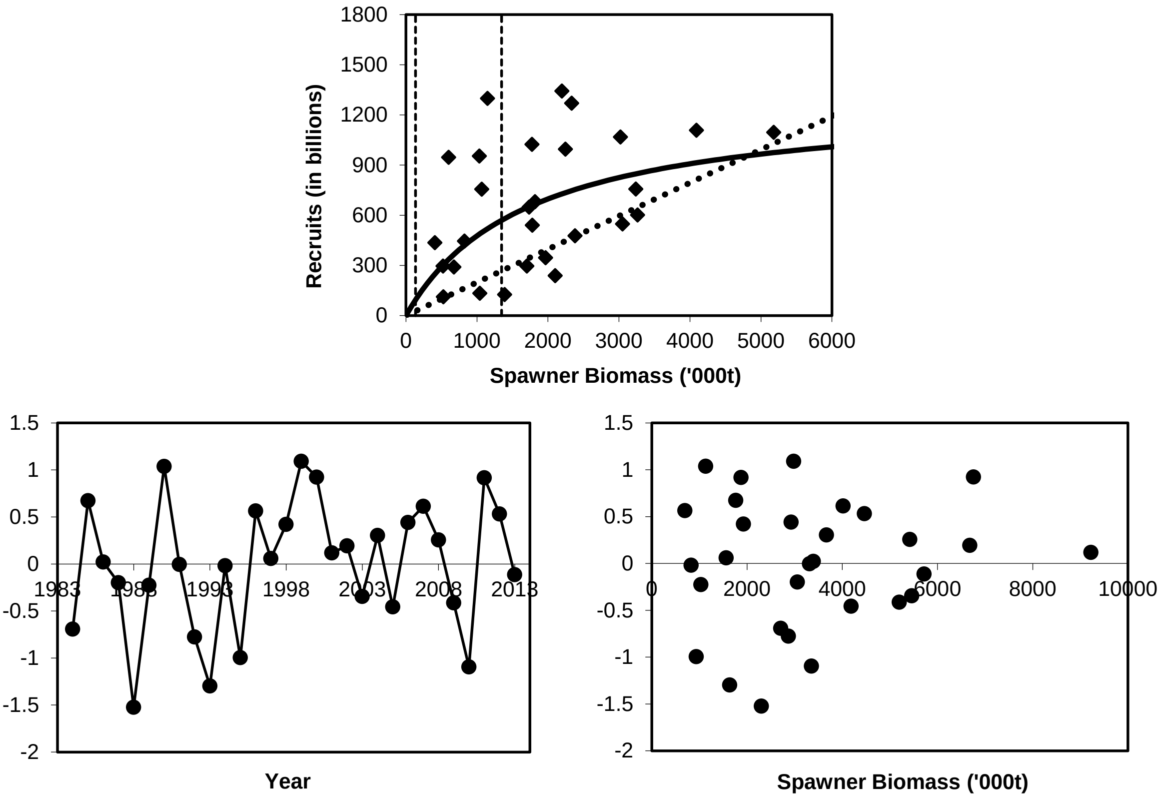

Figure 1. Model predicted anchovy recruitment (in November) plotted against spawner biomass from November 1984 to November 2013 for A_{BH} with the Beverton Holt stock recruitment relationship. The vertical thin dashed lines indicates the average 1984 to 1999 spawner biomass and 10% of that average (used in the definition of risk in OMP tuning). The dotted line indicates the replacement line. The standardised residuals from the fit are given in the lower plots, against year and against spawner biomass.

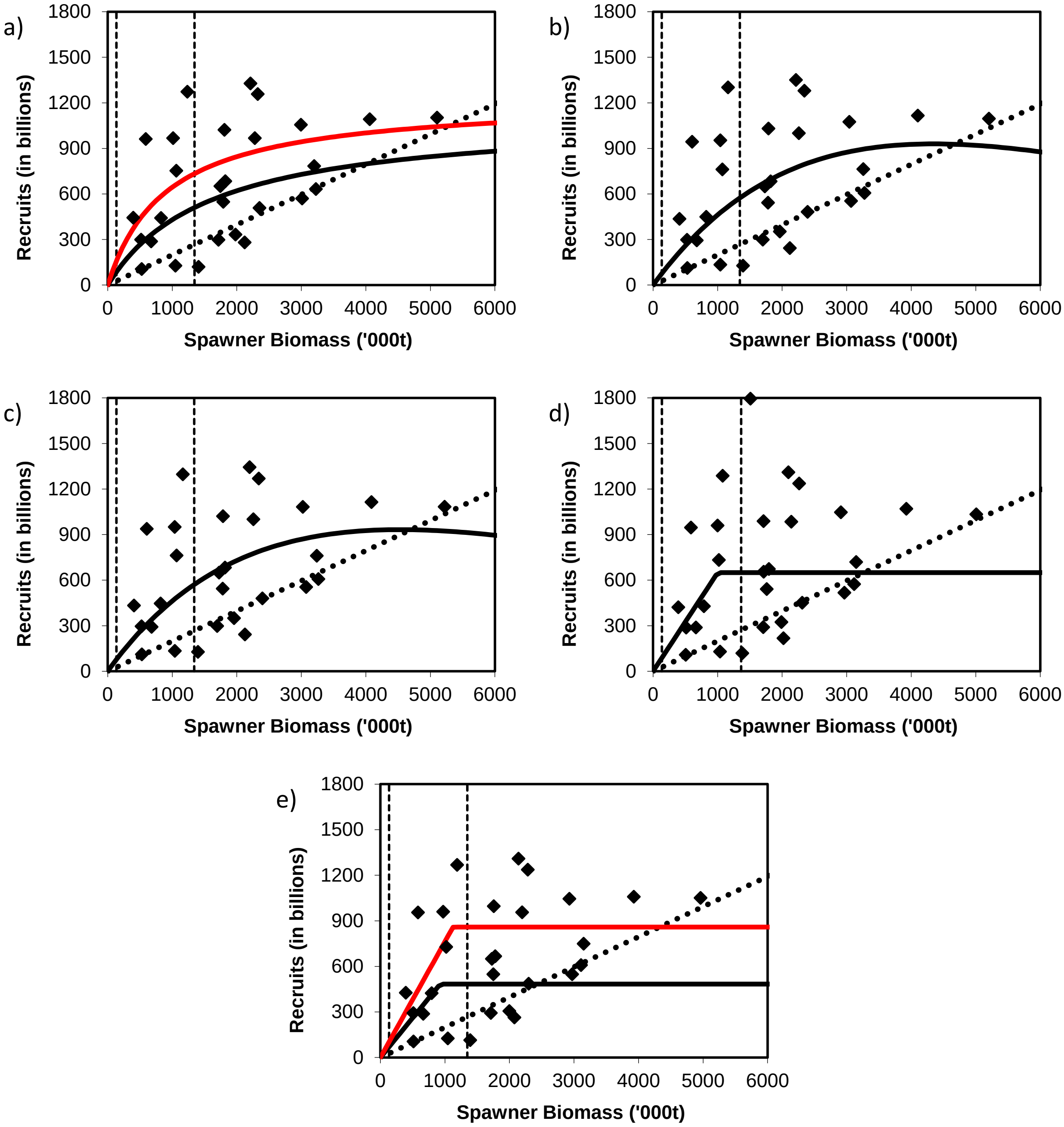

Figure 2. Stock-recruit relationships for a) A_{2BH} (red curve being the 2000+ relationship), b) A_{R}, c) A_{ModR}, d) A_{HS}, and e) A_{2HS} (red curve showing the 2000+ relationship).

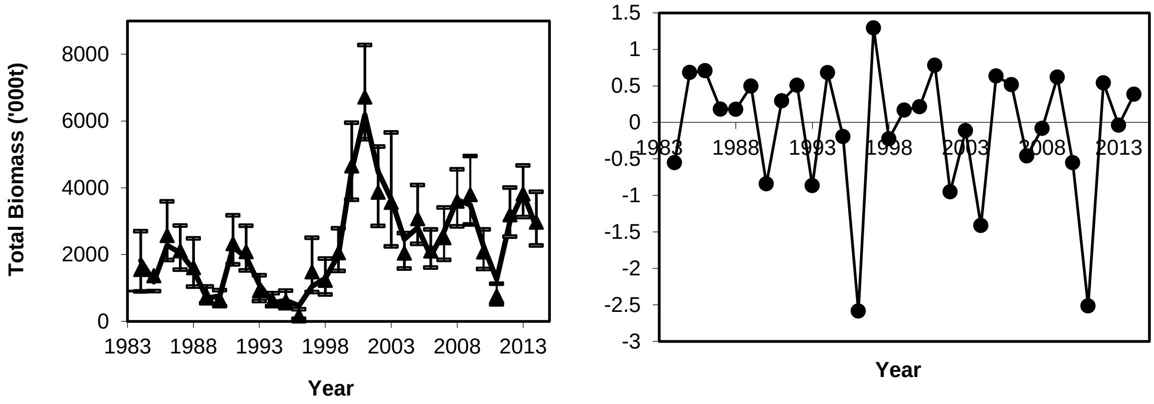

Figure 3. Acoustic survey results and model estimates for November anchovy spawner biomass from 1984 to 2014 for A_{BH}. The survey indices are shown with 95% confidence intervals reflecting survey inter-transect variance. The standardised residuals (i.e. the residual divided by the corresponding standard deviation, including additional variance where appropriate, calculated using equation (A.22)) are given in the right hand plot.

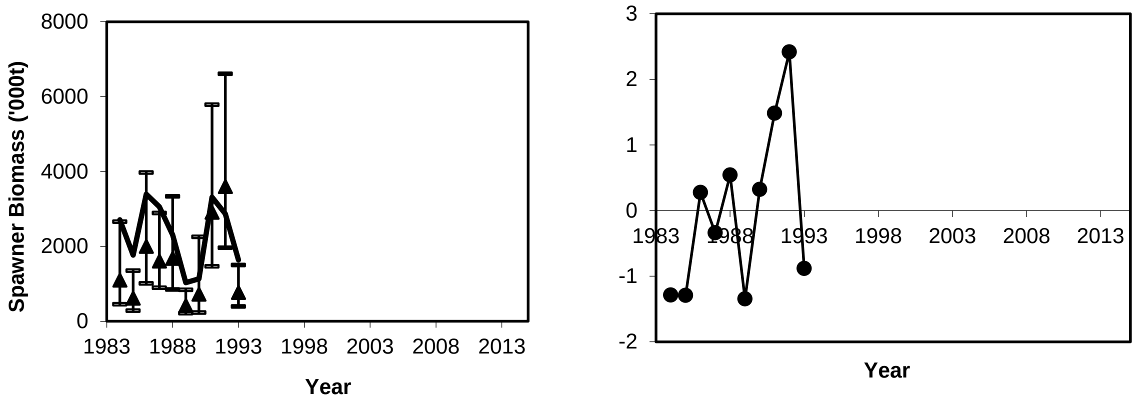

Figure 4. Egg survey results and model estimates for November anchovy spawner biomass from 1984 to 1993 for A_{BH}. The survey indices are shown with 95% confidence intervals. The standardised residuals are given in the right hand plot.

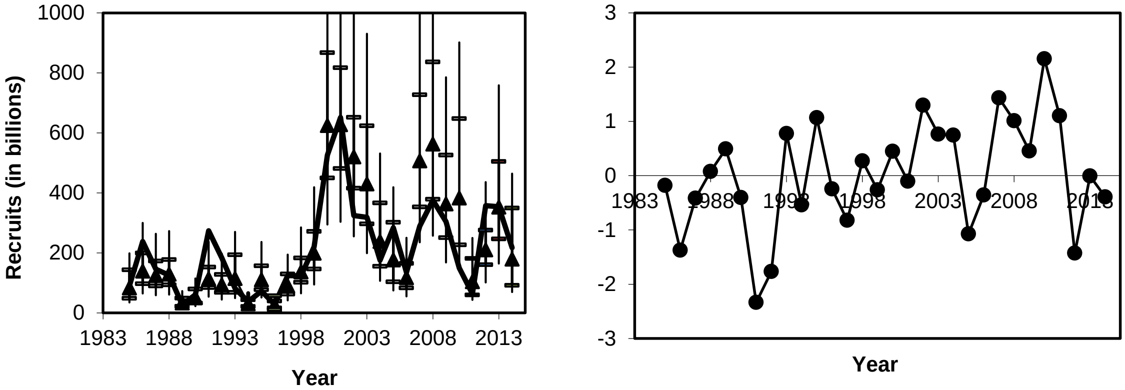

Figure 5. Acoustic survey results and model estimates for anchovy recruitment numbers from May 1985 to May 2014 for A_{BH}. The survey indices are shown with 95% confidence intervals reflecting survey inter-transect and additional variance. The horizontal bars on these vertical lines reflect the 95% confidence intervals from the survey inter-transect variance only. The standardised residuals (i.e. the residual divided by the corresponding standard deviation, including additional variance where appropriate, as specified in equation (A.24)) are given in the right hand plot.

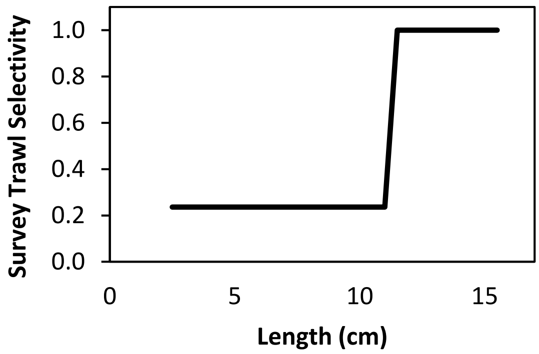

Figure 6. Model estimated trawl survey selectivity at length for A_{BH}.

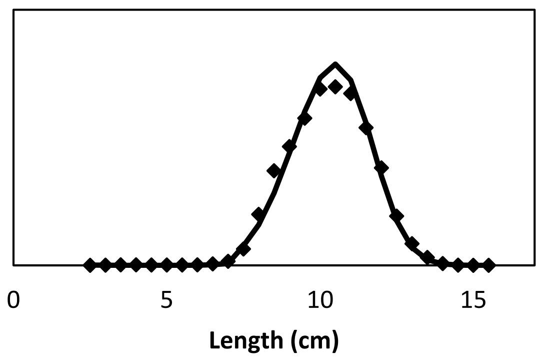

Figure 7. Average (over all years) model predicted and observed proportions-at-length in the November survey trawls for A_{BH}.

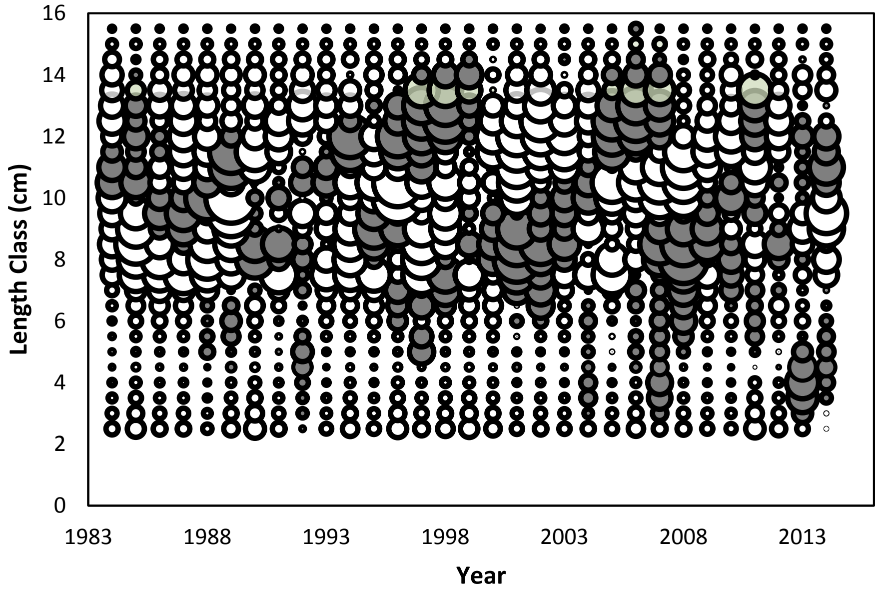

Figure 8. Standardised residuals for proportions-at-length in the November survey trawls for A_{BH}. The size of the bubbles are proportional to the absolute value of the residuals, while the shaded bubbles show positive and the unshaded bubbles show negative residuals.

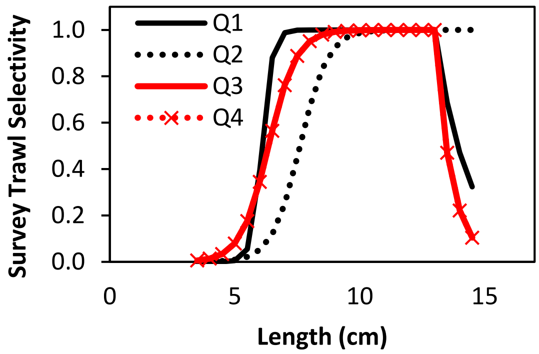

Figure 9. Model estimated quarterly commercial survey selectivity at length for A_{BH}.

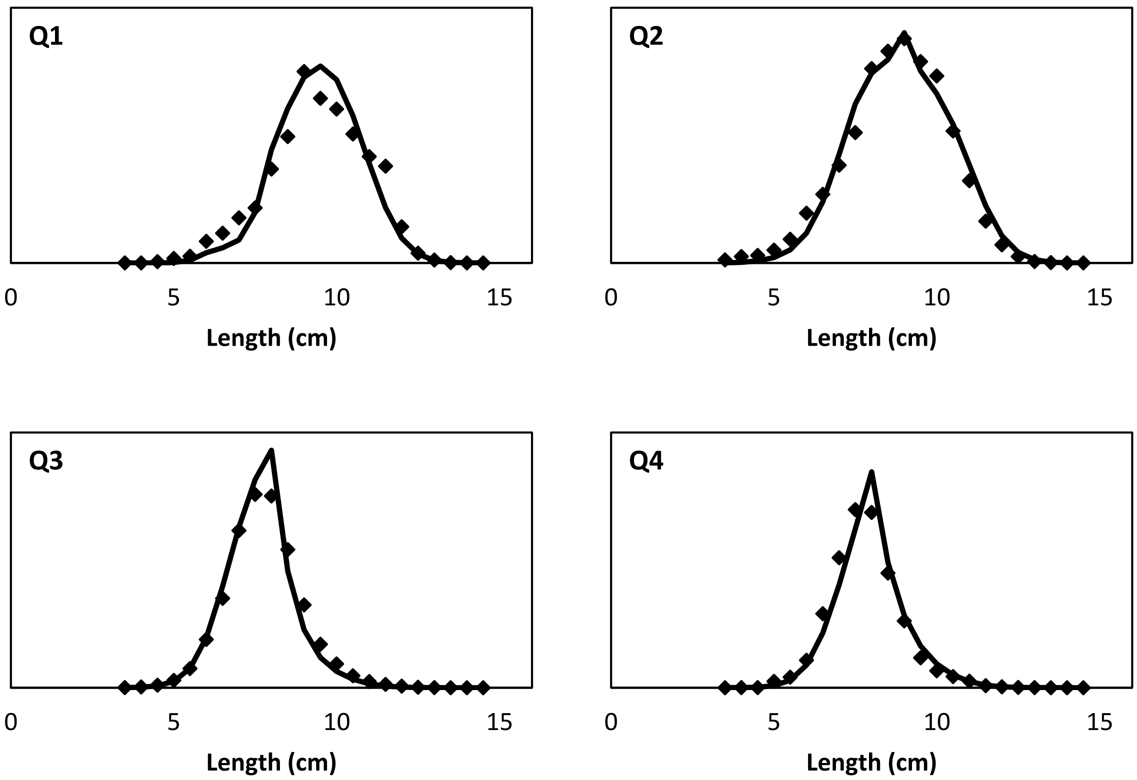

Figure 10. Average (over all years) model predicted and observed proportions-at-length in the quarterly commercial catch for A_{BH}.

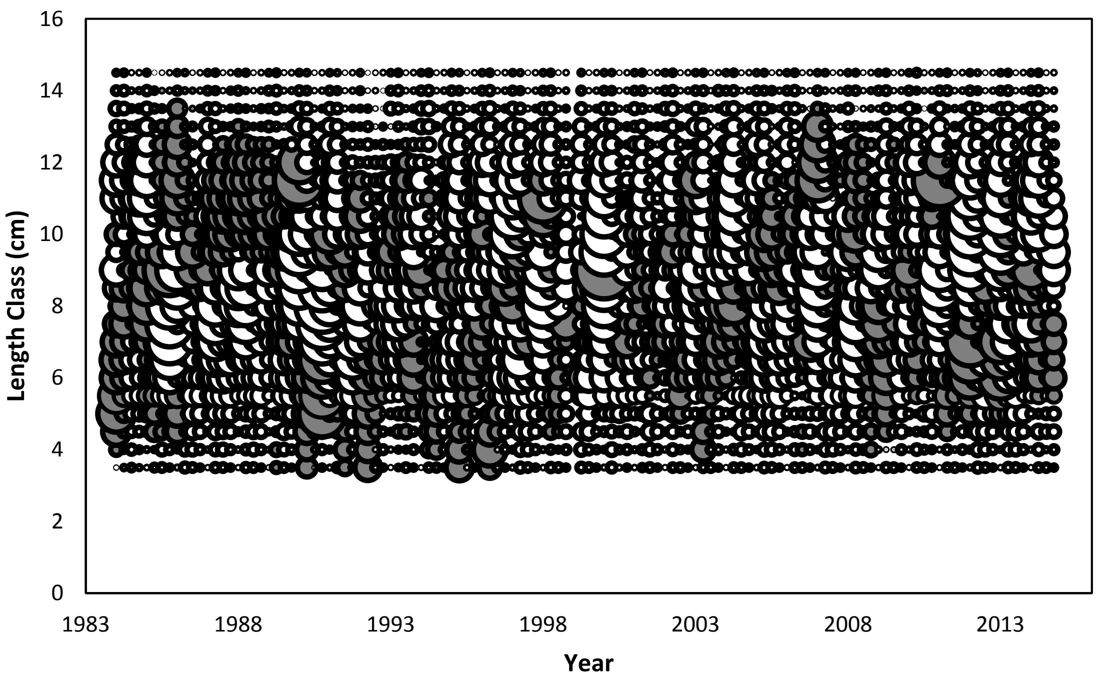

Figure 11. Standardised residuals for proportions-at-length in the quarterly commercial catch for A_{BH}. The size of the bubbles are proportional to the absolute value of the residuals, while the shaded bubbles show positive and the unshaded bubbles show negative residuals.

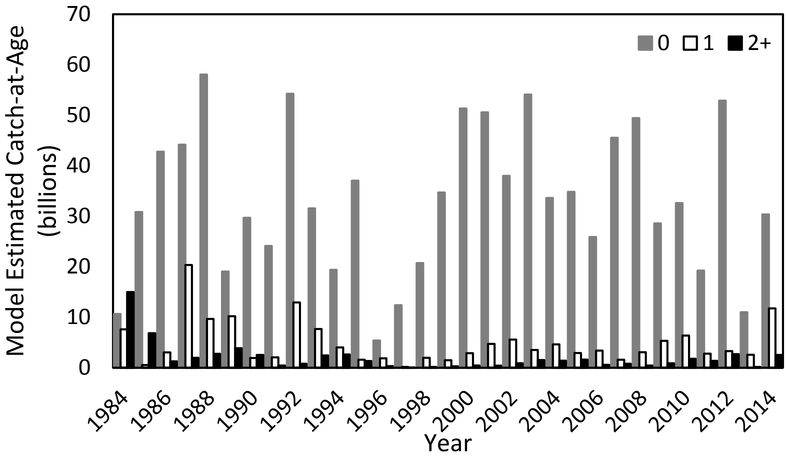

Figure 12. The model estimated quarterly catch-at-age for A_{BH}.

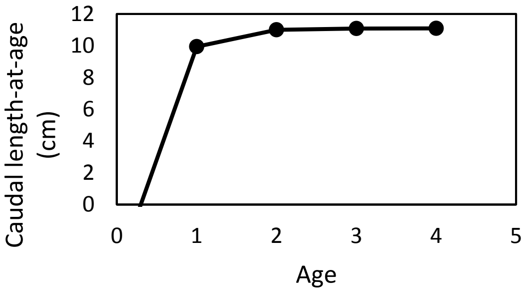

Figure 13. The model estimated von Bertalanffy growth curve, where integer ages are taken to correspond to November each year for A_{BH}.

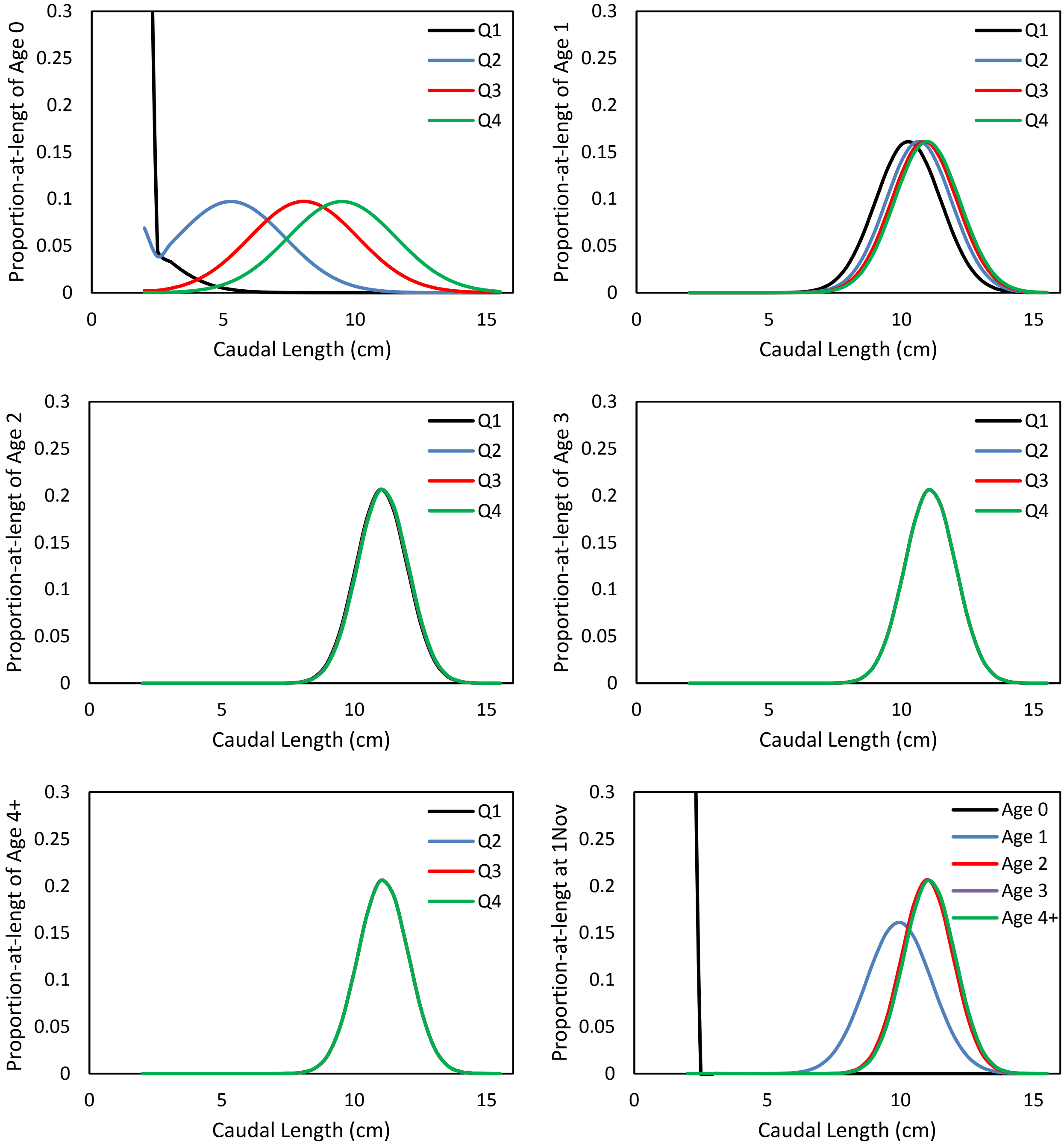

Figure 14. The model estimated distributions of proportions-at-length for each age for A_{BH}, given at the middle of each quarter of the year (corresponding to the times commercial catch is modelled to be taken). The last plot compares the distributions for all ages at 1 November.

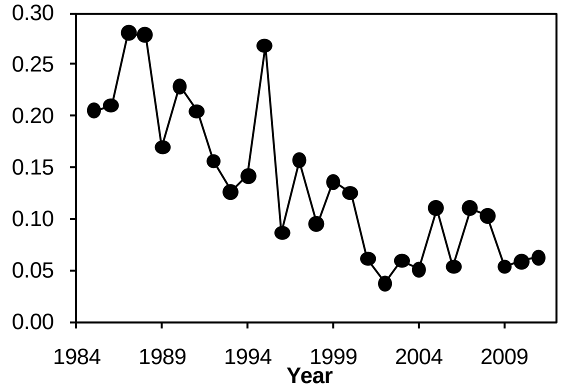

Figure 15. The model estimated historical harvest proportion (catch by mass as a proportion of total biomass) for anchovy for A_{BH}.

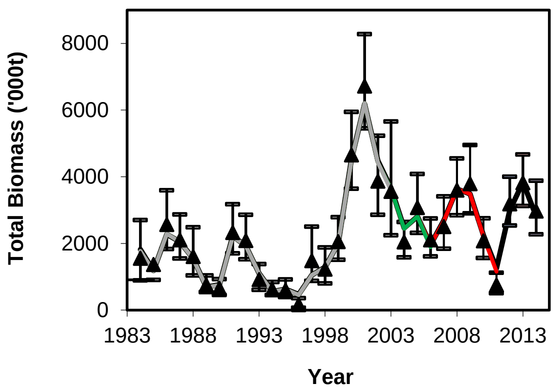

Figure 16. The model predicted November anchovy total biomass for A_{BH} and the retrospective runs A_{2011} using data up to 2011 (red line), A_{2006} using data up to 2006 (green line), and A_{2003} using data up to 2003 (grey line). Note that for earlier years these estimates overlap with only the grey line visible.

Appendix A: Bayesian operating model for the South African anchovy resource

In the below equations a " ^ " is used to represent an estimate of a quantity (e.g. biomass) from a source external to this model (e.g. a survey). Model predicted quantities are represented by terms without any additional super-/sub-scripts other than dependencies on, for example, year, length etc.

Model Assumptions

1) All fish have a birthdate of 1 November.
2) Anchovy mature according to a length-based ogive with an L_{50} of 10.6cm.
3) A plus group of age 4 is used, thus assuming that all population dynamics aspects are the same for age 4 and older.
4) A minus length class of 2cm and a plus length class of 16cm is used.
5) Natural mortality is age-invariant for fish aged 1 and older.
6) Two acoustic surveys are held each year: the first takes place in November and provides an index of abundance of the total stock; the second is in May/June (known as the recruit survey) and provides an index of abundance of juvenile anchovy only.
7) The November and recruit acoustic surveys provide relative indices of abundance of unknown bias.
8) The egg survey observations (derived from data collected during the earlier November surveys) provide estimates of abundance in absolute terms.
9) The survey designs have been such that they result in survey estimates of abundance whose bias is invariant over time.
10) Pulse fishing occurs four times a year, in the middle of each quarter of the assessment year (November to October).

Population Dynamics

The basic dynamic equations for anchovy, based on Pope's approximation (Pope, 1984), are as follows, where $y_1 = 1984$ and $y_n = 2014$.

Numbers-at-age at 1 November

$$N^A_{y,a} = \left(\left(\left(\left(N^A_{y-1,a-1} e^{-M^A_{a-1,y}/8} - C^A_{y,1,a-1}\right) e^{-M^A_{a-1,y}/4} - C^A_{y,2,a-1}\right) e^{-M^A_{a-1,y}/4} - C^A_{y,3,a-1}\right) e^{-M^A_{a-1,y}/4} - C^A_{y,4,a-1}\right) e^{-M^A_{a-1,y}/8} \quad y_1 \le y \le y_n \quad , \quad 1 \le a \le 3$$

$$\begin{aligned} N^A_{y,4+a} = &\left(\left(\left(\left(N^A_{y-1,3} e^{-M^A_{3,y}/8} - C^A_{y,1,3}\right) e^{-M^A_{3,y}/4} - C^A_{y,2,3}\right) e^{-M^A_{3,y}/4} - C^A_{y,3,3}\right) e^{-M^A_{3,y}/4} - C^A_{y,4,3}\right) e^{-M^A_{3,y}/8} \\ &+ \left(\left(\left(\left(N^A_{y-1,4+} e^{-M^A_{4+,y}/8} - C^A_{y,1,4+}\right) e^{-M^A_{4+,y}/4} - C^A_{y,2,4+}\right) e^{-M^A_{4+,y}/4} - C^A_{y,3,4+}\right) e^{-M^A_{4+,y}/4} - C^A_{y,4,4+}\right) e^{-M^A_{4+,y}/8} \\ & y_1 \le y \le y_n \qquad \text{(A.1)} \end{aligned}$$

Numbers-at-length at 1 November

The model estimated numbers-at-length range from a 2cm minus group to a 16cm plus group, denoted 2^- and 16^+, respectively, in the remaining text. The model predicted numbers-at-length at the time of the survey are:

$$N^A_{y,l} = \sum_{a=0}^{4+} A^{sur}_{a,l} N^A_{y,a} \qquad y_1 \le y \le y_n,\ 2^- cm \le l \le 16^+ cm \qquad \text{(A.2)}$$

The model predicted numbers-at-length of ages 1+ only are given by:

$$N^{A,1+}_{y,l} = \sum_{a=1}^{4+} A^{sur}_{a,l} N^A_{y,a} \qquad y_1 \le y \le y_n,\ 2^- cm \le l \le 16^+ cm \qquad \text{(A.3)}$$

The proportion of anchovy of age a that fall in the length group l at 1 November matrix, $A^{sur}_{a,l}$, is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A^{sur}_{a,l} \sim N\left(L_\infty\left(1-e^{-\kappa(a-t_0)}\right), \vartheta^2_a\right)^7 \qquad 0 \le a \le 4^+, 2^- cm \le l \le 16^+ cm \qquad \text{(A.4)}$$

Natural mortality

Natural mortality is modelled to vary annually around a median as follows:

$$M^A_{0,y} = \overline{M}^A_j e^{\varepsilon_{j,y}} \text{ with } \varepsilon^j_{1984} = \eta^j_{1984} \text{ and } \varepsilon^j_y = \rho\varepsilon^j_{y-1} + \sqrt{1-\rho^2}\eta^j_y,\ y > y_1 \qquad \text{(A.5)}$$

$$M^A_{1+,y} = \overline{M}^A_{ad} e^{\varepsilon_{ad,y}} \text{ with } \varepsilon^{ad}_{1984} = \eta^{ad}_{1984} \text{ and } \varepsilon^{ad}_y = \rho\varepsilon^{ad}_{y-1} + \sqrt{1-\rho^2}\eta^{ad}_y,\ y > y_1 \qquad \text{(A.6)}$$

Biomass associated with the November survey

$$B^A_y = \sum_{l=2^-}^{16^+} N^A_{y,l} w^A_{y,l} \qquad y_1 \le y \le y_n \qquad \text{(A.7)}$$

November spawner biomass

Anchovy are assumed to mature from age 1 and thus the spawning stock biomass is:

$$SSB^A_y = \sum_{l=2^-}^{16^+} f_l^A N^{A,1+}_{y,l} w^A_{y,l} \qquad y_1 \le y \le y_n \qquad \text{(A.8)}$$

Commercial selectivity

Commercial selectivity-at-length is assumed to follow the logistic shape, with a dome at high lengths. Commercial selectivity is assumed to vary by quarter, but remain unchanged over time. Selectivity-at-lengths less than the smallest observed length class (3.5cm) and greater than the largest observed length class (14.5cm) are taken to be zero. Thus we have:

[7] The proportion is calculated as the area under the curve between the mid-point of length class *l*-1 and length class *l*. The lower and upper tails are included in the proportions calculated for the minus and plus groups, respectively.

$$S_{y,q,l} = \begin{cases} 0 & if \quad 2^{-}cm \le l \le 3cm \\ 1/1+e^{\psi_q\left(l-l50_q\right)} & if \quad 3.5cm \le l \le S_q^{break} \\ S_{y,q,l-1}e^{\delta_q} & if \quad S_q^{break} < l \le 14.5cm \\ 0 & if \quad 15cm \le l \le 16^{+}cm \end{cases} \qquad y_1 \le y \le y_n,\ 1 \le q \le 4 \qquad \text{(A.9)}$$

Commercial selectivity-at-age is given by:

$$S_{y,q,a} = \sum_{l=2^-}^{16^+} A_{q,a,l}^{com} S_{y,l} \qquad y_1 \le y \le y_n,\ 1 \le q \le 4,\ 0 \le a \le 4^{+} \qquad \text{(A.10)}$$

Commercial catch

Anchovy quarterly pulse catches are split between ages using a model estimated selectivity:

$$C_{y,1,a}^{A} = N_{y-1,a}^{A} e^{-M_{a,y}^{A}/8} S_{y,1,a} F_{y,1}$$

$$C_{y,2,a}^{A} = \left(N_{y-1,a}^{A} e^{-M_{a,y}^{A}/8} - C_{y,1,a}^{A}\right) e^{-M_{a,y}^{A}/4} S_{y,2,a} F_{y,2}$$

$$C_{y,3,a}^{A} = \left(\left(N_{y-1,a}^{A} e^{-M_{a,y}^{A}/8} - C_{y,1,a}^{A}\right) e^{-M_{a,y}^{A}/4} - C_{y,2,a}^{A}\right) e^{-M_{a,y}^{A}/4} S_{y,3,a} F_{y,3}$$

$$C_{y,4,a}^{A} = \left(\left(\left(N_{y-1,a}^{A} e^{-M_{a,y}^{A}/8} - C_{y,1,a}^{A}\right) e^{-M_{a,y}^{A}/4} - C_{y,2,a}^{A}\right) e^{-M_{a,y}^{A}/4} - C_{y,3,a}^{A}\right) e^{-M_{a,y}^{A}/4} S_{y,4,a} F_{y,4}$$

$$y_1 \le y \le y_n,\ 0 \le a \le 4^{+} \qquad \text{(A.11)}$$

In the equations above the difference in the year subscript between the catch-at-age and initial numbers-at-age is because these numbers-at-age pertain to November of the previous year.

The fished proportion of the available biomass from the anchovy fishery is estimated by:

$$F_{y,1} = \frac{\sum_{m=11}^{12}\sum_{l=3.5}^{14.5} C_{y-1,m,l}^{RLF} + \sum_{l=3.5}^{14.5} C_{y,1,l}^{RLF}}{\sum_{a=0}^{4+} N_{y-1,a}^{A} e^{-M_{a,y}^{A}/8} S_{y,1,a}}$$

$$F_{y,2} = \frac{\sum_{m=2}^{4}\sum_{l=3.5}^{14.5} C_{y,m,l}^{RLF}}{\sum_{a=0}^{4+} \left(N_{y-1,a}^{A} e^{-M_{a,y}^{A}/8} - C_{y,1,a}^{A}\right) e^{-M_{a,y}^{A}/4} S_{y,2,a}}$$

$$F_{y,3} = \frac{\sum_{m=5}^{7}\sum_{l=3.5}^{14.5} C_{y,m,l}^{RLF}}{\sum_{a=0}^{4+} \left(\left(N_{y-1,a}^{A} e^{-M_{a,y}^{A}/8} - C_{y,1,a}^{A}\right) e^{-M_{a,y}^{A}/4} - C_{y,2,a}^{A}\right) e^{-M_{y,a}^{A}/4} S_{y,3,a}}$$

$$F_{y,4} = \frac{\sum_{m=8}^{10}\sum_{l=3.5}^{14.5} C_{y,m,l}^{RLF}}{\sum_{a=0}^{4+} \left(\left(\left(N_{y-1,a}^{A} e^{-M_{a,y}^{A}/8} - C_{y,1,a}^{A}\right) e^{-M_{a,y}^{A}/4} - C_{y,2,a}^{A}\right) e^{-M_{y,a}^{A}/4} - C_{y,3,a}^{A}\right) e^{-M_{y,a}^{A}/4} S_{y,4,a}}\,[8] \qquad y_1 \le y \le y_n \qquad \text{(A.12)}$$

A penalty is imposed within the model to ensure that $S_{y,l}F_{y,q} < 0.95$.

[8] The range of length classes used in these summation matches the range of length classes in the observations which is a smaller range than the 2^{-}cm to 16^{+}cm used in the model.

Recruitment

Recruitment at the beginning of November is assumed to fluctuate lognormally about a stock-recruitment curve (see Table 1):

$$N^A_{y,0} = f\left(SSB^A_y\right)e^{\varepsilon^A_y} \qquad y_1 \le y \le y_{n-1} \qquad \text{(A.13)}$$

Number of recruits at the time of the recruit survey

The following equation projects $N^A_{y,0}$ to the start of the recruit survey, taking natural and fishing mortality into account:

$$N^A_{y,r} = \left(\left(\left((N^A_{y-1,0}e^{-M^A_{0,y}/8} - C^A_{y,1,0})e^{-M^A_{0,y}/4}\right) - C^A_{y,2,0}\right)e^{-\left(1/8+0.5\times t^S_y/12\right)M^A_{y,0}} - C^A_{y,0bs}\right)e^{-0.5\times t^A_y x M^A_{0,y}/12} \qquad y_2 \le y \le y_n \qquad \text{(A.14)}$$

The juvenile catch from 1 May to the day before the survey is calculated as follows

$$C^A_{y,0bs} = \left(\left(N^A_{y-1,0}e^{-M^A_{0,y}/8} - C^A_{y,1,0}\right)e^{-M^A_{0,y}/4} - C^A_{y,2,0}\right)e^{-\left(1/8+0.5\times t^A_y/12\right)M^A_{0,y}}S_{y,3,0}F_{y,bs} \qquad y_2 \le y \le y_n \qquad \text{(A.15)}$$

where

$$F_{y,bs} = \frac{\sum_{l=3.5}^{14.5} C^{RLF}_{y,bs,l}}{\sum_{a=0}^{4+}\left(\left(N^A_{y-1,a}e^{-M^A_{a,y}/8} - C^A_{y,1,a}\right)e^{-M^A_{a,y}/4} - C^A_{y,2,a}\right)e^{-\left(1/8+0.5\times t^A_y/12\right)M^A_{y,a}}S_{y,3,a}} \qquad y_2 \le y \le y_n \qquad \text{(A.16)}$$

A penalty is imposed within the model to ensure that $S_{y,l}F_{y,bs} < 0.95$.

Proportion-at-length associated with the November survey

The model predicted proportion-at-length associated with the November survey is[9]:

$$p^A_{y,l} = \frac{N^A_{y,l}S^{survey}_l}{\sum_{l=2.5}^{15.5} N^A_{y,l}S^{survey}_l} \qquad y_1 \le y \le y_n,\ 2.5cm \le l \le 15.5cm \qquad \text{(A.17)}$$

Proportion-at-length associated with the commercial catch

The commercial catch-at-length from the anchovy fishery is:

$$C^A_{y,1,l} = \sum_{a=0}^{4+} N^A_{y-1,a}e^{-M^A_{a,y}/8}A^{com}_{1,a,l}S_{y,l}F_{y,1}$$

$$C^A_{y,2,l} = \sum_{a=0}^{4+}\left(N^A_{y-1,a}e^{-M^A_{a,y}/8} - C^A_{y,1,a}\right)e^{-M^A_{a,y}/4}A^{com}_{2,a,l}S_{y,l}F_{y,2}$$

$$C^A_{y,3,l} = \sum_{a=0}^{4+}\left(\left(N^A_{y-1,a}e^{-M^A_{a,y}/8} - C^A_{y,1,a}\right)e^{-M^A_{a,y}/4} - C^A_{y,2,a}\right)e^{-M^A_{a,y}/4}A^{com}_{3,a,l}S_{y,l}F_{y,3}$$

---

[9] Note the model predicted survey proportion of lengths 2^{-}cm and 16^{+}cm is zero, given a zero survey trawl selectivity in Table A.1. This is consistent with the range of length classes in the observed trawl survey proportions-at-length.

$$C^{A}_{y,4,l}=\sum_{a=0}^{4+}\left(\left(\left(N^{A}_{y-1,a}e^{-M^{A}_{a,y}/8}-C^{A}_{y,1,a}\right)e^{-M^{A}_{a,y}/4}-C^{A}_{y,2,a}\right)e^{-M^{A}_{a,y}/4}-C^{A}_{y,3,a}\right)e^{-M^{A}_{a,y}/4}A^{com}_{4,a,l}S_{y,l}F_{y,4}$$

$$y_1 \le y \le y_n,\ 2^{-}cm \le l \le 16^{+}cm \qquad \text{(A.18)}$$

The model predicted proportion-at-length by quarter in the commercial catch[10] is:

$$p^{coml,A}_{y,q,l}=\frac{C^{A}_{y,q,l}}{\sum_{l=3.5}^{14.5}C^{A}_{y,q,l}} \qquad y_1 \le y \le y_n,\ 1\le q\le 4,\ 3.5cm \le l \le 14.5cm \quad \text{(A.19)}$$

The proportion of anchovy of age a that fall in the length group l in quarter q, $A^{com}_{q,a,l}$, is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$A^{com}_{q,a,l} \sim N\left(L_{\infty}\left(1-e^{-\kappa(a+(2q-1)/8-t_0)}\right),\vartheta_a^{\ 2}\right)^{11} \qquad 1\le q\le 4,\ 0\le a\le 4^{+},\ 2^{-}cm \le l \le 16^{+}cm \quad \text{(A.20)}$$

Fitting the Model to Observed Data (Likelihood)

The survey observations of abundance are assumed to be log-normally distributed. The standard errors of the log-distributions for the survey observations of adult biomass and recruitment numbers are approximated by the CVs of the untransformed distributions and a further additional variance parameter. A “sqrt(p)” formulation, rather than the “adjusted lognormal” (“Punt-Kennedy”, Punt and Kennedy 1997) error distribution formulation, is assumed for the estimated proportions-at-length particularly as it can deal with occasional zero observations more easily. This “sqrt(p)” formulation mimics a multinomial form for the error distribution by forcing near-equivalent variance-mean relationship for the error distributions. The negative log-likelihood function is given by:

$$-\ln L=-\ln L^{Nov}-\ln L^{Egg}-\ln L^{rec}-\ln L^{sur\ propl}-\ln L^{com\ propl} \qquad \text{(A.21)}$$

where

$$-\ln L^{Nov}=\tfrac{1}{2}\sum_{y=y1}^{yn}\left\{\frac{\left(\ln\hat{B}^{A}_{y}-\ln(k^{A}_{N}B^{A}_{y})\right)^2}{(\sigma^{A}_{y,N})^2+(\lambda^{A}_{N})^2}+\ln\left[2\pi\left((\sigma^{A}_{y,N})^2+(\lambda^{A}_{N})^2\right)\right]\right\} \qquad \text{(A.22)}$$

$$-\ln L^{Egg}=\tfrac{1}{2}\sum_{y=y1}^{1993}\left\{\frac{\left(\ln\hat{B}^{A}_{y,egg}-\ln(k^{A}_{g}SSB^{A}_{y})\right)^2}{(\sigma^{A}_{y,egg})^2}+\ln\left[2\pi(\sigma^{A}_{y,egg})^2\right]\right\} \qquad \text{(A.23)}$$

$$-\ln L^{rec}=\tfrac{1}{2}\sum_{y=y1+1}^{yn}\left\{\frac{\left(\ln\hat{N}^{A}_{y,r}-\ln(k^{A}_{r}N^{A}_{y,r})\right)^2}{(\sigma^{A}_{y,r})^2+(\lambda^{A}_{r})^2}+\ln\left[2\pi\left((\sigma^{A}_{y,r})^2+(\lambda^{A}_{r})^2\right)\right]\right\} \qquad \text{(A.24)}$$

$$-\ln L^{sur\ propl}=w^{sur}_{propl}\sum_{y=y1}^{yn}\sum_{l=2.5}^{15.5}\left\{\frac{\left(\sqrt{\hat{p}^{A}_{y,l}}-\sqrt{p^{A}_{y,l}}\right)^2}{2\left(\sigma^{A}_{sur}\right)^2}+\ln\left(\sigma^{A}_{sur}\right)\right\}^{12} \qquad \text{(A.25)}$$

[10] Note there model predicted commercial catch of lengths <3.5cm and >14.5cm is zero, from a zero commercial selectivity in equation (A.9). This is consistent with the range of length classes in the observed commercial proportions-at-length.
[11] The proportion is calculated as the area under the curve between the mid-point of length class *l*-1 and length class *l*. The lower and upper tails are included in the proportions calculated for the minus and plus groups, respectively.
[12] Although strictly there may be bias in the proportions of length-at-age data, no bias is assumed in this assessment. The effect of such a bias is assumed to be small.

$$-\ln L^{com\ propl} = w^{com}_{propl} \sum_{y=y1}^{yn} \sum_{q=1}^{4} \sum_{l=3.5}^{14.5} \left\{ \frac{\left(\sqrt{\hat{p}^{A,coml}_{y,q,l}} - \sqrt{p^{A,coml}_{y,q,l}} \right)^2}{2\left(\sigma^{A}_{com}\right)^2} + \ln\left(\sigma^{A}_{com}\right) \right\} \qquad \text{(A.26)}$$

Table A.1. Assessment model parameters and variables.

	Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
Annual numbers and biomass	$N_{y,a}^{A}$	Model predicted numbers-at-age a at the beginning of November in year y	Billions		A.1	
	$N_{y,l}^{A}$	Model predicted numbers-at-length l at the beginning of November in year y	Billions		A.2	
	$N_{y,l}^{A,1+}$	Model predicted numbers-at-length length l at the beginning of November in year y of anchovy ages 1+ only	Billions		A.3	
	B_{y}^{A}	Model predicted total biomass at the beginning of November in year y, associated with the November survey	Thousand tons		A.7	
	$w_{y,l}^{A}$	Mean mass of anchovy of length l (in cm) sampled during the November survey of year y	Grams	$w_{y,l}^{A} = 0.0077 \times l^{3.09}$		de Moor and Butterworth (2015)
	SSB_{y}^{A}	Model predicted spawning biomass at the beginning of November in year y	Thousand tons		A.8	
	f_{l}^{A}	Proportion of anchovy of length l (in cm) that are mature	-	$f_{l}^{A} = 1/\left(1+e^{-(l-10.61)/0.66}\right)$		Figure A.1
Natural Mortality	M_{a}^{A}	Rate of natural mortality of age a	Year^{-1}		A.5 and A.6	Selected based on maximized joint posterior, and subject to a compelling reason to modify from previous assessment
	$\overline{M}_{j}^{A}$	Median juvenile rate of natural mortality	Year^{-1}	1.2		
	$\overline{M}_{ad}^{A}$	Median rate of natural mortality for 1+ anchovy	Year^{-1}	1.2		
	ε_{y}^{j}	Annual residuals about juvenile natural mortality rate	-		A.5	
	ε_{y}^{ad}	Annual residuals about natural mortality rate for 1+ anchovy	-		A.6	
	η_{y}^{j}	Normally distributed error in calculating ε_{y}^{j}	-	$N\left(0,\sigma_{j}^{2}\right)$		
	η_{y}^{ad}	Normally distributed error in calculating ε_{y}^{ad}	-	$N\left(0,\sigma_{ad}^{2}\right)$		
	σ_{j}	Standard deviation in the annual residuals about juvenile natural mortality	-	0		See robustness tests
	σ_{ad}	Standard deviation in the annual residuals about natural mortality for ages 1+	-	0		See robustness tests
	ρ	Annual autocorrelation coefficient	-	0		See robustness tests

Table A.1 (Continued).

	Parameter/ Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
Recruitment	h^A	Steepness associated with the stock-recruitment curve[13]	-	Table 1		
	K^A	Carrying capacity	Thousand tons	Table 1		
	a^A	Maximum median recruitment in the Hockey Stick stock-recruitment curve	Billions	Table 1		
	b^A	Biomass above which median recruitment is constant and independent of spawning biomass in the Hockey Stock stock-recruitment curve	Thousand tons	Table 1		
	α^A	Stock-recruitment curve parameter, related to h^A and K^A, for Beverton Holt and Ricker curves	-		Table 1	
	β^A	Stock-recruitment curve parameter, related to h^A and K^A, for Beverton Holt and Ricker curves	-		Table 1	
	ε_y^A	Annual lognormal deviation of recruitment	-	$\sim N\left(0,\left(\sigma_r^A\right)^2\right)$, $y_1 \le y \le 1999$ $\sim N\left(0,\left(\sigma_{r,2000+}^A\right)^2\right)$, $2000 \le y \le y_{n-1}$		Reflects the assumption of a different distribution applying pre- and post-2000
	$\left(\sigma_r^A\right)^2$	Variance in the residuals (lognormal deviation) about the stock recruitment curve pre-2000	-	$\sim U(0.16,10)$		Lower bound chosen to restrict the influence of the stock recruitment curve on the assessment results
	$\left(\sigma_{r,2000+}^A\right)^2$	Variance in the residuals (lognormal deviation) about the stock recruitment curve post-2000	-	$\sim U(0.16,10)$		
	$N_{y,r}^A$	Model predicted number of juveniles at the time of the recruit survey in year y	Billions		A.13	
Initial values	$N_{1983,a}^A$	Initial numbers-at-age a	Billions	$N_{1983,0}^A \sim N\left(51,30^2\right)$ $N_{1983,1}^A \sim N\left(143,20^2\right)$ $N_{1983,0}^A \sim N\left(349.6,5^2\right)$	$N_{1983,3}^A = N_{1983,2}^A e^{-M_{2,1983}^A}$ $N_{1983,4+}^A = N_{1983,3}^A \dfrac{e^{-M_{3,1983}^A}}{1-e^{-M_{3,1983}^A}}$	Assumed $M_{ad,1983}^A = M_{ad,1984}^A$

[13] The proportion of the median virgin recruitment that is realised at a spawning biomass level of 20% of average pre-exploitation (virgin) spawning biomass, K^A.

Table A.1 (Continued).

	Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
Multiplicative bias	k_N^A	Multiplicative bias associated with the November acoustic survey	-	$\ln(k_N^A) \sim U(-100, 0.7)$		Uninformative, corresponds to upper bound of $k_N^A \sim 2$
	k_g^A	Multiplicative bias associated with the November egg survey	-	1.0		See robustness tests
	k_r^A	Multiplicative bias associated with the recruit survey	-	$k_r^A / k_N^A \sim U(0,1)$		Recruit survey assumed to cover less of the recruits than the November survey covers of the total biomass
Proportions-at-length and growth curve	$p_{y,l}^A$	Model predicted proportion-at-length l associated with the November survey in year y	-		A.17	
	$A_{a,l}^{sur}$	Proportion of anchovy-at-age a that fall in the length group l in November	-		A.4	
	$p_{y,q,l}^{comA}$	Model predicted proportion-at-length l in the commercial catch during quarter q of year y	-		A.19	
	$A_{q,a,l}^{com}$	Proportion of anchovy-at-age a that fall in the length group l in quarter q	-		A.20	
	L_∞	Maximum length (in expectation) of anchovy	Cm	$\sim N(11.05, 1.105^2)$		See Appendix B
	κ	Annual somatic growth rate of anchovy	Year^{-1}	$\kappa \times L_\infty \sim N(2.915, 0.292^2)$		See Appendix B
	t_0	Age at which the length (in expectation) is zero	Year	$\sim N(0.112, 0.1^2)$		See Appendix B
	ϑ_a	Standard deviation of the distribution about the mean length for age a	-	$\vartheta_0 \sim N(2.0, 0.15^2)$ $\vartheta_1 \sim N(1.2, 0.18^2)$ $\vartheta_{2+} \sim N(1.0, 0.1^2)$		See Appendix B

Table A.1 (Continued).

	Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
Selectivity	S_l^{survey}	November survey trawl selectivity-at-length l	-	0 , $l = 2^- cm, 16^+ cm$ $\sim U(0,1)$, $2.5cm \le l \le 7cm$ 1, $7.5cm \le l \le 15.5cm$		Set to 0 outside the length range observed. Assumed 1 for most length classes due to survey design. Estimated less than 1 for smaller length classes due to net selectivity
	$S_{y,q,l}$	Commercial selectivity-at-length l during quarter q of year y	-		A.9	
	$S_{y,q,a}$	Commercial selectivity-at-age a during quarter q of year y	-		A.10	
	ψ_q	Steepness of ascending limb of logistic part of commercial selectivity curve during quarter q	-	$\sim U(-10,0)$, $\psi_2 = \psi_3 = \psi_4$		Uninformative
	$l50_q$	Length at which ascending limb of logistic part of commercial selectivity is 50% during quarter q	Cm	$\sim U(3,10)$, $l50_3 = l50_4$		Uninformative
	δ_q	Rate of exponential decrease in commercial selectivity at large lengths during quarter q	-	$\delta_1 = \delta_2 \sim N(0.38, 0.5^2)$ $\delta_3 = \delta_4 \sim N(0.75, 0.04^2)$		See Appendix B
	S_q^{break}	Length at which commercial selectivity starts to decrease during quarter q	Cm	13, $q = 1,3,4$ 15, $q = 2$		Informed by initial results

Table A.1 (Continued).

	Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
Catch	$C^{A}_{y,q,a}$	Model predicted number of anchovy of age a caught during quarter q [14] from 1 November $y-1$ to 31 October y	Billions		A.11	
	$F_{y,q}$	Fished proportion in quarter q of year y for a fully selected length class l	-		A.12	
	$C^{A}_{y,0bs}$	Number of juveniles caught between 1 May and the day before the start of the recruit survey in year y	Billions		A.15	
	$F_{y,bs}$	Fished proportion between 1 May and the day before the start of the recruit survey in year y	-		A.16	
Further output	s^{A}_{cor}	Recruitment serial correlation	-		$\dfrac{\sum_{y=y1}^{yn-2}\varepsilon_y\varepsilon_{y+1}}{\sqrt{\left(\sum_{y=y1}^{yn-2}\varepsilon_y^2\right)\left(\sum_{y=y1}^{yn-2}\varepsilon_{y+1}^2\right)}}$	
	η^{A}_{yn-1}	Standardised recruitment residual value for final year	-		$\dfrac{\varepsilon^{A}_{yn-1}}{\sigma^{A}_{r,2000+}}$	

[14] The quarters are $q=1$: November-January; $q=2$: February-April; $q=3$: May-July; $q=4$: August-October.

Table A.1 (Continued).

	Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
	$-\ln L^{Nov}$	Contribution to the negative log likelihood from the model fit to the November total survey biomass data	-		A.22	
	$-\ln L^{Egg}$	Contribution to the negative log likelihood from the model fit to the November egg survey spawner biomass data			A.23	
	$-\ln L^{rec}$	Contribution to the negative log likelihood from the model fit to the recruit survey data	-		A.24	
	$-\ln L^{surpropl}$	Contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data	-		A.25	
	$-\ln L^{compropl}$	Contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data	-		A.26	
Likelihood	$\left(\lambda_N^A\right)^2$	Additional variance, over and above $\left(\sigma_{y,N}^A\right)^2$, associated with the November survey	-	0		See robustness tests
	$\left(\lambda_r^A\right)^2$	Additional variance, over and above $\left(\sigma_{y,r}^A\right)^2$, associated with the recruit survey		$\sim U\left(0,100\right)$		Uninformative
	w_{propl}^{sur}	Weighting applied to the survey proportion-at-length data	-	0.2		To allow for autocorrelation[15]
	σ_{sur}^A	Standard deviation associated with the survey proportion-at-length data	-	$\sqrt{\sum_{y=y1}^{yn}\sum_{l=7}^{13}\left(\sqrt{\hat{p}_{y,l}^A}-\sqrt{p_{y,l}^A}\right)^2 \Big/ \sum_{y=y1}^{yn}\sum_{l=7}^{13}1}$		Closed form solution[16]
	w_{propl}^{com}	Weighting applied to the commercial proportion-at-length data	-	0.05		To allow for autocorrelation[17]
	σ_{com}^A	Standard deviation associated with the commercial proportion-at-length data	-	$\sqrt{\sum_{y=y1}^{yn}\sum_{q=1}^{4}\sum_{l=5}^{12}\left(\sqrt{\hat{p}_{y,q,l}^{A,coml}}-\sqrt{p_{y,q,l}^{A,coml}}\right)^2 \Big/ \sum_{y=y1}^{yn}\sum_{q=1}^{4}\sum_{l=5}^{12}1}\ 8$		Closed form solution[18]

[15] Based upon data being available ~5 times more frequently than annual age data which contain maximum information content on this
[16] A shorter range of lengths is used given the near absence of data outside this range, resulting in small/zero residuals, which would negatively bias this estimate.
[17] Based upon data being available ~4x5 times more frequently than annual age data which contain maximum information content on this
[18] A shorter range of lengths is used given the near absence of data outside this range, resulting in small/zero residuals, which would negatively bias this estimate.

Table A.2. Assessment model data, detailed in de Moor et al. (2015).

Quantity	Description	Units / Scale	Shown in Figure
$C^{RLF}_{y,m,l}$	Observed number of anchovy in length class l caught during month m of year y [19]	Billions	
$C^{RLF}_{y,bs}$	Observed number of anchovy in length class l caught from 1 May to the day before the start of the recruit survey in year y	Billions	
t^{A}_{y}	Time lapsed between 1 May and the start of the recruit survey in year y	Months	
$\hat{B}^{A}_{y}$	Acoustic survey estimate of total biomass from the November survey in year y	Thousand tons	Figure 3
$\sigma^{A}_{y,Nov}$	Survey sampling CV associated with $\hat{B}^{A}_{y}$ that reflects survey inter-transect variance	-	Figure 3
$\hat{B}^{A}_{y,egg}$	Egg survey estimate of spawner biomass from the November survey in year y	Thousand tons	Figure 4
$\sigma^{A}_{y,egg}$	Survey sampling CV associated with $\hat{B}^{A}_{y,egg}$ estimated from inter-transect variance		Figure 4
$\hat{N}^{A}_{y,r}$	Acoustic survey estimate of recruitment from the recruit survey in year y	Billions	Figure 5
$\sigma^{A}_{y,r}$	Survey sampling CV associated with $\hat{N}^{A}_{y,r}$ that reflects survey inter-transect variance	-	Figure 5
$\hat{p}^{A}_{y,l}$	Observed proportion (by number) of anchovy in length group l in the November survey of year y	-	
$\hat{p}^{A,coml}_{y,q,l}$	Observed proportion (by number) of anchovy commercial catch in length group l during quarter q of year y		

[19] This is the observed length-frequency adjusted such that the expected mass calculated using the weight-at-length relationship matches the observed catch in tons. The weight-at-length relationship applied to these commercial data is taken to vary by month, as obtained from fitting an inverted normal distribution for the “a parameter” to monthly commercial data from 1984 to 1996 (de Moor and Butterworth 2015).

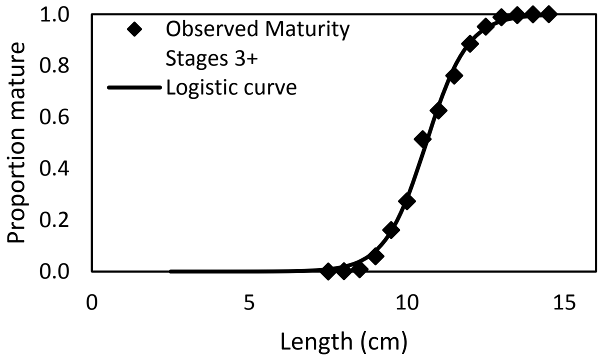

Figure A.1. The logistic curve fitted to stages 3+ proportions of sexually mature male and female anchovy sampled during the November surveys in 1985 and 1986 (Melo 1992). Sexual maturity was assumed for maturity stages 3 and higher (Melo pers. comm.). The four sets of data were combined for each length class into the single observation used in this plot. This was done by weighting each of the four observations of numbers of sexually mature males/females by the total numbers of males/females observed by length class, i.e. $f_l^{A,obs} = \frac{\sum_i mature^i \times total^i}{\sum_i total^i}$, where *i*=1,…,4 denotes each of the four data sets.

Appendix B: "Hardly informative" prior distributions

The model constantly demonstrated some problems attaining convergence to the joint posterior mode (a positive definite Hessian) for some parameters when initially these were given uninformative uniform prior distributions. Initial testing indicated estimation of these parameters was pushing the extremes of data limitation. "Hardly informative" prior distributions were thus used which do no more than simply aid the software to compute a Hessian and thus conduct MCMC simulations.

The process used was, while fixing other growth parameters, to separately develop likelihood profiles over the parameters L_∞, t_0, $\kappa \times L_\infty$ and ϑ_a. Normal prior distributions were then assigned to these parameters with means roughly corresponding to the parameters values giving the minimum objective function value[20]. The standard deviations for these prior distributions were chosen such that the Hessian-based SE resulting from the model fit was less (as much less as possible) than that of the prior distribution.

Normal prior distributions chosen in a similar manner were used for the commercial selectivity parameters, δ_q , and for the initial numbers-at-ages 0, 1 and 2. Alternative formulations for the initial numbers-at-age were also attempted. This included assuming a decreasing equilibrium age structure based purely on natural mortality, or on both natural mortality and an estimated equilibrium fishing mortality. The formulation implemented offered the best fit to the data, which was likely informed by the decrease in survey estimated anchovy total biomass between Novembers 1984 and 1985, while recruitment was observed to increase from May 1984 to 1985.

For the parameters where the Hessian-based SE was close to the standard deviation of the distribution (and convergence to the joint posterior mode was not possible with a larger standard deviation), i.e. ϑ_0, $\delta_3 = \delta_4$, $N^A_{1983,2}$, robustness tests were undertaken for alternative fixed values for these parameters.

[20] With the reservation that estimating these parameters jointly will likely result in a different combination of 'best values' than when the likelihood profiles are estimated with the other parameters fixed.

Appendix C: Calculation of annual total proportion fished and loss to predation of anchovy

The assessment model assumes catch is taken in a four pulses during the year. For simplicity, this catch is totalled and assumed to be taken mid-year when calculating the loss of anchovy to predation. The loss in numbers of age a in year y is calculated by:

$$P_{y,a}^{A,num} = N_{y-1,a}^{A}\left(1-e^{-0.5M_{a,y}^{A}}\right)+\left(N_{y-1,a}^{A}e^{-0.5M_{a,y}^{A}}-C_{y,a}^{A}\right)\left(1-e^{-0.5M_{a,y}^{A}}\right) \qquad 0\le a\le 4^{+},\ y_1\le y\le y_n$$

where $C_{y,a}^{A}=\sum_{q}C_{y,q,a}^{A}$

The loss in biomass of fish of age a to predation in year y is therefore given by:

$$P_{y,a}^{A} = \left[N_{y-1,a}^{A}\left(1-e^{-0.5M_{a,y}^{A}}\right)+\left(N_{y-1,a}^{A}e^{-0.5M_{a,y}^{A}}-C_{y,a}^{A}\right)\left(1-e^{-0.5M_{a,y}^{A}}\right)\right]\frac{1}{2}\left(w_{y-1,a}+w_{y,a+1}\right) \qquad y_1\le y\le y_n,\ 0\le a\le 3$$

[21]

$$P_{y,4^+}^{A} = \left[N_{y-1,4^+}^{A}\left(1-e^{-0.5M_{4+,y}^{A}}\right)+\left(N_{y-1,4^+}^{A}e^{-0.5M_{4+,y}^{A}}-C_{y,4^+}^{A}\right)\left(1-e^{-0.5M_{4+,y}^{A}}\right)\right]\frac{1}{2}\left(w_{y-1,4^+}+w_{y,4^+}\right) \qquad y_1\le y\le y_n$$

The assumption is made that $w_{1983,a}=w_{1984,a}$, $0\le a\le 4^{+}$.

The total loss in anchovy biomass to predation in year y is then given by:

$$P_{y}^{A}=\sum_{a=0}^{4+}P_{y,a}^{A}\ .$$

The anchovy biomass mid-way through the year is given by:

$$B_{a}^{Mid-year}=N_{y-1,a}^{A}e^{-0.5M_{a,y}^{A}}\,\frac{1}{2}\left(w_{y-1,a}^{A}+w_{y,a+1}^{A}\right) \qquad y_1\le y\le y_n,\ 0\le a\le 3$$

$$B_{4^+}^{Mid-year}=N_{y-1,4^+}^{A}e^{-0.5M_{4+,y}^{A}}\,\frac{1}{2}\left(w_{y-1,4^+}^{A}+w_{y,4^+}^{A}\right) \qquad y_1\le y\le y_n$$

The annual total proportion fished (catch/biomass) mortality is thus given by:

$$F_{y}^{A}=\frac{\sum_{a=0}^{4^+}C_{y,a}^{A}\,\frac{1}{2}\left(w_{y-1,a-1}^{A}+w_{y,a}^{A}\right)}{\sum_{a=0}^{4+}B_{a}^{Mid-year}}\ . \qquad y_1\le y\le y_n$$

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[21] The assumption is made that $w_{y,0}^{A}=0$ for all years. Also note that, since a time-invariant length-weight relationship is used for this assessment, in practice these weights-at-ages do not differ by year for this assessment.