# The stock assessment model for South African sardine 

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The stock assessment model for South African sardine is detailed in the Appendix. The following assumptions are made:

1) All infection occurs at 1 November; after all catch and before movement. Thus at the time of the recruit survey, all recruits are assumed to be uninfected.
2) All movement occurs at 1 November; after all catch is removed from the population and after infection by the parasite.
3) Permanent west-to-east movement is allowed for all ages.
4) No east-to-west movement is assumed ${ }^{1}$.
5) Infection only happens to west stock fish (hypothesised region of parasite host)
6) No difference in growth, maturity, natural or fishing mortality or movement is assumed between sardine that are uninfected or infected with the parasite.

Initial results from fitting this model to available data are presented in de Moor and Butterworth (2015b).

## References

de Moor CL, Butterworth DS. 2015a. Assessing the South African sardine resource: two stocks rather than one? African Journal of Marine Science. 37:41-51.
de Moor CL, Butterworth DS. 2015b. Initial results from fitting the revised sardine two-mixing stock model to data from 1984-2014, including consideration of parasite prevalence-by-length sampled from November surveys 2010-2014. MARAM International Fisheries Stock Assessment Workshop MARAM IWS/DEC15/Sardine/P3.
de Moor CL, Coetzee J, Merkle D, van der Westhuizen JJ, van der Lingen C. 2015. A record of the generation of data used in the 2015 sardine and anchovy assessments. Department of Agriculture, Forestry and Fisheries Report No FISHERIES/2015/NOV/SWG-PEL/42. 19pp. Also MARAM IWS/DEC15/Sardine/BG3. van der Lingen CD, Fréon P, Fairweather TP, van der Westhuizen JJ. 2006. Density-dependent changes in reproductive parameters and condition of southern Benguela sardine Sardinops sagax. African Journal of Marine Science 28:625-636.

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## Appendix: Bayesian assessment for the South African sardine resource

The assessment is run from November $y_{1}=1984$ to November $y_{n}=2014$, with quarters $q=1$ denoting November $y-1$ to January $y, q=2$ denoting February to April $y, q=3$ denoting May to July $y$ and $q=4$ denoting August to October $y$. All parameters are defined in Tables A. 1 and A.2.

The subscripts $j=W$ or $j=S$ denote the west and south stocks, respectively, where only the 'west' stock equations are used in the single stock hypothesis. The subscripts $p=N I$ or $p=I$ denote the sardine uninfected and infected with the digenean 'tetracotyle-type' metacercarian endoparasite, respectively.

## Population Dynamics

Numbers-at-age at 1 November before movement or infection

$$
\begin{gathered}
N_{j, p, y, a}^{S^{*}}=\left(\left(\left(\left(\left(N_{j, p, y-1, a-1}^{S} e^{-M_{y, a-1}^{S} / 8}-C_{j, p, y, 1, a-1}^{S}\right) e^{-M_{y, a-1}^{S} / 4}\right)-C_{j, p, y, 2, a-1}^{S}\right) e^{-M_{y, a-1}^{S} / 4}-C_{j, p, y, 3, a-1}^{S}\right) e^{-M_{y, a-1}^{S} / 4}-C_{j, p, y, 4, a-1}^{S}\right) e^{-M_{y, a-1}^{S}} \\
y_{1} \leq y \leq y_{n}, 1 \leq a \leq 4
\end{gathered}
$$

$$
\begin{gather*}
N_{j, p, y, a=5+}^{S *}=\left(\left(\left(\left(\left(N_{j, p, y-1,4}^{S} e^{-M_{y, 4}^{S} / 8}-C_{j, p, y, 1,4}^{S}\right) e^{-M_{y, 4}^{S} / 4}\right)-C_{j, p, y, 2,4}^{S}\right) e^{-M_{y, 4}^{S} / 4}-C_{j, p, y, 3,4}^{S}\right) e^{-M_{y, 4}^{S} / 4}-C_{j, p, y, 4,4}^{S}\right) e^{-M_{y, 4}^{S} / 8} \\
+\left(\left(\left(\left(\left(N_{j, p, y-1,5+}^{S} e^{-M_{5+}^{S} / 8}-C_{j, p, y, 1,5+}^{S}\right) e^{-M_{y, 5+/ 4}^{S}}\right)-C_{j, p, y, 2,5+}^{S}\right) e^{-M_{y, 5+}^{S} / 4}-C_{j, p, y, 3,5+}^{S}\right) e^{-M_{y, 5+}^{S} / 4}-C_{j, p, y, 4,5+}^{S}\right) e^{-M_{y, 5+}^{S} / 8} \\
y_{1}^{S} \leq y \leq y_{n} \tag{A.1}
\end{gather*}
$$

Infection of west stock sardine in the two stock hypothesis; in the single stock hypothesis $I_{y}=0$ as the parasite data have no influence so that they are not included in the likelihood

$$
\begin{array}{ll}
N_{W, N I, y, a}^{S * *}=\left(1-I_{y}\right) N_{W, N I, y, a}^{S^{*}} & y_{1} \leq y \leq y_{n}, 1 \leq a \leq 4 \\
N_{W, I, y, a}^{S^{* *}}=N_{W, I, y, a}^{S+}+I_{y} N_{W, N I, y, a}^{S^{*}} & y_{1} \leq y \leq y_{n}, 1 \leq a \leq 4 \\
N_{S, p, y, a}^{S^{* *}}=N_{S, p, y, a}^{S^{*}} & p=I, N I, y_{1} \leq y \leq y_{n}, 1 \leq a \leq 4
\end{array}
$$

Movement of west stock $(j=W)$ sardine to the south stock $(j=S)$ in the two stock hypothesis; in the single stock hypothesis move ${ }_{y, a}=0$

$$
\begin{array}{ll}
N_{W, p, y, a}^{S}=\left(1-\text { move }_{y, a}\right) N_{W, p, y, a}^{S+^{*}} & y_{1} \leq y \leq y_{n}, 1 \leq a \leq 5^{+} \\
N_{s, p, y, a}^{S}=N_{s, p, y, a}^{S *}+\operatorname{move}_{y, a} N_{W, p, y, a}^{S^{*}} & y_{1} \leq y \leq y_{n}, 1 \leq a \leq 5^{+} \tag{A.3}
\end{array}
$$

Numbers-at-age mid-way through each quarter (for use in catch equations)

$$
\begin{align*}
& N_{j, p, y, 1, a}^{S}=N_{j, p, y-1, a}^{S} e^{-M_{y, a}^{S} / 8} \\
& N_{j, p, y, 2, a}^{S}=\left(N_{j, p, y, 1, a}^{S}-C_{j, p, y, 1, a}^{S}\right) e^{-M_{y, a}^{S} / 4} \\
& N_{j, p, y, 3, a}^{S}=\left(N_{j, p, y, 2, a}^{S}-C_{j, p, y, 2, a}^{S}\right) e^{-M_{y, a}^{S} / 4} \\
& N_{j, p, y, 4, a}^{S}=\left(N_{j, p, y, 3, a}^{S}-C_{j, p, y, 3, a}^{S}\right) e^{-M_{y, a}^{S} / 4} \tag{A.4}
\end{align*}
$$

Numbers-at-length at 1 November (after infection and movement)
The model estimated numbers-at-length range from a 2.5 cm minus group to a 24 cm plus group, denoted $2.5^{-}$and $24^{+}$, respectively, in the remaining text.
$N_{j, p, y, l}^{S}=\sum_{a=0}^{5+} A_{j, a, l}^{\text {sur }} N_{j, p, y, a}^{S} \quad y_{1} \leq y \leq y_{n}, 2.5^{-} \mathrm{cm} \leq l \leq 24^{+} \mathrm{cm}$
The model predicted numbers-at-length of ages 1+ only are given by:
$N_{j, p, y, l}^{S, 1+}=\sum_{a=1}^{5+} A_{j, a, l}^{\text {sur }} N_{j, p, y, a}^{S} \quad y_{1} \leq y \leq y_{n}, 2.5^{-} \mathrm{cm} \leq l \leq 24^{+} \mathrm{cm}$
The proportion of sardine of age $a$ in stock $j$ that fall in length group $l$ at 1 November, $A_{j, a, l}^{\text {sur }}$, is calculated under the assumption that length-at-age is normally distributed about a von Bertalanffy growth curve:

$$
\begin{equation*}
A_{j, a, l}^{s u r} \sim N\left(L_{j, \infty}\left(1-e^{-\kappa_{j}\left(a-t_{0}\right)}\right), \vartheta_{j, a}^{2}\right) \quad 0 \leq a \leq 5^{+}, 2.5^{-} \mathrm{cm} \leq l \leq 24^{+} \mathrm{cm} \tag{A.7}
\end{equation*}
$$

## Natural mortality

Natural mortality is modelled to vary annually in an autocorrelated manner around a median as follows (although the baseline assumes no such correlation - Table A.1):
$M_{y, a=0}^{S}=\bar{M}_{j}^{S} e^{\varepsilon_{y}^{j}}$ with $\varepsilon_{1984}^{j}=\eta_{1984}^{j}$ and $\varepsilon_{y}^{j}=\rho \varepsilon_{y-1}^{j}+\sqrt{1-\rho^{2}} \eta_{y}^{j}, y>y_{1}$
$M_{y, a=1+}^{S}=\bar{M}_{a d}^{S} e^{\varepsilon_{y}^{a d}}$ with $\varepsilon_{1984}^{a d}=\eta_{1984}^{a d}$ and $\varepsilon_{y}^{a d}=\rho \varepsilon_{y-1}^{a d}+\sqrt{1-\rho^{2}} \eta_{y}^{a d}, y>y_{1}$

Spawning biomass and biomass associated with the November survey

$$
\begin{array}{ll}
S S B_{j, y}^{S}=\sum_{p} \sum_{l=2.5^{-}}^{24^{+}} f_{j, y, l}^{S} N_{j, p, y, l}^{S, 1+} w_{j, y, l}^{S} & y_{1} \leq y \leq y_{n} \\
B_{j, y}^{S}=k_{j, N}^{S} \sum_{p} \sum_{l=2.5^{-}}^{24^{+}} N_{j, p, y, l}^{S} w_{j, y, l}^{S} & y_{1} \leq y \leq y_{n} \tag{A.11}
\end{array}
$$

where $w_{j, y, l}^{S}=w_{j, l}^{S} \times \frac{\tilde{w}_{j, y}}{\left(\sum_{p} \sum_{l=2.5^{-}}^{24^{+}} N_{j, p, y, l}^{S} w_{j, l}^{S}\right) /\left(\sum_{p} \sum_{l=2.5^{-}}^{24^{+}} N_{j, p, y, l}^{S}\right)} \quad y_{1} \leq y \leq y_{n}, 2.5^{-} \mathrm{cm} \leq l \leq 24^{+} \mathrm{cm}$

Commercial selectivity

$$
\begin{equation*}
y_{1} \leq y \leq y_{n} \tag{A.13}
\end{equation*}
$$

$$
\begin{equation*}
S_{j, y, q, a}=\sum_{l=3^{-}}^{23.5^{+}} A_{j, q, a, l}^{c o m} S_{j, y, l} \tag{A.14}
\end{equation*}
$$

$$
y_{1} \leq y \leq y_{n}, 1 \leq q \leq 4,0 \leq a \leq 5^{+}
$$

$$
\begin{equation*}
\text { where } A_{j, q, a, l}^{c o m} \sim N\left(L_{j, \infty}\left(1-e^{-\kappa_{j}\left(a+(2 q-1) / 8-t_{0}\right)}\right), \vartheta_{j, a}^{2}\right) \quad 0 \leq a \leq 5^{+}, 2.5^{-} \mathrm{cm} \leq l \leq 24^{+} \mathrm{cm} \tag{A.15}
\end{equation*}
$$

and the 23.5 cm is one length class above the maximum for which observations can be predicted.

Bycatch in the anchovy directed fishery
$C_{j, p, y, q=1, a=0}^{\text {bycatch }}=\frac{N_{j, p, y, q=1, a=0}^{S}}{\sum_{p} N_{j, p, y, 1,0}^{S}} \times\left\{\sum_{m=11}^{12} \sum_{l<l c u t}^{y, m}, ~ C_{j, y-1, m, l}^{R L F, f l e e t=3}+\sum_{l<l c u t_{y, m}} C_{j, y, 1, l}^{R L F, f l e e t=3}\right\}$

$$
C_{j, p, y, q=1, a=1}^{b y c a t c h}=\frac{N_{j, p, y, q=1, a=1}^{S}}{\sum_{p} N_{j, p, y, 1,1}^{S}} \times\left\{\sum_{m=11}^{12} \sum_{l>=l c u t_{y, m}} C_{j, y-1, m, l}^{R L F, \text { fleet }=3}+\sum_{l>=l c u t}^{y, m} 1 C_{j, y, 1, l}^{R L F, f l e e t=3}\right\}
$$

$C_{j, p, y, q=2, a=0}^{\text {bycatch }}=\frac{N_{j, p, y, q=2, a=0}^{S}}{\sum_{p} N_{j, p, y, 2,0}^{S}} \times \sum_{m=2}^{4} \sum_{l<l c u t}^{y, m} 1 C_{j, y, m, l}^{R L F, \text { fleet }=3}$ $C_{j, p, y, q=2, a=1}^{\text {bycatch }}=\frac{N_{j, p, y, q=2, a=1}^{S}}{\sum_{p} N_{j, p, y, 2,1}^{S}} \times \sum_{m=2}^{4} \sum_{l>=l c u t}^{y, m}<1 C_{j, y, m, l}^{R L F, f l e e t=3}$
$C_{j, p, y, q=3, a=0}^{b y c a t c h}=\frac{N_{j, p, y, q=3, a=0}^{S}}{\sum_{p} N_{j, p, y, 3,0}^{S}} \times \sum_{m=5}^{7} \sum_{l<l c u t_{y, m}} C_{j, y, m, l}^{R L F, \text { fleet }=3}$

$$
C_{j, p, y, q=3, a=1}^{\text {bycatch }}=\frac{N_{j, p, y, q=3, a=1}^{S}}{\sum_{p} N_{j, p, y, 3,1}^{S}} \times \sum_{m=5}^{7} \sum_{l>=l c u t}^{y, m} 1 C_{j, y, m, l}^{R L F, f l e e t=3}
$$

$C_{j, p, y, q=4, a=0}^{\text {bycatch }}=\frac{N_{j, p, y, q=4, a=0}^{S}}{\sum_{p} N_{j, p, y, 4,0}^{S}} \times \sum_{m=8}^{10} \sum_{l<l c u t}^{y, m} 1 C_{j, y, m, l}^{R L F, \text { fleet }=3}$

$$
C_{j, p, y, q=4, a=1}^{\text {bycatch }}=\frac{N_{j, p, y, q=4, a=1}^{S}}{\sum_{p} N_{j, p, y, 4,1}^{S}} \times \sum_{m=8}^{10} \sum_{l>=\text { lcut }_{y, m}} C_{j, y, m, l}^{R L F, \text { fleet }=3}
$$

$C_{j, p, y, q, a}^{\text {bycatch }}=0$

$$
\begin{equation*}
y_{1} \leq y \leq y_{n}, 1 \leq q \leq 4,2 \leq a \leq 5^{+} \tag{A.16}
\end{equation*}
$$

Catch in the directed sardine and round herring bycatch fisheries
$C_{j, p, y, q, a}^{\text {dir }}=\left(N_{j, p, y, q, a}^{S}-C_{j, p, y, q, a}^{\text {bycatch }}\right) S_{j, y, q, a} F_{j, y, q}$

$$
\begin{equation*}
y_{1} \leq y \leq y_{n}, 1 \leq q \leq 4,0 \leq a \leq 5^{+} \tag{A.17}
\end{equation*}
$$

Total catch

$$
\begin{equation*}
C_{j, p, y, q, a}^{S}=C_{j, p, y, q, a}^{\text {bycatch }}+C_{j, p, y, q, a}^{d i r} \tag{A.18}
\end{equation*}
$$

$$
y_{1} \leq y \leq y_{n}, 1 \leq q \leq 4,0 \leq a \leq 5^{+}
$$

$81 \quad F_{j, y, q=4}=\frac{\sum_{\text {fleet }=1}^{2} \sum_{m=81 \geq 6 c m}^{10} C_{j, y, m, l}^{R F L, f l e e t}}{\sum_{p} \sum_{a=0}^{5+}\left(N_{j, p, y, 4, a}^{S}-C_{j, y, 4, a}^{\text {bycatch }}\right) S_{j, y, 4, a}}$

$F_{j, y, q=2}=\frac{\sum_{\text {fleet }=1}^{2} \sum_{m=2 l \geq 6 c m}^{4} \sum_{j, y, m, l}^{R F L, \text { fleet }}}{\sum_{p} \sum_{a=0}^{5+}\left(N_{j, p, y, 2, a}^{S}-C_{j, y, 2, a}^{b y c a t c h}\right) S_{j, y, 2, a}}$
$F_{j, y, q=3}=\frac{\sum_{\text {fleet }=1}^{2} \sum_{m=51 \geq 6 c m}^{7} \sum_{j, y, m, l}^{R F L, \text { fleet }}}{\sum_{p} \sum_{a=0}^{5+}\left(N_{j, p, y, 3, a}^{S}-C_{j, y, 3, a}^{b y c a t c h}\right) S_{j, y, 3, a}}$

$$
\begin{equation*}
N_{j, I, y, a=0}^{S}=0 \tag{A.20}
\end{equation*}
$$

90 Carrying Capacity
$91 \quad K_{j}^{S}=a_{j}^{S} e^{-0.5\left(\sigma_{j, r}^{S}\right)^{2}} \sum_{a=1}^{4} \bar{w}_{j, a}^{S} e^{-M_{j}^{S}-(a-1) \bar{M}_{a d}^{S}}+\bar{w}_{j, 5+} e^{-M_{j}^{S}-4 \bar{M}_{a d}^{S}} \frac{1}{1-e^{-\bar{M}_{a d}^{S}}}$
A penalty is imposed within the model to ensure that $S_{j, y, l} F_{j, y, q}<0.95$. Fish $<6 \mathrm{~cm}$ were caught in less than $10 \%$ of the quarters and were thus not used in fitting this model. Commercial selectivity-at-length is fixed to zero for length classes $<6 \mathrm{~cm}$ (equation A.13)

## Recruitment

$$
N_{j, N I, y, a=0}^{S}=\left\{\begin{array}{cc}
a_{j}^{S} e^{\varepsilon_{j, y}^{S}-0.5\left(\sigma_{j, r}^{S}\right)^{2}} & \text { if } S S B_{j, y}^{S} \geq b_{j}^{S} \\
\frac{a_{j}^{S}}{b_{j}^{S}} S S B_{j, y}^{S} e^{\varepsilon_{j, y}^{S}-0.5\left(\sigma_{j, r}^{S}\right)^{2}} & \text { if } S S B_{j, y}^{S}<b_{j}^{S}
\end{array} \quad y_{1} \leq y \leq y_{n}{ }^{2}\right.
$$

$$
\begin{equation*}
K_{\text {peak }}^{S}=a_{j}^{S} e^{-0.5\left(\sigma_{r, p e a k}^{S}\right)^{2}} \sum_{a=1}^{4} \bar{w}_{j, a}^{S} e^{-M_{j}^{S}-(a-1) \bar{M}_{a d}^{S}}+\bar{w}_{j, 5+} e^{-M_{j}^{S}-4 \bar{M}_{a d}^{S}} \frac{1}{1-e^{-\bar{M}_{a d}^{S}}} \tag{A.21}
\end{equation*}
$$

[^1]Number of recruits associated with the recruit survey

$$
\begin{equation*}
N_{j, y, r}^{S}=k_{j, r}^{S}\left(\sum_{p}\left(N_{j, p, y, 2,0}^{S}-C_{j, p, y, 2,0}^{S}\right) e^{-\left(1 / 8+0.5 t_{y}^{S} / 12\right) M_{y, 0}^{S}}-\tilde{C}_{j, y, 0 b s}^{S}\right) e^{-0.5 t_{y}^{S} \times M_{y, 0}^{S} / 12} \quad y_{1} \leq y \leq y_{n} \tag{A.22}
\end{equation*}
$$

Multiplicative survey bias
$k_{j, N}^{S}=k_{a c}^{S}$
$k_{1, r}^{S}=k_{\mathrm{cov}}^{S} \times k_{a c}^{S}$
$k_{2, r}^{S}=k_{\mathrm{cov} S}^{S} \times k_{\mathrm{cov}}^{S} \times k_{a c}^{S}$ (for the two stock hypothesis only)

Survey trawl selectivity

$$
S_{j, y, l}=\left\{\begin{array}{ccc}
0 & l=2.5^{-} \mathrm{cm} &  \tag{A.26}\\
{\left[1+\exp \left\{-\left(l-S_{50}\right) / \delta\right\}\right]^{-1}} & 3 \mathrm{~cm} \leq l \leq 24^{+} \mathrm{cm}
\end{array} \quad y_{1} \leq y \leq y_{n}\right.
$$

Proportion-at-length associated with the November survey

$$
\begin{align*}
& p_{j, y, 6^{-}}^{S}=\frac{\sum_{p} \sum_{l \leq 6 c m} N_{j, p, y, l}^{S} S_{j, l}^{\text {survey }}}{\sum_{p}^{23.5} \sum_{l=3}^{23} N_{j, p, y, l}^{S} S_{j, l}^{\text {survey }}}  \tag{A.27}\\
& y_{1} \leq y \leq y_{n} \\
& p_{j, y, l}^{S}=\frac{\sum_{p} N_{j, p, y, l}^{S} S_{j, l}^{\text {survey }}}{\sum_{p} \sum_{l=3}^{23.5} N_{j, p, y, l}^{S} S_{j, l}^{\text {survey }}}  \tag{A.28}\\
& p_{j, y, 21-23.3}^{S}=\frac{\sum_{p} \sum_{l=21}^{23.5} N_{j, p, y, l}^{S} S_{j, l}^{\text {survey }}}{\sum_{p} \sum_{l=3}^{23.5} N_{j, p, y, l}^{S} S_{j, l}^{\text {survey }}}  \tag{A.29}\\
& y_{1} \leq y \leq y_{n}, 6.5 \mathrm{~cm} \leq l \leq 20.5 \mathrm{~cm} \\
& p_{j, y, 24^{+}}^{s}=\frac{\sum_{p} N_{j, p, y, 24^{+}}^{s} S_{j, 24^{+}}^{\text {survey }}}{\sum_{p} N_{j, p, y, 24^{+}}^{S} S_{j, 24^{+}}^{\text {surve }}} 3  \tag{A.30}\\
& y_{1} \leq y \leq y_{n}
\end{align*}
$$

Proportion-at-length of fish infected with the parasite in November

$$
\begin{equation*}
P_{j, y, l}^{S}=\frac{N_{j, l, y, l}^{S}}{\sum_{p} N_{j, p, y, l}^{S}} \tag{A.31}
\end{equation*}
$$

$$
y_{1} \leq y \leq y_{n}, 5 \mathrm{~cm} \leq l \leq 22.5 \mathrm{~cm}
$$

[^2]Initial numbers-at-age

$$
\begin{array}{ll}
N_{1, N, 1983, a}^{s}=N_{1, N N, 1983, a-1}^{s} e^{- \text {Finit }_{1}-M_{a}^{s}} & 3 \leq a \leq 4 \\
N_{2, N, 1983, a}^{s}=N_{2, N, 1983, a-1}^{s} e^{- \text {Finit }_{2}-M_{a}^{s}} & 1 \leq a \leq 4 \\
N_{j, N, 1983, a=5+}^{s}=N_{j, N 1,1983,4}^{s} \frac{e^{- \text {Finit }_{j}-M_{5+}^{s}}}{1-e^{- \text {Finit }_{j}-M_{5+}^{s}}} & \\
N_{j,, 1,198, a}^{s}=0 & 0 \leq a \leq 5^{+}
\end{array}
$$

Fitting the Model to Observed Data (Likelihood)

$$
\begin{equation*}
-\ln L=-\ln L^{\text {Nov }}-\ln L^{\text {rec }}-\ln L^{\text {sur propl }}-\ln L^{\text {com propl }}-\ln L^{\text {prev }} \tag{A.35}
\end{equation*}
$$

where
$-\ln L^{\text {Nov }}=\frac{1}{2} \sum_{j} \sum_{y=y 1}^{y n}\left\{\left\{\frac{5^{5}\left(\frac{\left|\ln \left(\hat{B}_{j, y}^{S}\right)-\ln \left(B_{j, y}^{S}\right)\right|}{\sqrt{\left(\sigma_{j, y, N o v}^{S}\right)^{2}+\left(\phi_{a c}^{S}\right)^{2}+\left(\lambda_{j, N}^{S}\right)^{2}}}\right)^{5}}{\left.\left.\left(\frac{\left|\ln \left(\hat{B}_{j, y}^{S}\right)-\ln \left(B_{j, y}^{S}\right)\right|}{5^{5}+\left(\frac{\sqrt{\left(\sigma_{j, y, N o v}^{S}\right)^{2}+\left(\phi_{a c}^{S}\right)^{2}+\left(\lambda_{j, N}^{S}\right)^{2}}}{5}\right.}\right\}^{2 / 5}\right\}^{5}+\ln \left[2 \pi\left(\left(\sigma_{j, y, N o v}^{S}\right)^{2}+\left(\phi_{a c}^{S}\right)^{2}+\left(\lambda_{j, N}^{S}\right)^{2}\right)\right]\right\}}\right\}\right.$
$-\ln L^{r e c}=\frac{1}{2} \sum_{j} \sum_{y=y 1+1}^{y n}\left\{\left\{\frac{5^{5}\left(\frac{\left|\ln \left(\hat{N}_{j, y, r}^{S}\right)-\ln \left(N_{j, y, r}^{S}\right)\right|}{\sqrt{\left(\sigma_{j, y, r e c}^{S}\right)^{2}+\left(\phi_{a c}^{S}\right)^{2}+\left(\lambda_{j, r}^{S}\right)^{2}}}\right)^{5}}{5^{5}+\left(\frac{\left|\ln \left(\hat{N}_{j, y, r}^{S}\right)-\ln \left(N_{j, y, r}^{S}\right)\right|}{\sqrt{\left(\sigma_{j, y, r e c}^{S}\right)^{2}+\left(\phi_{a c}^{S}\right)^{2}+\left(\lambda_{j, r}^{S}\right)^{2}}}\right)^{5}}\right\}^{2 / 5}+\ln \left[2 \pi\left(\left(\sigma_{j, y, r e c}^{S}\right)^{2}+\left(\phi_{a c}^{S}\right)^{2}+\left(\lambda_{j, r}^{S}\right)^{2}\right)\right]\right\}$
$133-\ln L^{\text {sur propl }}=w_{\text {propl }}^{\text {sur }} \sum_{y=y 1}^{y n} \sum_{l=6}^{21}\left\{\frac{\left(\sqrt{\hat{p}_{j, y, l}^{S}}-\sqrt{p_{j, y, l}^{S}}\right)^{2}}{2\left(\sigma_{j, \text { ur }}^{S}\right)^{2}}+\ln \left(\sigma_{j, \text { sur }}^{S}\right)\right\}$
$-\ln L^{\text {com propl }}=W_{\text {propl }}^{\text {com }} \sum_{y=y 1}^{y n} \sum_{q=1}^{4} \sum_{l=5.5}^{21}\left\{\frac{\left(\sqrt{\left.\hat{p}_{j, y, q, l}^{S, \text { coml }}-\sqrt{p_{j, y, q, l}^{S, \text { coml }}}\right)^{2}}\right.}{2\left(\sigma_{j, \text { com }}^{S}\right)^{2}}+\ln \left(\sigma_{j, \text { com }}^{S}\right)\right\}$
$-\ln L^{\text {prev }}=\sum_{j} \sum_{y=2010}^{2014} \sum_{l=5 c m}^{23 c m}\left\{-\hat{N}_{j, y, l}^{\text {prev }} \ln \left(P_{j, y, l}^{S}\right)-\left(n_{j, y, l}^{\text {prev }}-\hat{N}_{j, y, l}^{S}\right) \ln \left(1-P_{j, y, l}^{S}\right)\right\}$
A "robustified likelihood" is used for the contributions from the hydro-acoustic surveys to ensure no undue influence from any extreme (outlying) values for residuals. The functional form chosen to robustify makes negligible difference for standardised residuals of magnitude three or less, but essentially treats large standardised residuals as if they do not exceed five in magnitude.

Table A.1. Assessment model parameters and variables. As the majority of prior distributions are uninformative, notes are provided only for informative priors and/or bounds.


## Table A. 1 (Continued).

| Parameter / <br> Variable |  | Description | Units / Scale | Fixed Value / Prior Distribution | Equation | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w_{j, l}$ | Mean mass of sardine of stock $j$ in length class $l$ | Grams | $1.1639 \times 10^{-5} \times l^{3.03155}$ |  |  |
|  | $w_{j, y, l}^{s}$ | Mean mass of sardine of stock $j$ in length class $l$ at the beginning of November in year $y$ | Grams | A. 12 |  |  |
|  | $\widetilde{w}_{j, y}$ | Mean mass of sardine sampled from stock $j$ during the November survey of year $y$ | Grams | $\sum_{l=3}^{23.5} N_{j, y, l}^{s} \times \frac{\hat{B}_{j, y}^{s}}{\sum_{l=3}^{23.5} N_{j, y, l}^{s} w_{j, l}^{s}}$ |  |  |
|  | $\bar{w}_{j, a}^{S}$ | Mean mass of age $a$ from stock $j$ sampled during each November survey, averaged over all years | Grams | $\sum_{l=2.5^{-}}^{24^{+}} A_{j, a, l}^{s u r} W_{j, l}^{s}$ |  |  |
|  | $M_{y, a}^{S}$ | Rate of natural mortality of age $a$ in year $y$ | Year ${ }^{-1}$ |  | $\begin{gathered} \text { A. } 8 \text { and } \\ \text { A. } 9 \end{gathered}$ | Selected based on maximized joint posterior, and subject to a |
|  | $\bar{M}_{j}^{s}$ | Median juvenile rate of natural mortality | Year ${ }^{-1}$ | 1.0 |  | compelling reason to modify from |
|  | $\bar{M}_{a d}^{s}$ | Median rate of natural mortality for 1+ sardine | Year ${ }^{-1}$ | 0.8 |  | previous assessment |
|  | $\varepsilon_{y}^{j}$ | Annual residuals about juvenile natural mortality rate | - |  | A. 8 |  |
|  | $\varepsilon_{y}^{a d}$ | Annual residuals about natural mortality rate for 1+ sardine | - |  | A. 9 |  |
|  | $\eta_{y}^{j}$ | Normally distributed error in calculating $\varepsilon_{y}^{j}$ | - | $N\left(0, \sigma_{j}^{2}\right)$ |  |  |
|  | $\eta_{y}^{a d}$ | Normally distributed error in calculating $\varepsilon_{y}^{\text {ad }}$ | - | $N\left(0, \sigma_{a d}^{2}\right)$ |  |  |
|  | $\sigma_{j}$ | Standard deviation in the annual residuals about juvenile natural mortality | - | 0 |  | See robustness tests |
|  | $\sigma_{a d}$ | Standard deviation in the annual residuals about natural mortality for ages 1+ | - | 0 |  | See robustness tests |
|  | $\rho$ | Annual autocorrelation coefficient | - | 0 |  | See robustness tests |

Table A. 1 (Continued).

|  | arameter / <br> Variable | Description | Units / Scale | Fixed Value / Prior Distribution | Equation | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $N_{j, p, y, l}^{S}$ | Model predicted numbers-at-length $l$ at the beginning of November in year $y$ of stock $j$ that are uninfected ( $p=0$ ) or infected ( $p=1$ ) with the endoparasite | Billions |  | A. 5 |  |
|  | $p_{j, y, l}^{s}$ | Model predicted proportion-at-length $l$ of stock $j$ associated with the November survey in year $y$ | - |  | A. $27-\mathrm{A} .30$ |  |
|  | $A_{j, a, l}^{\text {sur }}$ | Proportion of age $a$ of stock $j$ that falls in the length group l in November | - |  | A. 7 |  |
|  | $L_{\text {j, }}$ | Maximum length (in expectation) of stock $j$ | Cm | $\sim U(10,30)$ |  | $\kappa_{j} \times L_{j, \infty}$ assumed same for both stocks. Bounds informed by data |
|  | $\kappa_{j}$ | Somatic growth rate parameter for stock $j$ | Year ${ }^{-1}$ | $\kappa_{j} \times L_{j, \infty} \sim U(0,10)$ |  |  |
|  | $t_{0}$ | Age at which the length (in expectation) is zero | Year | $\sim U(-4,4)$ |  |  |
|  | $\vartheta_{j, a}$ | Standard deviation of the distribution about the mean length for age $a$ of stock $j$ | - | $\sim U(0.01,3), a=0,1,2+$ |  | Assumed same for both stocks. Upper bound chosen to preclude unrealistically large lengths for very young fish |
|  | $p_{j, y, q, l}^{\text {com }}$ | Model predicted proportion-at-length $l$ of stock $j$ in the directed catch and round herring bycatch during quarter $q$ of year $y$ | - |  | A. 33 |  |
|  | $A_{j, q, a, l}^{\text {com }}$ | Proportion of age $a$ of stock $j$ that falls in the length group $l$ mid-way through quarter $q$ | - |  | A. 15 |  |
|  | $P_{j, y, l}^{S}$ | Model predicted proportion-at-length $l$ of stock $j$ that are infected with the endoparasite, at the time of the November survey in year $y$ |  |  | A. 31 |  |

Table A. 1 (Continued).

|  | Parameter / Variable | Description | Units / Scale | Fixed Value / Prior Distribution | Equation | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| N <br> $\stackrel{y}{4}$ <br> $\frac{0}{0}$ <br>  | $S_{j, l}^{\text {survey }}$ | Survey selectivity-at-length $l$ in the November survey for stock $j$ | - |  | A. 26 | Some smaller fish escape through the trawl net |
|  | $S_{50}$ | Length at which survey selectivity is $50 \%$ | Cm | $\sim U(2.5,7)$ |  |  |
|  | $\delta$ | Slope of survey selectivity-at-length ogive when selectivity is $50 \%$ | - | $\sim U(0.1,1)$ |  |  |
|  | $S_{j, y, l}$ | Commercial selectivity-at-length $l$ during year $y$ of stock $j$ | - |  | A. 13 |  |
|  | $S_{j, y, q, a}$ | Commercial selectivity-at-age $a$ during quarter $q$ of year $y$ of stock $j$ | - |  | A. 14 |  |
|  | $\chi_{j}$ | Height of the near-normal curve component for stock $j$ relative to the height of the near-lognormal component | - | $\sim U(0,1)$ |  |  |
|  | $\bar{l}_{1, j}$ | Mean of the near-normal distribution for stock $j$ | Cm | $\sim U(5,15)$ |  | Bounds reflect a |
|  | $\bar{l}_{2, j}$ | Median of the near-lognormal distribution for stock $j$ | Cm | $\bar{l}_{2, j}-\bar{l}_{1, j} \sim U(0,15)$ |  | not wanting to influence the data |
|  | $\left(\sigma_{1}^{\text {sel }}\right)^{2}$ | Variance parameter of the near-normal distribution | Cm | $\sim U(2,7)$ |  | and ensuring that |
|  | $\left(\sigma_{2}^{\text {sel }}\right)^{2}$ | Variance parameter of the near-lognormal distribution | Cm | $\sim U(0,2)$ |  | results remained realistic |


| Parameter / Variable | Description | Units / Scale | Fixed Value / Prior Distribution | Equation | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $C_{j, p, y, q, a}^{S}$ | Model predicted number of age $a$ fish of stock $j$ caught during quarter $q$ of year $y$ that are uninfected ( $p=0$ ) or infected ( $p=1$ ) with the endoparasite | Billions |  | A. 18 |  |
| lcut $_{y, m}$ | Cut off length for recruits in month $m$ of year $y$ | Cm | de Moor et al. 2015a |  | Differ by month and year as informed by the recruit surveys |
| $C^{\text {bycatch }}$ | Number of age $a$ fish of stock $j$ bycaught in the anchovy-directed fishery in quarter $q$ of year $y$ that are uninfected ( $p=0$ ) or infected $(p=1)$ with the endoparasite | Billions |  | A. 16 |  |
| $C_{j, p, y, q, a}^{\text {dir }}$ | Number of age $a$ fish of stock $j$ caught in the sardine-directed and round herring bycatch fisheries in quarter $q$ of year $y$ that are uninfected ( $p=0$ ) or infected ( $p=1$ ) with the endoparasite | Billions |  | A. 17 |  |
| $C_{j, p, y, q, l}^{\text {dir }}$ | Number of length $l$ fish of stock $j$ caught in the sardine-directed and round herring bycatch fisheries in quarter $q$ of year $y$ | Billions |  | A. 32 |  |
| $F_{j, y, q}$ | Fished proportion in quarter $q$ of year $y$ for a fully selected age class $a$ of stock $j$, by the directed and round herring bycatch fisheries | - |  | A. 19 |  |

## Table A. 1 (Continued).

| Parameter/ Variable | Description | Units / Scale | Fixed Value / Prior Distribution | Equation | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{j}^{S}$ | Maximum recruitment of stock $j$ in the hockey stick model | Billions | $\ln \left(a_{j}^{S}\right) \sim U(0,5.6)$ |  | Uninformative on log-scale as scale is not known a priori, with the maximum corresponding to about 10 million tons for $K_{j}^{S}$ |
| $b_{j}^{S}$ | Spawner biomass below which the expectation for recruitment is reduced below the maximum for stock $j$ | Thousand tons | $b_{j}^{S} / K_{j}^{S} \sim U(0,1)$ |  |  |
| $K_{j}^{S}$ | Carrying capacity for stock $j$ | Thousand tons |  | A. 21 |  |
| $\stackrel{ \pm}{\sim} K_{\text {peak }}^{\text {S }}$ | Carrying capacity during "peak" years (single stock hypothesis only) | Thousand tons |  | A. 21 |  |
|  $\varepsilon_{j, y}^{S}$ | Lognormal deviation of recruitment of stock $j$ in year $y$ | - | $\varepsilon_{j, y}^{S} \sim N\left(0,\left(\sigma_{j, r}^{S}\right)^{2}\right)$ <br> Except for $\varepsilon_{1, y}^{S} \sim N\left(0,\left(\sigma_{r, p e a k}^{S}\right)^{2}\right)$ <br> $2000 \leq y \leq 2004$ for single stock hypothesis |  | Reflects the assumption of a different distribution applying over the peak period |
| $\left(\sigma_{j, r}^{S}\right)^{2}$ | Variance in the residuals (lognormal deviation) about the stock recruitment curve of stock $j$ | - | $\sim U(0.16,10)$ |  | Lower bound chosen to restrict the influence of the |
| $\left(\sigma_{r, \text { peak }}^{S}\right)^{2}$ | Variance in the residuals (lognormal deviation) about the stock recruitment curve during "peak" years (single stock hypothesis only) | - | $\sim U(0.16,10)$ |  | stock recruitment curve on the assessment results |
| $N_{j, y, r}^{S}$ | Model predicted number of juveniles of stock $j$ at the time of the recruit survey in year $y$ | Billions |  | A. 22 |  |

Table A. 1 (Continued).

|  | Parameter/ Variable | Description | Units / Scale | Fixed Value / Prior Distribution | Equation | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $k_{j, N}^{S}$ | Multiplicative bias associated with the November survey of stock $j$ | - |  | A. 23 |  |
|  | $k_{j, r}^{S}$ | Multiplicative bias associated with the recruit survey of stock $j$ | - |  | $\begin{gathered} \text { A. } 24- \\ \text { A. } 25 \end{gathered}$ |  |
|  | $k_{a c}^{S}$ | Multiplicative bias associated with the hydro-acoustic survey | - | $\sim N\left(0.714,0.077^{2}\right)$ |  | de Moor and Butterworth (2015a) |
|  | $k_{\mathrm{cov}}^{S}$ | Multiplicative bias associated with the coverage of the recruits by the recruit survey in comparison to the $1+$ biomass by the November survey | - | $\sim U(0.3,1)$ |  | Lower bound selected in discussions with scientists on these surveys and their field experience |
|  | $k_{\text {covS }}^{S}$ | Multiplicative bias associated with the coverage of the south stock recruits by the recruit survey in comparison to the west stock recruits during the same survey | - | $\sim U(0,1)$ |  |  |
|  | $N_{j, 1983, a}^{S}$ | Initial numbers-at-age $a$ in stock $j$ | Billion | $\begin{gathered} N_{j, 1983, a}^{S} \sim U(0,50) \text { for } \\ j=1,0 \leq a \leq 2 \text { and } \\ j=2, a=0 \end{gathered}$ | A. 34 |  |
|  | Finit $_{\text {j }}$ | Rate of fishing mortality assumed in the initial year for stock $j$ |  | $\sim U(0,1)$ |  |  |



[^3]Table A.2. Assessment model data, detailed in de Moor et al. (2015).

| Quantity | Description | Units / Scale |
| :---: | :---: | :---: |
| $t_{y}^{S}$ | Time lapsed between 1 May and the start of the recruit survey in year $y$ | Months |
| $\tilde{C}_{j, y, 0 b s}^{s}$ | Number of juveniles of stock $j$ caught between 1 May and the day before the start of the recruit survey in year $y$ | Billions |
| $C_{j, y, m, l}^{\text {RFL, fleet }}$ | Number of fish in length class $l$ landed by fleet in month $m$ of year $y$ of stock $j$. fleet $=1$ denotes the sardine directed fishery, fleet $=2$ denotes the sardine bycatch with round herring (1984-2011) or $\geq 14 \mathrm{~cm}$ sardine bycatch (2012-14) and fleet $=3$ denotes the juvenile sardine bycatch with anchovy (1984-2011) or $<14 \mathrm{~cm}$ sardine bycatch (2012-14) | Billions |
| $\hat{B}_{j, y}^{S}$ | Acoustic survey estimate of biomass of stock $j$ from the November survey in year $y$ | Thousand tons |
| $\sigma_{j, y, N o v}^{S}$ | Survey sampling CV associated with $\hat{B}_{j, y}^{S}$ that reflects survey inter-transect variance | - |
| $\hat{N}_{j, y, r}^{S}$ | Acoustic survey estimate of recruitment of stock $j$ from the recruit survey in year $y$ | Billions |
| $\sigma_{j, y, \text { rec }}^{S}$ | Survey sampling CV associated with $\hat{N}_{j, y, r}^{S}$ that reflects survey inter-transect variance | - |
| $\hat{p}_{j, y, l}^{S}$ | Observed proportion (by number) of stock $j$ in length group $l$ in the November survey of year $y$ | - |
| $\hat{p}_{j, y, q, l}^{\text {S,coml }}$ | Observed proportion (by number) of the directed catch and round herring bycatch of fish of stock $j$ and length group $l$ during quarter $q$ of year $y$ | - |
| $\hat{N}^{\text {j, }, \text {, }}$ l | Number of sardine of stock $j$ in length class $l$ sampled from the November survey in year $y$ that are infected with the endoparasite | Numbers |
| $n_{j, y, l}^{\text {prev }}$ | Number of sardine of stock $j$ in length class $l$ sampled from the November survey in year $y$ that were tested for infection with the endoparasite | Numbers |


[^0]:    * MARAM (Marine Resource Assessment and Management Group), Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch, 7701, South Africa.
    ${ }^{1}$ One two-mixing stock hypothesis allowed for south stock sardine to be distributed west of Cape Agulhas for some time each year, but that hypothesis is not considered here.

[^1]:    ${ }^{2} \sigma_{j, r}^{S}$ is replaced with $\sigma_{r, p e a k}^{S}$ during the peak years of 2000-2004 in the single stock hypothesis (see Table A.1).

[^2]:    ${ }^{3}$ The inclusion of model predicted proportion-at-length $24^{+} \mathrm{cm}$ is deliberate to take into account the zero sampling of $24^{+} \mathrm{cm}$ sardine in the survey.

[^3]:    ${ }^{4}$ Based upon data being available $\sim 4 x 6$ times more frequently than annual age data which contain maximum information content on this

