# Current methodology for assessing the South African hake resource: A genderdisaggregated assessment fitting directly to age-length keys 

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## Executive summary

The assessment methodology used in the development of the current OMP for the hake resource is a species- and gender-disaggregated Age-Structure Production Model which is fitted directly to age-length keys (ALKs) and length frequencies, estimating growth curve parameters internally in the likelihood maximisation process. This approach proves able to remove the conflict between fits to catch-at-age and catch-at-length data, and growth curves input, that was evident in earlier assessments.

The reason for the gender-disaggregation is that very clear gender-specific differences in somatic growth exist for both M. paradoxus and M. capensis. Because of gender-differential growth, larger sized males are not well represented in the catch, this could confound estimates based on catch-at-age data developed from a genderaggregated ALKs, which might consequently under-represent the number of older hake present. This is turn could affect estimates of natural mortality as well as distort estimates of year-class strength. The comparatively low variability of previous estimates of the recruitment has been a puzzle and concern, which might be resolved by gender differentiation of the assessments. Furthermore there is a sex-imbalance in certain components of the fishery.

The model assesses the two species as two independent stocks. For each species, the fish on the west and south coasts are assumed to be part of a single population. However coast-specific fishing selectivities (both commercial and survey) allow for the fact that the size structure of the population differ on the two coasts. The model is fitted to species-disaggregated data as well as species-combined data. Survey biomass and size structure information is available disaggregated by species (and in some cases, by gender). The species-split of the catches is carried out external to the model. From 1978 onwards, size-based proportion-by-depth relationships for the west and south coasts (based on research survey data) is used to split by species the offshore trawl fleet catches (making up more than $80 \%$ of the total catch). Pre-1978, a logistic function is assumed for the proportion of $M$. capensis caught since the depth information for the landings are not available. The basis for this assumption is that trawling was concentrated in inshore areas when the fishery began (i.e. probably catching $M$. capensis exclusively) and progressively moved offshore. On the south coast, the inshore, longline and handline fleet are assumed to catch $M$. capensis exclusively, while on the west coast longline catches are assumed to consist of $30 \% M$. capensis.

A Reference Set (RS) of 12 scenarios was selected as the primary basis to be used to simulation test candidates for the revised OMP for hake, OMP-2011. The principal uncertainty axes spanned by this RS are the central year for the switch from a primarily M. capensis to a primarily M. paradoxus fishery, values for natural mortality at age, and the form of the stock-recruitment relationship. This document gives details of the first of these assessments, the Reference Case (RC).

## Introduction

In updating the assessments used in the development of OMP-2007, the first step has been to incorporate catch-at-length information directly when fitting the population model. Indeed, it was discovered that in years for which age-length keys were not available, catches-at-length had been converted to catches-at-age
based on averages of age-length keys for other years (a problematic and potentially biased approach). This work resulted in the development of a "New Baseline" assessment in 2008, in which catch-at-length information was used directly in years for which ALK are not available (Rademeyer and Butterworth 2008). However, the "New Baseline" fits to the commercial and survey catches-at-age and catches-at-length data were poor (Figures 1 and 2), the likely reason for this being a conflict between the catch-at-age and catch-at-length information. For example, for the offshore trawl fleet (west coast and both coasts), the observed length distribution of the catch is very narrow compared to a relatively wide range of ages observed in the catches-at-age (Figure 1).

The Review Panel at the December 2008 International Stock Assessment Workshop (Punt and Smith 2008) recommended regarding the hake resource that: "A sex-structured population dynamics model should be fit to the conditional age-at-length data (age-length keys) and length-frequency data (by sex when such data are available). The growth curves (and the variation in length-at-age) should be estimated within the assessment." The recommendation arose in large part from the need to resolve this conflict between catch-at-age and catch-atlength data, and growth curves input to the then current 2008 "New Baseline" assessment (Rademeyer and Butterworth 2008).

The reason for the gender-disaggregation is that very clear gender-specific differences in somatic growth exist for both M. paradoxus and M. capensis, in fact more so than between species (as evidenced by the growth curve fits discussed below). Routine application of age-length keys to obtain catch-at-age proportions had been conducted without attention to gender-specific differences, but gender-differential growth means that larger sized males are not well represented in the catch. This could confound estimates based on catch-at-age data developed from a gender-aggregated age-length key, which might consequently under-represent the number of older hake present. This is turn could affect estimates of natural mortality as well as distort estimates of year-class strength. The comparatively low variability of previous estimates of the recruitment has been a puzzle and concern, which might be resolved by gender differentiation of the assessments. Furthermore there is a seximbalance in certain components of the fishery: for example Geromont et al. (1995) estimated a female proportion in the south coast longline catches as high as $83 \%$.

In the light of these considerations, the assessment of the hake resource has been refined to incorporate gender- as well as species-differentiation. Thus in this analysis, the genders are modelled separately. As recommended above, the model is also fit directly to age-length keys (ALKs) and length frequencies (as e.g. in Punt et al. 2006), rather than to the age frequency information which multiplying the two would provide. There are three reasons for this:
a) ALKs are not available for all years and surveys or fisheries, so that length distribution data have to be fitted directly in those cases.
b) The fishery selectivity is essentially length- rather than age-specific; age-specific selectivities as assumed when fitting to age-distribution data will lead to mis-fitting of length distribution data in these circumstances (e.g. the lower tails of the length distributions of younger fish are not present in catches, but an age-specific selectivity requires them to be).
c) The feature of the data described in b) leads to a bias in the estimation of hake growth curves if estimated directly from hake age data, leading to the lengths at younger ages being positively biased; growth curve parameters need to be estimated internally within the assessment to correct for this bias.

This document reports the results from this refined assessment, which is termed the "new Reference Case" (RC). This assessment has formed the basis from which the Operating Models that provide the basis for simulation testing of the revised OMP (OMP-2011) for hake have been developed.

## Data and Methods

Appendix I details the data used in this analysis, while the specifications and equations of the overall model are set out in Appendix II.

## Results and Discussion

Values of components of the negative log likelihood and estimates of management quantities for the new Reference Case are given in Table 1, while Figure 3 plots the spawning biomass trajectories. The spawning biomass trajectories for $M$. paradoxus and $M$. capensis show little gender difference, with the current depletion estimated to be at $15 \%$ and $54 \%$ for $M$. paradoxus and $M$. capensis respectively. For $M$. capensis the female spawning biomass is estimated to be above its MSY level, whereas the corresponding component of the M. paradoxus population is estimated at $59 \%$ of that level.

The estimated commercial and survey selectivities are shown in Figures 4 and 5 respectively. Apart for M. paradoxus on the South Coast, the male and female selectivities at length are assumed to be the same and are then converted to gender specific selectivities at age. Because of selectivity difference between males and females apparent in the South Coast surveys for M. paradoxus, gender-specific selectivities are estimated for this species in the South Coast autumn and spring surveys, with the female selectivities scaled downwards for these two surveys (for M. paradoxus only) by a common factor across lengths which is estimated in the model fitting procedure. This gender difference is assumed to affect the commercial fleet as well, and the female selectivity for the South Coast offshore trawl fleet (the only fleet assumed to catch $M$. paradoxus on the south coast) is therefore also scaled downwards by a factor estimated in the model fitting. The female selectivity scaling factors estimated for the South Coast spring and autumn surveys and the offshore trawl fleet are $0.13,0.27$ and 0.10 respectively.

All the commercial selectivities show a decrease for large fish. This decrease is estimated for all fleets apart from the offshore trawl and south coast handline fleets for $M$. capensis. For the M. capensis offshore trawl fleet, the selectivity slope is fixed to $1 / 3$ of the inshore trawl fleet slope estimated, while for the handline fleet, the selectivity slope is taken as the average of the estimated longline and inshore fleet slopes.

A penalty has been added to the $-\operatorname{lnL}$ to constrain the survey $q$ 's for each species not to exceed 1 (see equation App.II.34). This amounts to the assumption that there is no substantial herding effect which is biasing the swept-area estimates of abundance from these trawl surveys, and precludes survey abundance estimates from (on average) exceeding the underlying available biomass. Huse et al. (2001) estimated a swept area survey $q$ of about 0.8 for Namibian hake by comparing acoustic and swept area data. For the Reference Case, the $M$. paradoxus $q$ for the West Coast summer survey hits the upper boundary of 1 (see Table 1).

Figure 6 plots the gender-specific growth curves estimated in the model, as well as the estimated length-at-age distributions. The difference between male and female growth curves is estimated to be more important for $M$. paradoxus than for $M$. capensis. M. capensis is estimated to grow larger than M. paradoxus.

Figure 7 plots the estimated stock-recruitment relationships, and the time series of residuals about these relationships for both species. A penalty was added to the $-\ln \mathrm{L}$ so that the mean of the estimated recruitment residuals is close to zero (see equation App.II.43). The reason for this is that fits to the last some 30 years for which these residuals could be estimated generally showed averages appreciably below zero for $M$. capensis. If such fits had been taken through to projections, this would have meant that immediate future recruitment for $M$. capensis would have been higher on average that over recent years, thus giving a likely spuriously positive impression of resource production. It was felt more appropriate to force this average level of future production to be similar to that over the past three decades. While the possibility that those decades constitute a regime of generally low $M$. capensis productivity cannot be excluded, the associated effective changes in $K$ are considered better reserved for consideration in robustness tests. The extent of residual variability indicated for Figure 7 remains low compared to the norm for populations of similar demersal species (the output $\sigma_{\mathrm{R}}$ values are 0.26 and 0.29 for $M$. paradoxus and $M$. capensis respectively).

Figures 8 and 9 show the fits the CPUE and survey abundance series. The fits are reasonable for all series.

The fits to the commercial catch-at-length data are shown in Figure 10. The length-at-age distributions are assumed to follow a log-normal distribution rather than a normal distribution, as plots of these data are indicative of skew distributions (see Figures App.I.2-4). The fits as averaged over the years for which data are available are reasonable for all the data sets. There are however some patterns evident in the bubble plots of residuals which could perhaps be improved by having further periods between which selectivities change.

The fits to the survey gender-aggregated and gender-disaggregated catch-at-age data are shown in Figures 11 and 12 respectively. These fits are also broadly reasonable. In particular, the problem of the lack of fit evident in the corresponding plots for the 2008 "New Baseline" assessment (Rademeyer and Butterworth 2008) appears to be resolved.

The fit to the ALKs is shown in Figure 13. The observed and predicted ALKs are compared aggregated over all ALKs, first summed over ages, which should be exactly equal by construction and then summed over lengths. The ALK likelihood downweighting factor, $w$, (see equation App.II.41) was set to 0.01 as fits from earlier runs with $w=0.001$ indicated an apparent lack of influence (underweighting) of these data.

The maturity-at-age ogive used in the new Reference Case is shown in Figure 14. Since resource depletion estimates can be sensitive to the manner in which spawning biomass is defined, Table 2 compares the estimated 2009 depletions for this ogive with those for fish of age $3+$ and of age $4+$ (definitions used for earlier assessments). Results for the ogive lie between those for fish of age $3+$ and of age $4+$.

Table 1: Values of components of the negative $\log$ likelihood and estimates of management quantities for the Reference Case. The modified Ricker $\gamma$ values are given in parenthesis next to the $b$ values. The survey slopes given are for the West Coast summer and South Coast autumn surveys respectively (the two longest series). Note: MSY and related quantities have been calculated assuming a fishing pattern that is the average over the last five years (2005-2009). The natural mortality values shown in bold are fixed.

| $-\operatorname{lnL}$ total CPUE historic CPUE GLM Survey Commercial CAL <br> Survey CAL (sex-aggr.) <br> Survey CAL (sex-disaggr.) <br> ALK <br> Recruitment penalty <br> Selectivity smoothing penalty | $\begin{gathered} \hline-94.5 \\ -36.9 \\ -136.4 \\ -33.9 \\ -51.3 \\ -6.6 \\ 20.4 \\ 124.4 \\ 9.1 \\ 16.2 \end{gathered}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | Both | Males | Females |
| $K^{s p}$ | 1363 | 649 | 715 |
| $h$ | 1.08 | (0.18) |  |
| \% $B^{s p}{ }_{2009}$ | 208 | 107 | 102 |
| $\underset{\mathbb{O}}{ } \quad B^{s p}{ }_{2009} / K^{s p}$ | 0.15 | 0.16 | 0.14 |
| $B^{s p}{ }_{M S Y}$ |  |  | 174 |
| $\dot{\mathcal{E}} B^{s p}{ }_{M S Y} / K^{s p}$ |  |  | 0.24 |
| $B^{s p}{ }_{2009} / B^{s p}{ }_{M S Y}$ |  |  | 0.59 |
| MSY | 113 |  |  |
| $M_{2}$. | 0.75 |  |  |
| $M_{5+}$ | 0.375 |  |  |
| survey slopes ( $\mathrm{cm}^{-1}$ ) | 0.002 | 0.141 |  |
| $K^{s p}$ | 516 | 254 | 262 |
| $h$ | 1.01 | (1.58) |  |
| $\cdots B^{s p}{ }_{2009}$ | 279 | 142 | 137 |
| $\text { む } B^{s p}{ }_{2009} / K^{s p}$ | 0.54 | 0.56 | 0.52 |
| - $B^{s p}{ }_{M S Y}$ |  |  | 122 |
| 之 $B^{s p}{ }_{M S Y} / K^{s p}$ |  |  | 0.47 |
| $B^{s p}{ }_{2009} / B^{s p}{ }_{M S Y}$ |  |  | 1.12 |
| MSY | 69 |  |  |
| $M_{2}$ | 0.75 |  |  |
| $M_{5+}$ | 0.375 |  |  |
| survey slopes ( $\mathrm{cm}^{-1}$ ) | 0.008 | 0.071 |  |
| 2009 species ratio $B^{s p}$ | 1.34 | 1.33 | 1.34 |



Table 2: Estimated 2009 depletion for the new Reference Case spawning biomass, 3+ biomass and 4+ biomass for M. paradoxus and M. capensis.

|  | M. paradoxus | M. capensis |
| :---: | :---: | :---: |
| $B^{s{ }_{2}{ }_{2009} / K^{s p}}$ | 0.15 | 0.54 |
| $B^{3+}{ }_{2009} / K^{s p}$ | 0.18 | 0.55 |
| $B^{4+}{ }_{2009} / K^{s p}$ | 0.13 | 0.50 |



Figure 1: Fit to the commercial CAA and CAL data for the "New Baseline" assessment of 2008 (Rademeyer and Butterworth 2008).
Figure 2: Fit to survey CAA and CAL for M. paradoxus for the "New Baseline" assessment of 2008 (Rademeyer and Butterworth 2008).







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Figure 3: Estimated spawning biomass trajectories for M. paradoxus and M. capensis, both in absolute terms and relative to the pre-exploitation level for the RC.


Selectivity-at-age (from gender independent selectivity-at-length)
M. paradoxus





Figure 4: Commercial gender-independent selectivities-at-length estimated in the model-fitting and commercial gender-dependent selectivities-at-age that follow from those. Note that because of space constraints, the offshore and longline legends have been omitted for the south coast M. capensis selectivity-at-age plots. The legends for these two fleets are as for those for the west coast.

Figure 5: Survey selectivity-at-length (estimated) for the 'old' and 'new' Africana gear and derived gender-dependent survey selectivity-at-age for each of the four surveys for $M$. paradoxus and $M$. capensis. Note that for the south coast spring and autumn surveys, gender-specific selectivities have been estimated for $M$. paradoxus. For all the other combinations of species and surveys, the selectivities-at-length are gender independent



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Figure 6: Estimated length-at-age relationship and resulting length-at-age distributions for males and females $M$. paradoxus and M. capensis. In the lower plots, the distributions, starting from the left, correspond to ages $0,1,2$, $\ldots$ The 0 -year old group has a distribution overlapping zero, which is accumulated into a minus-group of 1 cm length; though such a model for this length distribution is clearly unrealistic, this hardly matters as in implementation the lowest minus-group considered for length is 10 cm , so that any implied structure below that length is ignored.


Figure 7: Estimated stock-recruitment relationships and time series of standardised stock-recruitment residuals for the RC.


Figure 8: Fit of the RC to the CPUE data.


Figure 9: Fit of the RC to the survey abundance indices. The triangles represent surveys that have been conducted with the new gear on the Africana. These are rescaled by the calibration factor for the species concerned to make them comparable to the other survey results.


Figure 10: Fit of the RC to the commercial proportion-at-length data, aggregated over years for which data are available for the plots on the left.
Figure 11: Fit of the RC to the survey gender-aggregated surveys proportion-at-length data (in some plots, aggregated over years for which data are available).

Figure 12a: Fit of the RC to the west coast summer survey gender- disaggregated proportion-at-length data (in some plots, aggregated over years for which data are available).

Figure 12b: Fit of the RC to the south coast spring survey gender- disaggregated proportion-at-length data (in some plots, aggregated over years for which data are available).


Figure 12c: Fit of the RC to the south coast autumn survey gender- disaggregated proportion-at-length data (in some plots, aggregated over years for which data are available).

M. paradoxus




Figure 13: Fit of the RC to the ALKs, aggregated over all ALKs. The first two rows are summed over ages (so that model estimate and observation should be exactly equal by construction) and second two rows summed over lengths.



Figure 14: Maturity-at-age used in the new Reference Case for M. paradoxus and M. capensis (Fairweather and Leslie 2008)
Figure 15a: Observed vs predicted mean age-at-length for $M$. paradoxus males and females for the west coast summer surveys










ALK10
West coast sum
2007







ALK8
West coast summ
1999





ALK7
West coast sur
1996



Figure 15b: Observed vs predicted mean age-at-length for M. paradoxus males and females for the West Coast winter, South Coast spring and South Coast autumn surveys.




outh Coast spring an
$\begin{gathered}\text { ALK18 } \\ \text { South coast spring }\end{gathered}$




 West Coast winter,
ALK17
South crasis spring


















Figure 15c: Observed vs predicted mean age-at-length for M. paradoxus males and females for the South Coast autumn surveys, offshore commercial trawl and
commercial longline.

$\begin{array}{cc}\text { Lemsth (cm) } \\ & \\ & \text { ALK36 } \\ & \text { Longline } \\ 1994\end{array}$












- Femates. predicted

Figure 16a: Observed vs predicted mean age-at-length for $M$. capensis males and females for the West Coast summer surveys.







ALK4
West coast summer
1993


ALK12

而

$=$
$=$
$=$
$=$
$=$
$=$
















$\xrightarrow{\text { ALK2 }}$


Figure 16b: Observed vs predicted mean age-at-length for M. capensis males and females for the West Coast winter, South Coast spring and South Coast autumn surveys.

Figure 16c: Observed vs predicted mean age-at-length for $M$. capensis males and females for the South Coast autumn surveys, offshore commercial trawl and











ALK29
South coast autumn
1999


ALK35
Offshore
1993


ALK28
South coast autumn
1999





ALK27
South coast autumn
1997


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## APPENDIX I - The data utilized

## I. 1 Annual catches

The species-split of the catches is carried out external to the model as described in Rademeyer and Butterworth (2010). The size-based species proportion-by-depth relationships for the west and south coasts which are used, from 1978 onwards, to split by species the offshore trawl fleet catches have been updated by Gaylard and Bergh (2009) from research survey data. Furthermore, for the New Reference Case, the logistic function assumed for the proportion of M. capensis caught for the period 1917-1977 has its centre at $1958\left(P_{1}\right)$, with $P_{2}=1.5$.

The total catch in 2009 is assumed equal to the TAC for that year (118 600 t ); it is split between the different fleets and species assuming the same proportions as in 200 The reported or assumed/estimated catches by fleet and species are given in Table App.I. 1 and plotted in Figure App.I.1.

## I. 2 Abundance indices

Six CPUE time-series are available for assessing the status of the resource (Table App. I.2): a CPUE series for each of the south and west coasts developed by the International Commission for South East Atlantic Fisheries (ICSEAF 1989) and a GLM-standardised CPUE series for each coast, for each of M. paradoxus and M. capensis (Table App.I.2) from the offshore trawl fleet (Glazer and Butterworth 2009). The two historical CPUE series cannot be disaggregated by species, as there are no effort-by-depth data available for this pre-1978 period. The GLM standardized CPUE indices are species-specific (the catch data utilised being based on the Gaylard and Bergh (2009) algorithm).

Research surveys have been conducted on board the FRS Africana from 1986 in spring and/or autumn on the south coast and from 1985 in summer and/or winter on the west coast, and provide fully species-specific information. Since 2003, new fishing gear has occasionally been used on the Africana, for which a calibration factor is available. Survey biomass estimates and their estimated (sampling) standard errors are listed in Tables App.I.3-4 (Fairweather 2009). Only surveys extending to the deepest depth ( 500 m ) which is normally included in the survey design are considered for reasons of comparability.

The surveys conducted on the West Coast in 2000 and 2001 by the Nansen research vessel have not been used in this analysis. As no calibration experiments were conducted at the time, these would have to be considered as a separate series and with only two data points would hardly be informative.

## I. 3 Length frequencies

Survey length frequencies are available disaggregated by species and in some years disaggregated by gender (Table App.I.5) (Fairweather et al. 2009b).

Sex-aggregated proportions-at-length for each survey stratum ( $p_{y l}^{\text {surv,i}}$ ) are provided in 1 cm length classes (Fairweather et al. 2009b). In some instances, the proportions of males and females for a particular survey stratum and length class are available $\left(q_{y l}^{g, s u r v, i}\right.$, where $\left.\sum_{g} q_{y l}^{g, s u r v, i}=1\right)$. These are converted to survey specific (i.e. aggregated over all strata for a particular cruise) proportions-at-lengths for males ( $g=1$ ), females $(g=2)$ and unsexed $(g=0)\left(\right.$ with $\left.\sum_{g=0}^{2} p_{y l}^{g, \text { surv }, i}=1\right)$ as follows:

The proportions-at-length are grouped into 2 cm length classes.
a. For all length classes $<21 \mathrm{~cm}$, the proportions-at-length are assumed to be unsexed;
b. For length classes $>20 \mathrm{~cm}$ :

- If there is no sex-information for either of the two 1 cm length classes to group (i.e. $\sum_{g} q_{y l}^{g, s u r v, i}=0$ and $\left.\sum_{g} q_{y, l+1}^{g, s u r v, i}=0\right)$, then the proportion for the resulting 2 cm length class is assumed to be unsexed:

$$
p_{y L}^{g, s u r v, i}=\left\{\begin{array}{cc}
p_{y l}^{s u r v, i}+p_{y, l+1}^{s u r v, i} & \text { for } g=0  \tag{App.I.1}\\
0 & \text { for } g=1 / 2
\end{array}\right.
$$

- If there is sex-information for one of the two 1 cm length classes to group (i.e. if $\sum_{g} q_{y l}^{g, s u r v, i}=1$ or $\left.\sum_{g} q_{y, l+1}^{g, s u r v, i}=1\right)$, then the sex-information from the one length class is used for both:

$$
p_{y L}^{g, s u r v, i}=\left\{\begin{array}{cc}
0 & \text { for } g=0  \tag{App.I.2}\\
q_{y l}^{g, s u r v, i}\left(p_{y l}^{\text {surv }, i}+p_{y, l+1}^{\text {surv,i}}\right) & \text { for } g=1 / 2
\end{array}\right.
$$

- If there is sex-information for both of the two 1 cm length classes to group (i.e. if $\sum_{g} q_{y l}^{g, s u r v, i}=1$ and

$$
\begin{align*}
& \left.\sum_{g} q_{y, l+1}^{g, s u r v, i}=1\right), \text { then the sex-information is used directly: } \\
& p_{y L}^{g, s u r v, i}=\left\{\begin{array}{cc}
0 & \text { for } g=0 \\
q_{y l}^{g, s u r v, i} p_{y l}^{s u r v, i}+q_{y, l+1}^{g, \text { surv,i}} p_{y, l+1}^{s u r v} & \text { for } g=1 / 2
\end{array}\right. \tag{App.I.3}
\end{align*}
$$

c. The strata proportions-at-length are weighted by the estimated total number in the stratum concerned to obtain a survey specific gender-disaggregated proportions-at-length $\left(p_{y l}^{g, \text { surv }}\right)$. The estimated total number in each stratum is calculated as:

$$
\begin{equation*}
N_{y}^{s u r v, i}=B_{y}^{s u r v, i} / \bar{W}_{y}^{s u r v, i} \tag{App.I.4}
\end{equation*}
$$

where
$B_{y}^{s u r v, i}$ is the survey biomass estimate for stratum $i$ in survey surv, and
$\bar{W}_{y}$ surv,i is the mean weight of fish for stratum $i$ in survey surv, with

$$
\begin{equation*}
\bar{W}_{y}^{s u r v, i}=\sum_{l}\left(p_{y l}^{s u r v, i} \alpha l^{\beta}\right) \tag{App.I.5}
\end{equation*}
$$

d. For each 2 cm length class, if the unsexed proportion is less than $20 \%$ of the total proportion in that length class, the sexed proportion is used to split the unsexed proportion into males and females.

Figures. App.I.2-3 plot the survey length frequencies available.
Length frequency information from the commercial catch is not available by species, the reason being that it is often based on cleaned (headed and gutted) fish, which cannot be easily identified by species. As a result it is not possible to disaggregate the commercial length frequencies by species. Commercial catches-at-length for the offshore and for the inshore and longline fleets are shown in Tables App.II.10-12. The south coast inshore and longline fleet catches are assumed to consist of M. capensis only.

## I. 4 Age-Length Keys

Table App.I. 6 lists the age-length keys available. Data from animals with frills on gills (FOG) have been discarded ( $<3 \%$ of the total). All aged animals less or equal to 20 cm in length are assumed to be juveniles, i.e. of unknown gender. The few unsexed data from animals greater than 20 cm have been discarded ( $<1 \%$ of the total), as well as the outliers, defined as the data points lying outside the mean $\pm 3$ s.d. for each age (mean and s.d. calculated across all years and surveys). Three ALKs for M. paradoxus have been totally ignored in the model fitting as they seemed completely inconsistent with the other ALKs: i) 1997 West Coast summer survey, ii) 2004 West Coast summer and iii) 2006 West Coast summer. The data for these three sets are shown in Figure App.I. 5 .

Most otoliths are read more than once; however only one reading for each otoliths is used in the fitting procedure. Table App.I. 7 show which reader was selected in the event of more than one reading.

Table App.I.1a: Species-disaggregated offshore trawl catches (in thousand tons) of South African hake from the south and west coasts (see text for details), assuming 1958 as the centre year of the shift from a primarily $M$. capensis to a primarily M. paradoxus

|  | M. paradoxus |  | M. capensis |  | M. paradoxus |  |  | M. capensis |  | M. paradoxus |  |  | M. capensis |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | WC | SC | WC | SC | Year | WC | SC | WC | SC | Year | WC | SC | WC | SC |
| 1917 | - | - | 1.000 | - | 1948 | 0.056 | - | 58.744 | - | 1979 | 93.711 | 2.653 | 39.811 | 4.266 |
| 1918 | - | - | 1.100 | - | 1949 | 0.107 | - | 57.293 | - | 1980 | 100.723 | 2.833 | 32.805 | 3.628 |
| 1919 | - | - | 1.900 | - | 1950 | 0.260 | - | 71.740 | - | 1981 | 90.572 | 1.208 | 30.358 | 4.277 |
| 1920 | - | - | 0.000 | - | 1951 | 0.627 | - | 88.873 | - | 1982 | 84.030 | 4.063 | 29.319 | 7.294 |
| 1921 | - | - | 1.300 | - | 1952 | 1.201 | - | 87.599 | - | 1983 | 71.628 | 5.920 | 22.805 | 6.596 |
| 1922 | - | - | 1.000 | - | 1953 | 2.422 | - | 91.078 | - | 1984 | 82.940 | 4.689 | 28.316 | 6.246 |
| 1923 | - | - | 2.500 | - | 1954 | 5.149 | - | 100.251 | - | 1985 | 93.192 | 10.054 | 31.878 | 9.962 |
| 1924 | - | - | 1.500 | - | 1955 | 10.343 | - | 105.057 | - | 1986 | 105.097 | 9.974 | 28.708 | 5.991 |
| 1925 | - | - | 1.900 | - | 1956 | 18.540 | - | 99.660 | - | 1987 | 95.954 | 9.495 | 21.571 | 6.189 |
| 1926 | - | - | 1.400 | - | 1957 | 32.241 | - | 94.159 | - | 1988 | 83.910 | 7.184 | 22.672 | 7.332 |
| 1927 | - | - | 0.800 | - | 1958 | 49.136 | - | 81.564 | - | 1989 | 84.719 | 6.919 | 22.541 | 11.993 |
| 1928 | - | - | 2.600 | - | 1959 | 72.535 | - | 73.465 | - | 1990 | 89.976 | 11.636 | 13.660 | 11.155 |
| 1929 | - | - | 3.800 | - | 1960 | 95.147 | - | 64.753 | - | 1991 | 92.787 | 9.604 | 13.663 | 12.470 |
| 1930 | - | - | 4.400 | - | 1961 | 98.478 | - | 50.222 | - | 1992 | 89.638 | 19.260 | 13.649 | 7.202 |
| 1931 | - | - | 2.800 | - | 1962 | 103.768 | - | 43.832 | - | 1993 | 107.370 | 11.143 | 10.694 | 3.117 |
| 1932 | - | - | 14.300 | - | 1963 | 123.055 | - | 46.445 | - | 1994 | 112.355 | 7.842 | 11.512 | 3.210 |
| 1933 | - | - | 11.100 | - | 1964 | 119.837 | - | 42.463 | - | 1995 | 104.842 | 4.486 | 16.055 | 2.664 |
| 1934 | - | - | 13.800 | - | 1965 | 151.211 | - | 51.789 | - | 1996 | 119.889 | 10.467 | 9.286 | 2.822 |
| 1935 | - | - | 15.000 | - | 1966 | 145.914 | - | 49.086 | - | 1997 | 108.917 | 12.902 | 8.237 | 2.934 |
| 1936 | - | - | 17.700 | - | 1967 | 132.530 | 5.391 | 44.170 | 8.795 | 1998 | 115.290 | 11.165 | 12.363 | 2.988 |
| 1937 | - | - | 20.200 | - | 1968 | 107.834 | 10.619 | 35.766 | 17.289 | 1999 | 90.030 | 12.749 | 13.731 | 2.597 |
| 1938 | - | - | 21.100 | - | 1969 | 124.056 | 14.442 | 41.044 | 23.489 | 2000 | 91.366 | 8.777 | 26.336 | 4.753 |
| 1939 | - | - | 20.000 | - | 1970 | 107.108 | 9.035 | 35.392 | 14.688 | 2001 | 98.164 | 8.213 | 19.433 | 7.944 |
| 1940 | - | - | 28.600 | - | 1971 | 151.855 | 11.472 | 50.145 | 18.644 | 2002 | 95.122 | 13.629 | 9.809 | 4.955 |
| 1941 | - | - | 30.600 | - | 1972 | 183.394 | 17.789 | 60.539 | 28.907 | 2003 | 95.062 | 20.503 | 10.314 | 4.530 |
| 1942 | 0.001 | - | 34.499 | - | 1973 | 118.629 | 27.566 | 39.153 | 44.790 | 2004 | 86.340 | 28.805 | 11.891 | 5.965 |
| 1943 | 0.001 | - | 37.899 | - | 1974 | 92.480 | 34.613 | 30.520 | 56.240 | 2005 | 88.722 | 24.374 | 6.545 | 4.872 |
| 1944 | 0.002 | - | 34.098 | - | 1975 | 67.381 | 25.703 | 22.236 | 41.760 | 2006 | 84.951 | 19.923 | 8.547 | 4.705 |
| 1945 | 0.004 | - | 29.196 | - | 1976 | 108.192 | 19.785 | 35.702 | 32.145 | 2007 | 96.426 | 14.899 | 12.444 | 2.345 |
| 1946 | 0.010 | - | 40.390 | - | 1977 | 76.939 | 14.086 | 25.389 | 22.886 | 2008 | 92.445 | 13.861 | 6.930 | 3.646 |
| 1947 | 0.020 | - | 41.380 | - | 1978 | 103.665 | 3.830 | 23.847 | 3.755 | 2009 | 85.357 | 12.798 | 6.399 | 3.366 |

Table App.I.1b: Inshore trawl catches of South African hake (assumed to be M. capensis exclusively) from the South Coast.

| Year M. capensis |  | Year M. capensis |  | Year M. capensis |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1960 | 1.000 | 1977 | 3.500 | 1994 | 9.569 |
| 1961 | 1.308 | 1978 | 4.931 | 1995 | 10.630 |
| 1962 | 1.615 | 1979 | 6.093 | 1996 | 11.062 |
| 1963 | 1.923 | 1980 | 9.121 | 1997 | 8.834 |
| 1964 | 2.231 | 1981 | 9.400 | 1998 | 8.283 |
| 1965 | 2.538 | 1982 | 8.089 | 1999 | 8.595 |
| 1966 | 2.846 | 1983 | 7.672 | 2000 | 10.906 |
| 1967 | 3.154 | 1984 | 9.035 | 2001 | 11.836 |
| 1968 | 3.462 | 1985 | 9.203 | 2002 | 9.581 |
| 1969 | 3.769 | 1986 | 8.724 | 2003 | 9.883 |
| 1970 | 4.077 | 1987 | 8.607 | 2004 | 10.004 |
| 1971 | 4.385 | 1988 | 8.417 | 2005 | 7.881 |
| 1972 | 4.692 | 1989 | 10.038 | 2006 | 5.524 |
| 1973 | 5.000 | 1990 | 10.012 | 2007 | 6.350 |
| 1974 | 10.056 | 1991 | 8.206 | 2008 | 5.496 |
| 1975 | 6.372 | 1992 | 9.252 | 2009 | 5.075 |
| 1976 | 5.740 | 1993 | 8.870 |  |  |

Table App.I.1c: Species-disaggregated longline trawl catches of South African hake from the south and west coasts.

| M. para |  |  | M. capensis |  | M. para |  |  | M. capensis |  |  | M. para |  |  | M. capensis |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | WC | WC | SC | Year | WC | WC | SC | Year | WC | WC | SC |  |  |  |  |
| 1983 | 0.161 | 0.069 | - | 1992 | - | - | 1.500 | 2001 | 2.793 | 1.197 | 1.688 |  |  |  |  |
| 1984 | 0.256 | 0.110 | 0.016 | 1993 | - | - | - | 2002 | 4.772 | 2.045 | 3.945 |  |  |  |  |
| 1985 | 0.817 | 0.350 | 0.292 | 1994 | 1.130 | 0.484 | 0.626 | 2003 | 4.668 | 2.000 | 4.878 |  |  |  |  |
| 1986 | 0.965 | 0.413 | 0.302 | 1995 | 0.670 | 0.287 | 0.650 | 2004 | 3.758 | 1.611 | 4.429 |  |  |  |  |
| 1987 | 2.500 | 1.071 | 0.353 | 1996 | 1.676 | 0.718 | 1.828 | 2005 | 4.172 | 1.788 | 4.559 |  |  |  |  |
| 1988 | 3.628 | 1.555 | 0.331 | 1997 | 1.806 | 0.774 | 1.872 | 2006 | 3.592 | 1.539 | 4.032 |  |  |  |  |
| 1989 | 0.203 | 0.087 | 0.032 | 1998 | 0.647 | 0.277 | 1.471 | 2007 | 3.151 | 1.350 | 3.834 |  |  |  |  |
| 1990 | 0.270 | 0.116 | - | 1999 | 1.963 | 0.841 | 4.144 | 2008 | 2.170 | 0.930 | 2.740 |  |  |  |  |
| 1991 | - | - | 3.000 | 2000 | 3.456 | 1.481 | 2.077 | 2009 | 2.004 | 0.859 | 2.530 |  |  |  |  |

Table App.I.1d: Handline catches of South African hake (assumed to be M. capensis exclusively) from the South Coast.

| Year |  | M. capensis | Year M. capensis |  | Year M. capensis |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 1985 | 0.065 | 1994 | 0.449 | 2003 | 3.000 |  |
| 1986 | 0.084 | 1995 | 0.756 | 2004 | 1.600 |  |
| 1987 | 0.096 | 1996 | 1.515 | 2005 | 0.700 |  |
| 1988 | 0.071 | 1997 | 1.404 | 2006 | 0.400 |  |
| 1989 | 0.137 | 1998 | 1.738 | 2007 | 0.400 |  |
| 1990 | 0.348 | 1999 | 2.749 | 2008 | 0.231 |  |
| 1991 | 1.270 | 2000 | 5.500 | 2009 | 0.213 |  |
| 1992 | 1.099 | 2001 | 7.300 |  |  |  |
| 1993 | 0.278 | 2002 | 3.500 |  |  |  |

Table App.I.2: South and west coast historic (ICSEAF, 1989) and GLM standardized CPUE data (GLM3 of Glazer and Butterworth, 2009) for M. paradoxus and M. capensis. The historic CPUE series are for M. capensis and M. paradoxus combined.

|  | ICSEAF CPUE $\left(\mathrm{t} \mathrm{hr}^{-1}\right)$ |  |  |  | GLM CPUE $\left(\mathrm{kg} \mathrm{min}^{-1}\right)$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Table App.I.3: Survey abundance estimates and associated standard errors in thousand tons for M. paradoxus for the depth range $0-500 \mathrm{~m}$ for the south coast and for the west coast. Values in bold are for the surveys conducted by the Africana with the new gear.

| Year | West coast |  |  |  | South coast |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Summer |  | Winter |  | Spring (Sept) |  | Autumn (Apr/May) |  |
|  | Biomass | (s.e.) | Biomass | (s.e.) | Biomass | (s.e.) | Biomass | (s.e.) |
| 1985 | 169.959 | (36.680) | 264.839 | (52.949) | - | - | - | - |
| 1986 | 196.111 | (36.358) | 172.477 | (24.122) | 13.758 | (3.554) | - | - |
| 1987 | 284.805 | (53.101) | 195.482 | (44.415) | 21.554 | (4.605) | - | - |
| 1988 | 158.758 | (27.383) | 233.041 | (64.003) | - | - | 30.316 | (11.104) |
| 1989 | - | - | 468.780 | (124.830) | - | - | - | - |
| 1990 | 282.174 | (78.945) | 226.862 | (46.007) | - | - | - | - |
| 1991 | 327.020 | (82.180) | - | - | - | - | 26.638 | (10.460) |
| 1992 | 226.687 | (32.990) | - | - | - | - | 24.304 | (15.195) |
| 1993 | 334.151 | (50.234) | - | - | - | - | 198.849 | (98.452) |
| 1994 | 330.270 | (58.319) | - | - | - | - | 111.469 | (34.627) |
| 1995 | 324.554 | (80.357) | - | - | - | - | 55.068 | (22.380) |
| 1996 | 430.908 | (80.604) | - | - | - | - | 85.546 | (25.484) |
| 1997 | 569.957 | (108.200) | - | - | - | - | 135.192 | (51.031) |
| 1998 | - | - | - | - | - | - | - | - |
| 1999 | 562.859 | (116.302) | - | - | - | - | 321.478 | (113.557) |
| 2000 | - | - | - | - | - | - | - | - |
| 2001 | - | - | - | - | 19.929 | (9.956) | - | - |
| 2002 | 267.487 | (35.068) | - | - | - | - | - | - |
| 2003 | 411.177 | (69.431) | - | - | 88.442 | (36.051) | 108.857 | (37.528) |
| 2004 | 259.527 | (56.021) | - | - | 63.900 | (17.894) | 48.898 | (20.343) |
| 2005 | 286.416 | (39.849) | - | - | - | - | 26.605 | (7.952) |
| 2006 | 315.310 | (49.490) | - | - | 72.415 | (15.500) | 34.799 | (8.325) |
| 2007 | 392.812 | (70.043) | - | - | 52.287 | (19.231) | 129.646 | (60.661) |
| 2008 | 246.542 | (51.973) | - | - | 24.816 | (8.775) | 39.505 | (11.408) |
| 2009 | 330.235 | (28.526) | - | - | - | - | 102.834 | (28.670) |

Table App.I.4: Survey abundance estimates and associated standard errors in thousand tons for M. capensis for the depth range $0-500 \mathrm{~m}$ for the south coast and for the west coast. Values in bold are for the surveys conducted by the Africana with the new gear.

| Year | West coast |  |  |  | South coast |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Summer |  | Winter |  | Spring (Sept) |  | Autumn (Apr/May) |  |
|  | Biomass | (s.e.) | Biomass | (s.e.) | Biomass | (s.e.) | Biomass | (s.e.) |
| 1985 | 124.647 | (22.707) | 181.487 | (27.476) | - | - | - | - |
| 1986 | 117.810 | (23.636) | 119.587 | (18.489) | 121.197 | (16.625) | - | - |
| 1987 | 75.693 | (10.241) | 87.391 | (11.198) | 159.088 | (17.233) | - | - |
| 1988 | 66.725 | (10.765) | 47.120 | (9.568) | - | - | 165.939 | (21.871) |
| 1989 | - | - | 323.833 | (67.295) | - | - | - | - |
| 1990 | 455.798 | (135.237) | 157.800 | (23.561) | - | - | - | - |
| 1991 | 77.357 | (14.995) | - | - | - | - | 274.298 | (44.395) |
| 1992 | 95.407 | (11.744) | - | - | - | - | 138.085 | (15.357) |
| 1993 | 92.598 | (14.589) | - | - | - | - | 158.340 | (13.733) |
| 1994 | 121.257 | (35.951) | - | - | - | - | 160.555 | (23.701) |
| 1995 | 199.142 | (26.812) | - | - | - | - | 236.025 | (31.840) |
| 1996 | 83.337 | (9.285) | - | - | - | - | 244.410 | (25.107) |
| 1997 | 257.293 | (46.056) | - | - | - | - | 183.087 | (18.906) |
| 1998 | - | - | - | - | - | - | - | - |
| 1999 | 198.716 | (32.467) | - | - | - | - | 191.203 | (14.952) |
| 2000 | - | - | - | - | - | - | - | - |
| 2001 | - | - | - | - | 133.793 | (20.858) | - | - |
| 2002 | 106.253 | (15.813) | - | - | - | - | - | - |
| 2003 | 75.960 | (13.314) | - | - | 82.928 | (9.010) | 128.450 | (20.062) |
| 2004 | 205.939 | (33.216) | - | - | 106.119 | (15.596) | 99.902 | (12.027) |
| 2005 | 70.983 | (13.845) | - | - | - | - | 76.932 | (5.965) |
| 2006 | 88.420 | (22.851) | - | - | 99.867 | (9.803) | 130.900 | (14.816) |
| 2007 | 82.270 | (11.441) | - | - | 74.615 | (7.383) | 70.940 | (5.615) |
| 2008 | 50.877 | (5.355) | - | - | 94.232 | (11.456) | 108.195 | (9.978) |
| 2009 | 175.289 | (39.920) | - | - | - | - | 124.004 | (11.808) |

Table App.I.5: Survey length frequencies available in February 2010.

| Year | West coast |  |  |  | South coast |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Summer |  | Winter |  | Spring (Sept) |  | Autumn (Apr/May) |  |
|  | Sex-ager. | By sex | Sex-aggr. | By sex | Sex-aggr. | By sex | Sex-ager. | By sex |
| 1985 | $\checkmark$ | - | $\checkmark$ | - | - | - | - | - |
| 1986 | $\checkmark$ | - | $\checkmark$ | - | $\checkmark$ | - | - | - |
| 1987 | $\checkmark$ | - | $\checkmark$ | - | $\checkmark$ | - | - | - |
| 1988 | $\checkmark$ | - | $\checkmark$ | - | - | - | $\checkmark$ | - |
| 1989 | - | - | $\checkmark$ | - | - | - | - | - |
| 1990 | $\checkmark$ | - | $\checkmark$ | - | - | - | - | - |
| 1991 | $\checkmark$ | - | - | - | - | - | $\checkmark$ | - |
| 1992 | $\checkmark$ | - | - | - | - | - | $\checkmark$ | - |
| 1993 | $\checkmark$ | $\checkmark$ | - | - | - | - | $\checkmark$ | $\checkmark$ |
| 1994 | $\checkmark$ | $\checkmark$ | - | - | - | - | $\checkmark$ | $\checkmark$ |
| 1995 | $\checkmark$ | $\checkmark$ | - | - | - | - | $\checkmark$ | $\checkmark$ |
| 1996 | $\checkmark$ | $\checkmark$ | - | - | - | - | $\checkmark$ | $\checkmark$ |
| 1997 | $\checkmark$ | $\checkmark$ | - | - | - | - | $\checkmark$ | $\checkmark$ |
| 1998 | - | - | - | - | - | - | - | - |
| 1999 | $\checkmark$ | $\checkmark$ | - | - | - | - | $\checkmark$ | - |
| 2000 | - | - | - | - | - | - | - | - |
| 2001 | - | - | - | - | $\checkmark$ | - | - | - |
| 2002 | $\checkmark$ | - | - | - | - | - | - | - |
| 2003 | $\checkmark$ | - | - | - | $\checkmark$ | - | $\checkmark$ | - |
| 2004 | $\checkmark$ | - | - | - | $\checkmark$ | - | $\checkmark$ | - |
| 2005 | $\checkmark$ | - | - | - | - | - | $\checkmark$ | - |
| 2006 | $\checkmark$ | $\checkmark$ | - | - | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 2007 | $\checkmark$ | $\checkmark$ | - | - | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 2008 | $\checkmark$ | $\checkmark$ | - | - | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 2009 | $\checkmark$ | $\checkmark$ | $-$ | - | - | - | $\checkmark$ | $\checkmark$ |

Table App.I.6: Species- and sex-disaggregated age and length data available in February 2010 by reader.


Table App.I.7: For each set of age readers, the reader shaded is the one whose otolith readings were used.
M. paradoxus

| 1 | UR |  |  |
| :---: | :---: | :---: | :---: |
| 2 | KG | JP | AP/DJ |
| 3 | UR | AD | LB |
| 4 | AD |  |  |
| 5 | JP | KB |  |

M. capensis

| 1 | UR |  |  |  |  |
| :---: | :---: | :---: | :---: | :--- | :--- |
| 2 | KG | JP | AP/DJ |  |  |
| 3 | JP | KB |  |  |  |
| 4 | UR | AD | LB |  |  |
| 5 | AD | LB | PM | TA | KB |
| 6 | PM | TA | KB |  |  |
| 5 | LB |  |  |  |  |

The readers are: Alexia Daniels (AD), Luke Bester (LB), 'Unknown Reader' (UR), Kevin Gradie (KG), John Prinsloo (JP), Andy Payne/Dave Japp (AP/DJ), Phoeby Mullins (PM), Teressa Akkers (TA) and Kashif Booley (KB).


Figure App.I.1: Annual catches, see text for details, assuming 1958 as the centre year of the shift from a primarily M. capensis to a primarily M. paradoxus offshore trawl catch..


Figure App.I.2: Sex-aggregated survey catch-at-length information. The vertical bars show the minus and plus groups used.


Fig App.I.3a: West coast summer gender-disaggregated survey catch-at-length information. The vertical bars show the minus and plus groups used.


Fig App.I.3b: South coast spring gender-disaggregated survey catch-at-length information. The vertical bars show the minus and plus groups used.


Fig App.I.3c: South coast autumn gender-disaggregated survey catch-at-length information. The vertical bars show the minus and plus groups used.


Fig App.I.4: Commercial catch-at-length information. The vertical bars show the minus and plus groups used.


Figure App.I.5: Data points for each gender separately for the three M. paradoxus ALKs that have been omitted from the model fitting. The average of the other ALKs is shown with the error bars representing the $\pm 2$ s.d. range.

## APPENDIX II- Gender-disaggregated, age-structured production model fitting to age-length keys

The model used is a gender-disaggregated Age-Structured Production Model (ASPM), which is fitted directly to age-length keys (ALKs) and length frequencies. The model also assesses the two species as two independent stocks and is fitted to species-disaggregated data as well as species-combined data. The general specifications and equations of the overall model are set out below, together with some key choices in the implementation of the methodology. Details of the contributions to the log-likelihood function from the different data considered are also given. Quasi-Newton minimisation is used to minimise the total negative loglikelihood function (implemented using AD Model Builder ${ }^{\mathrm{TM}}$, Otter Research, Ltd.).

## II. 1 Population Dynamics

## II.1.1 Numbers-at-age

The resource dynamics of the two populations (M. capensis and M. paradoxus) of the South African hake are modelled by the following set of equations.

Note: for ease of reading, the 'species' subscript $s$ has been omitted below where equations are identical for the two species.

$$
\begin{align*}
& N_{y+1,0}^{g}=R_{y+1}^{g} \\
& N_{y+1, a+1}^{g}=\left(N_{y a}^{g} e^{-M_{a}^{g} / 2}-\sum_{f} C_{f j a}^{g}\right) e^{-M_{a}^{g} / 2} \quad \text { for } 0 \leq a \leq m-2 \\
& N_{y+1, m}^{g}=\left(N_{y, m-1}^{g} e^{-M_{m-1}^{g} / 2}-\sum_{f} C_{f, y, m-1}^{g}\right) e^{-M_{m-1}^{g} / 2}+\left(N_{y m}^{g} e^{-M_{m}^{g} / 2}-\sum_{f} C_{f y m}^{g}\right) e^{-M_{m i}^{g} / 2} \tag{App.II.3}
\end{align*}
$$

where
$N_{y a}^{g} \quad$ is the number of fish of gender $g$ and age $a$ at the start of year $y^{\prime}$;
$R_{y}^{g} \quad$ is the recruitment (number of 0 -year-old fish) of fish of gender $g$ at the start of year $y$;
$m \quad$ is the maximum age considered (taken to be a plus-group);
$M_{a}^{g}$ denotes the natural mortality rate on fish of gender $g$ and age $a$; and
$C_{f y a}^{g} \quad$ is the number of hake of gender $g$ and age $a$ caught in year $y$ by fleet $f$.

## II.1.2 Recruitment

The number of recruits (i.e. new zero-year old fish) at the start of year $y$ is assumed to be related to the corresponding female spawning stock size (i.e., the biomass of mature female fish) by means of the BevertonHolt (Beverton and Holt 1957) or a modified (generalised) form of the Ricker stock-recruitment relationship. These forms are parameterized in terms of the "steepness" of the stock-recruitment relationship, $h$, the preexploitation equilibrium female spawning biomass, $K^{\circ}{ }^{\rho} s p$, and the pre-exploitation recruitment, $R_{0}$ and assuming a $50: 50$ sex-split at recruitment.

[^0]$R_{y}^{g}=\frac{4 h R_{0} B_{y}^{\rho, s p}}{K^{\rho, s p}(1-h)+(5 h-1) B_{y}^{\rho, s p}} e^{\left(\varsigma_{y}-\sigma_{k}^{2} / 2\right)}$
for the Beverton-Holt stock-recruitment relationship and
$R_{y}^{g}=\alpha B_{y}^{\odot, s p} \exp \left(-\beta\left(B_{y}^{\odot, s p}\right)^{\gamma}\right) e^{\left(\varsigma_{y}-\sigma_{R}^{2} / 2\right)}$
with
$\alpha=R_{0} \exp \left(\beta\left(K^{\rho, s p}\right)^{\gamma}\right) \quad$ and $\quad \beta=\frac{\ln (5 h)}{\left(K^{\rho, s p}\right)^{\gamma}\left(1-5^{-\gamma}\right)}$
for the modified Ricker relationship (for the true Ricker, $\gamma=1$ ) where
$\boldsymbol{\zeta}_{y} \quad$ reflects fluctuation about the expected recruitment in year $y$;
$\sigma_{R} \quad$ is the standard deviation of the log-residuals, which is input ( $\sigma_{R}=0.45$ and is taken to decrease from this value to 0.1 over the last five years to statistically stabilise estimates of recent recruitment).
$B_{y}^{\text {o } s p} \quad$ is the female spawning biomass at the start of year $y$, computed as:
$B_{y}^{\circ, s p}=\sum_{a=1}^{m} f_{a}^{\circ} w_{a}^{\circ} N_{y a}^{\circ}$
where
$w_{a}^{g} \quad$ is the begin-year mass of fish of gender $g$ and age $a$;
$f_{a}^{g} \quad$ is the proportion of fish of gender $g$ and age $a$ that are mature (converted from maturity-at-length, see equation App.II.47); and
$R_{0}=K^{\circ}, s p /\left[\sum_{a=1}^{m-1} f_{a}^{\circ} w_{a}^{\circ} e^{-\sum_{a=0}^{a-1} M_{a^{\prime}}^{g}}+f_{m}^{\circ} w_{m}^{\circ} \frac{e^{-\sum_{a=0}^{m-1} M_{a^{\prime}}^{g}}}{1-e^{-M_{m}^{g}}}\right]$
For the Beverton-Holt form, $b$ is bounded above by 0.98 to preclude high recruitment at extremely low spawning biomass, whereas for the modified Ricker form, $b$ is bounded above by 1.5 to preclude extreme compensatory behaviour.

## II.1.3 Total catch and catches-at-age

The fleet-disaggregated catch by mass, in year $y$ is given by:
$C_{f y}=\sum_{g} \sum_{a=0}^{m} w_{a+1 / 2}^{g} C_{f y a}^{g}=\sum_{g} \sum_{a=0}^{m} w_{a+1 / 2}^{g} N_{y a}^{g} e^{-M_{a}^{g} / 2} F_{f y} \widetilde{S}_{f y a}^{g}$
where
$C_{f y a}^{g}$ is the catch-at-age, i.e. the number of fish of gender $g$ and age $a$, caught in year $y$ by fleet $f$,
$F_{f y} \quad$ is the fishing mortality of a fully selected age class, for fleet $f$ in year $y$ (independent of $g$ );
$\tilde{S}_{f y a}^{g}=\tilde{w}_{f y, a+1 / 2}^{g} / w_{a+1 / 2}^{g}$
$\tilde{S}_{f y a}^{g} \quad$ is the effective commercial selectivity of gender $g$ at age $a$ for fleet $f$ and year $y$; with

$$
\begin{equation*}
\tilde{w}_{f y, a+1 / 2}^{g}=\sum_{l} S_{f y l}^{g} w_{l}^{g} P_{a+1 / 2, l}^{g} \tag{App.II.9}
\end{equation*}
$$

$\widetilde{w}_{f y, a+1 / 2}^{g}$ is the selectivity-weighted mid-year weight-at-age $a$ of gender $g$ for fleet $f$ and year $y$;
$w_{l}^{g} \quad$ is the weight of fish of gender $g$ and length $l ;$
$w_{a+1 / 2}^{g}$ is the mid-year weight of fish of gender $g$ and age $a$, at median length for that age;
$S_{f y l}^{g} \quad$ is the commercial selectivity of gender $g$ at length $l$ for year $y$, and fleet $f$;
$P_{a+1 / 2, l}^{g}$ is the mid-year proportion of fish of age $a$ and gender $g$ that fall in the length group $l$ (i.e., $\sum_{l} P_{a+1 / 2, l}^{g}=1$ for all ages $a$.

The matrix $P$ is calculated under the assumption that length-at-age is log-normally distributed about a mean given by the von Bertalanffy equation, i.e.:
$\ln l_{a} \sim N\left[\ln \left(l_{\infty}\left(1-e^{-\kappa\left(a-t_{0}\right)}\right)\right) ;\left(\frac{\theta_{a}}{l_{\infty}\left(1-e^{-\kappa\left(a-t_{0}\right)}\right)}\right)^{2}\right]$
where $\theta_{a}$ is the standard deviation of length-at-age $a$, which is estimated directly in the model fitting for age 0 , and for ages 1 and above a linear relationship applies:
$\theta_{a}=\left\{\begin{array}{cc}B_{0} / l_{0} & \text { for } a=0 \\ (\beta a+\alpha) / l_{a} & \text { for } 1 \leq a \leq m\end{array}\right.$
with species and gender-specific $B_{0}, \alpha$ and $\beta$ estimated in the model fitting procedure. A penalty is added so that $\theta_{a}$ is increasing with age, i.e. $\beta>0$.

## II.1.4 Exploitable and survey biomasses

The model estimate of the mid-year exploitable ("available") component of biomass for each species and fleet is calculated by converting the numbers-at-age into mid-year mass-at-age and applying natural and fishing mortality for half the year:
$B_{f y}^{e x}=\sum_{g} \sum_{a=0}^{m} w_{a+1 / 2}^{g} \tilde{S}_{f ; a}^{g} N_{y a}^{g} e^{-M_{a}^{g} / 2}\left(1-\sum_{f} S_{f j a}^{g} F_{f y} / 2\right)$
The model estimate of the survey biomass at the start of the year (summer) is given by:
$B_{y}^{\text {surv }}=\sum_{g} \sum_{a=0}^{m_{s}} w_{a}^{g} \tilde{S}_{a}^{g, s u m} N_{y a}^{g}$
and in mid-year (winter):
$B_{y}^{s u r v}=\sum_{g} \sum_{a=0}^{m} w_{a+1 / 2}^{g} \tilde{S}_{a}^{g, w i n} N_{y a}^{g} e^{-M_{a}^{g} / 2}\left(1-\sum_{f} S_{f y a}^{g} F_{f y} / 2\right)$
where
$\tilde{S}_{a}^{g, \text { sum/win }}$ is the effective survey selectivity of gender $g$ for age $a$, converted from survey selectivity-at-length and selectivity-weighted weight-at-age in the same manner as for the commercial selectivity (eqns App.II. 8 and App.II.9), taking account of the being-year $\left(\widetilde{w}_{y, a}^{g, \text { sum }}\right.$ from $\left.P_{a, l}^{g}\right)$ or mid-year $\left(\tilde{w}_{y, a+1 / 2}^{g, \text { win }}\right.$ from $\left.P_{a+1 / 2, l}^{g}\right)$ nature of the surveys

Note that both the spring and autumn surveys are taken to correspond to winter (mid-year).
It is assumed that the resource is at the deterministic equilibrium that corresponds to an absence of harvesting at the start of the initial year considered, i.e., $B_{1}^{g, s p}=K^{g, s p}$, and the year $y=1$ corresponds to 1917 when catches are taken to commence.

## II. 2 MSY and related quantities

The equilibrium catch for a fully selected fishing proportion $F^{*}$ is calculated as:

$$
\begin{equation*}
C\left(F^{*}\right)=\sum_{g} \sum_{a} w_{a+1 / 2}^{g} \widetilde{S}_{a}^{g} F^{*} N_{a}^{g}\left(F^{*}\right) e^{-\left(\left(M_{a}^{g}+S_{a}^{g} F^{*}\right) / 2\right)} \tag{App.II.14}
\end{equation*}
$$

where
$S_{a}^{g}$ and $\tilde{S}_{a}^{g}$ are respectively average selectivities and effective selectivities across all fleets, for the most recent five years;
$S_{a}^{g}=\frac{\sum_{y=2005}^{2009} \sum_{f} S_{f y a}^{g} F_{f y}}{\max \left(\sum_{y=2005}^{2009} \sum_{f} S_{f y a}^{g} F_{f y}\right)}$
$\tilde{S}_{a}^{g}=\frac{\sum_{y=2005}^{2009} \sum_{f} \tilde{S}_{f y a}^{g} F_{f y}}{\max \left(\sum_{y=2005}^{2009} \sum_{f} \tilde{S}_{f y a}^{g} F_{f y}\right)}$
where the maximum is taken over genders and ages; and with
$N_{a}^{g}\left(F^{*}\right)=\left\{\begin{array}{cc}R_{1}\left(F^{*}\right) & \text { for } a=1 \\ N_{a-1}^{g}\left(F^{*}\right) e^{-M_{a-1}^{g}\left(1-S_{a-1}^{g} F^{*}\right)} & \text { for } 1<a<m \\ \frac{N_{m-1}^{g}\left(F^{*}\right) e^{-M_{m-1}^{g}}\left(1-S_{m-1}^{g} F^{*}\right)}{\left(1-e^{-M_{m}^{g}}\left(1-S_{m}^{g} F^{*}\right)\right)} & \text { for } a=m\end{array}\right.$
where
$R_{1}\left(F^{*}\right)=\frac{\alpha B^{o, s p}\left(F^{*}\right)}{\beta+B^{o, s p}\left(F^{*}\right)}$
for a Beverton-Holt stock-recruitment relationship.
The maximum of $C\left(F^{*}\right)$ is then found by searching over $F^{*}$ to give $F_{\text {MSY }}^{*}$, with the associated female spawning biomass given by:

$$
B_{M S Y}^{\odot, s p}=\sum_{a} f_{a}^{\circ} w_{a}^{\odot} N_{a}^{\circ}\left(F_{\mathrm{MSY}}^{*}\right)
$$

## II. 3 The likelihood function

The model is fit to CPUE and survey abundance indices, commercial and survey length frequencies, survey age-length keys, as well as to the stock-recruitment curve to estimate model parameters. Contributions by each of these to the negative of the log-likelihood $(-\ell \mathrm{n} L)$ are as follows ${ }^{2}$.

## II.3.1 CPUE relative abundance data

The likelihood is calculated by assuming that the observed abundance index (here CPUE) is lognormally distributed about its expected value:
$I_{y}^{i}=\hat{I}_{y}^{i} e^{\varepsilon_{y}^{i}} \quad$ or $\quad \varepsilon_{y}^{i}=\ln \left(I_{y}^{i}\right)-\ln \left(\hat{I}_{y}^{i}\right)$
where
$I_{y}^{i} \quad$ is the abundance index for year $y$ and series $i$ (which corresponds to a specified species and fleet);
$\hat{I}_{y}^{i}=\hat{q}^{i} \hat{B}_{f y}^{e x}$ is the corresponding model estimate, where $\widehat{B}_{f y}^{e x}$ is the model estimate of exploitable resource biomass, given by equation App.II.11;
$\hat{q}^{i} \quad$ is the constant of proportionality for abundance series $I$; and
$\varepsilon_{y}^{i} \quad$ from $N\left(0,\left(\sigma_{y}^{i}\right)^{2}\right)$.
In cases where the CPUE series are based upon species-aggregated catches (as available pre-1978), the corresponding model estimate is derived by assuming two types of fishing zones: z1) an " $M$. capensis only zone", corresponding to shallow water and z2) a "mixed zone" (Figure App.II.1).

The total catch of hake of both species $(B S)$ by fleet $f$ in year $y\left(C_{B S, f y}\right)$ can be written as:
$C_{B S, f y}=C_{C, f y}^{z 1}+C_{C, f y}^{z 2}+C_{P, f y}$
where
$C_{C, f y}^{z 1} \quad$ is the $M$. capensis catch by fleet $f$ in year $y$ in the $M$. capensis only zone (z1);
$C_{C, f y}^{z 2} \quad$ is the $M$. capensis catch by fleet $f$ in year $y$ in the mixed zone (z2); and
$C_{P, f y}$ is the M. paradoxus catch by fleet $f$ in year $y$ in the mixed zone.
Catch rate is assumed to be proportional to exploitable biomass. Furthermore, let $\gamma$ be the proportion of the $M$. capensis exploitable biomass in the mixed zone $\left(\gamma=B_{C, f y}^{e x, z 2} / B_{C, f y}^{e x}\right)$ (assumed to be constant throughout the period for simplicity) and $\psi_{f y}$ be the proportion of the effort of fleet $f$ in the mixed zone in year $y$ $\left(\psi_{f y}=E_{f y}^{z 2} / E_{f y}\right)$, so that:
$C_{C, f y}^{z 1}=q_{C}^{i, z 1} B_{C f y}^{e x, z 1} E_{f y}^{z 1}=q_{C}^{i, z 1}(1-\gamma) B_{C, f y}^{e x}\left(1-\psi_{f y}\right) E_{f y}$

[^1]\[

$$
\begin{align*}
& C_{C, f y}^{z 2}=q_{C}^{i, z 2} B_{C, f y}^{e x, z 2} E_{f y}^{z 2}=q_{C}^{i, z 2} \gamma B_{C, f y}^{e x} \psi_{f y} E_{f y} \text { and }  \tag{App.II.22}\\
& C_{P, f y}=q_{P}^{i} B_{P, f y}^{e x} E_{f y}^{z 2}=q_{P}^{i} B_{P, f y}^{e x} \psi_{f y} E_{f y} \tag{App.II.23}
\end{align*}
$$
\]

where
$E_{f y}=E_{f y}^{z 1}+E_{f y}^{z 2}$ is the total effort of fleet $f$, corresponding to combined-species CPUE series $i$ which consists of the effort in the M. capensis only zone ( $E_{f y}^{z 1}$ ) and the effort in the mixed zone ( $E_{f y}^{z 2}$ );
$q_{C}^{i, z j} \quad$ is the catchability for $M$. capensis ( $C$ ) for abundance series $i$, and zone $s ;$; and
$q_{P}^{i} \quad$ is the catchability for $M$. paradoxus ( $P$ ) for abundance series $i$.
It follows that:
$C_{C, f y}=B_{C, f y}^{e x} E_{f y}\left[q_{C}^{i, z 1}(1-\gamma)\left(1-\psi_{f y}\right)+q_{C}^{i, z 2} \gamma \psi_{f y}\right]$
$C_{P, f y}=B_{P, f y}^{e x} E_{f y} q_{P}^{i} \psi_{f y}$
From solving equations App.II. 24 and App.II.25:
$s_{f y}=\frac{q_{C}^{i, z 1}(1-\gamma)}{\left\{\frac{C_{C, f y} B_{P, f y}^{e x} q_{P}^{i}}{B_{C, f y}^{e x} C_{P, f y}}-q_{C}^{i, z 2} \gamma+q_{C}^{i, z 1}(1-\gamma)\right\}}$
and:

$$
\begin{equation*}
\hat{I}_{y}^{i}=\frac{C_{f y}}{E_{f y}}=\frac{C_{f y} B_{P, f y}^{e x} q_{P}^{i} \psi_{f y}}{C_{P, f y}} \tag{App.II.27}
\end{equation*}
$$

| Zone 1 (z1): | Zone 2 (z2): |
| :---: | :---: |
| M. capensis only | Mixed zone |
| M. capensis: | M. capensis: |
| biomass $\left(B_{C}^{z 1}\right), \operatorname{catch}\left(C_{C}^{z 1}\right)$ | biomass $\left(B_{C}^{z 2}\right), \operatorname{catch}\left(C_{C}^{z 2}\right)$ |
|  | M. paradoxus: |
|  | biomass $\left(B_{P}\right), \operatorname{catch}\left(C_{P}\right)$ |
| Effort in zone 1 $\left(E^{z 1}\right)$ | Effort in zone 2 $\left(E^{z 2}\right)$ |

Figure App.II.1: Diagrammatic representation of the two conceptual fishing zones.

Two species-aggregated CPUE indices are available: the ICSEAF west coast and the ICSEAF south coast series. For consistency, $q$ 's for each species (and zone) are forced to be in the same proportion:

$$
\begin{equation*}
q_{s}^{S C}=r q_{s}^{W C} \tag{App.II.28}
\end{equation*}
$$

To correct for possible negative bias in estimates of variance $\left(\sigma_{y}^{i}\right)$ and to avoid according unrealistically high precision (and so giving inappropriately high weight) to the CPUE data, lower bounds on the standard
deviations of the residuals for the logarithm of the CPUE series have been enforced: for the historic ICSEAF CPUE series (separate west coast and south coast series) the lower bound is set to 0.25 , and to 0.15 for the recent GLM-standardised CPUE series, i.e.: $\sigma^{I C S E A F} \geq 0.25$ and $\sigma^{G L M} \geq 0.15$.

The contribution of the CPUE data to the negative of the log-likelihood function (after removal of constants) is then given by:
$-\ln L^{\text {CPUE }}=\sum_{i} \sum_{y}\left\lfloor\ln \left(\sigma_{y}^{i}\right)+\left(\varepsilon_{y}^{i}\right)^{2} / 2\left(\sigma_{y}^{i}\right)^{2}\right]$
where
$\sigma_{y}^{i} \quad$ is the standard deviation of the residuals for the logarithms of index $i$ in year $y$.
Homoscedasticity of residuals for CPUE series is customarily assumed ${ }^{3}$, so that $\sigma_{y}^{i}=\sigma^{i}$ is estimated in the fitting procedure by its maximum likelihood value:

$$
\begin{equation*}
\hat{\sigma}^{i}=\sqrt{1 / n_{i} \sum_{y}\left(\ln \left(I_{y}^{i}\right)-\ln \left(\hat{I}_{y}^{i}\right)\right)^{2}} \tag{App.II.30}
\end{equation*}
$$

where $n_{i}$ is the number of data points for abundance index $i$.
In the case of the species-disaggregated CPUE series, the catchability coefficient $q^{i}$ for abundance index $i$ is estimated by its maximum likelihood value, which in the more general case of heteroscedastic residuals is given by:
$\ln \hat{q}^{i}=\frac{\sum_{y}\left(\ln I_{y}^{i}-\ln \hat{B}_{f y}^{e x}\right) /\left(\sigma_{y}^{i}\right)^{2}}{\sum_{y} 1 /\left(\sigma_{y}^{i}\right)^{2}}$
In the case of the species-combined CPUE, $q_{C}^{W C, z 1}, q_{C}^{W C, z 2}, q_{P}^{W C}, r$ and $\gamma$ are estimated directly in the fitting procedure.

## II.3.2 Survey abundance data

Data from the research surveys are treated as relative abundance indices in a similar manner to the species-disaggregated CPUE series above, with survey selectivity function $S_{a}^{g, \text { sum/win }}$ replacing the commercial selectivity $S_{\text {fya }}^{g}$ (see equations App.II. 12 and App.II. 13 above, which also take account of the begin- or mid-year nature of the survey).

An estimate of sampling variance is available for most surveys and the associated $\sigma_{y}^{i}$ is generally taken to be given by the corresponding survey CV. However, these estimates likely fail to include all sources of variability, and unrealistically high precision (low variance and hence high weight) could hence be accorded to these indices. The contribution of the survey data to the negative log-likelihood is of the same form as that of the CPUE abundance data (see equation App.II.29). The procedure adopted takes into account an additional variance $\left(\sigma_{A}\right)^{2}$ which is treated as another estimable parameter in the minimisation process. This procedure is carried out enforcing the constraint that $\left(\sigma_{A}\right)^{2}>0$, i.e. the overall variance cannot be less than its externally input component.

[^2]In June 2003, the trawl gear on the Africana was changed and a different value for the multiplicative bias factor $q$ is taken to apply to the surveys conducted with the new gear. Calibration experiments have been conducted between the Africana with the old gear (hereafter referred to as the "old Africana") and the Nansen, and between the Africana with the new gear ("new Africana") and the Nansen, in order to provide a basis to relate the multiplicative biases of the Africana with the two types of gear ( $q_{\text {old }}$ and $q_{\text {new }}$ ). A GLM analysis assuming negative binomial distributions for the catches made (Brandão et al., 2004) provided the following estimates:
$\Delta \ell n q^{\text {capensis }}=-0.494$ with $\sigma_{\Delta \text { enq }}$ capensis $=0.141$
i.e. $\left(q^{\text {new }} / q^{\text {old }}\right)^{\text {capensis }}=0.610$ and
$\Delta \ell n q^{\text {paradoxus }}=-0.053$ with $\sigma_{\Delta \ell n q^{\text {paradoous }}}=0.117$
i.e. $\left(q^{\text {new }} / q^{\text {old }}\right)^{\text {paradoxus }}=0.948$
where
$\ell n q_{\text {new }}^{s}=\ell n q_{\text {old }}^{s}+\Delta \ell n q^{s} \quad$ with $s=$ capensis or paradoxus (App.II.32)

No plausible explanation has yet been found for the particularly large extent to which catch efficiency for $M$. capensis is estimated to have decreased for the new research survey trawl net. It was therefore recommended (BENEFIT, 2004) that the ratio of the catchability of the new to the previous Africana net be below 1 , but not as low as the ratio estimated from the calibration experiments. $\Delta l n q^{\text {capensis }}$ is therefore taken as -0.223 , i.e. $\left(q^{\text {new }} / q^{\text {old }}\right)^{\text {capensis }}=0.8$.

The following contribution is therefore added as a penalty (or a log prior in a Bayesian context) to the negative log-likelihood in the assessment:
$-\ell n L^{q-c h}=\left(\ell n q_{\text {new }}-\ell n q_{\text {old }}-\Delta \ell n q\right)^{2} / 2 \sigma_{\Delta \ell n q}^{2}$
A different length-specific selectivity is estimated for the "old Africana" and the "new Africana".
The survey's coefficients of catchability $q$ (for the survey with the old Africana gear) are constrained to values below 1 (i.e. it is assumed that the nets do not herd the hake):

$$
\begin{equation*}
\text { pen }^{q}=\sum_{i}\left(q_{\text {old }}^{i}-1\right)^{2} / 0.02^{2} \quad \text { if } \quad q_{\text {old }}^{i}>1 \tag{App.II.34}
\end{equation*}
$$

## II.3.3 Commercial proportions at length

Commercial proportions at length cannot be disaggregated by species and gender. The model is therefore fit to the proportions at length as determined for both species and gender combined.

The catches at length are computed as:
$C_{f y l}=\sum_{s} \sum_{g} \sum_{a=0}^{m} N_{s y a}^{g} F_{s f y} S_{s f y l}^{g} P_{s, a+1 / 2, l}^{g} e^{-M_{s a}^{g} / 2}\left(1-\sum_{f} S_{s f y a}^{g} F_{f y} / 2\right)$
with the predicted proportions at length:
$\hat{p}_{y l}^{i}=C_{f y l} / \sum_{l^{\prime}} C_{f y l^{\prime}}$
The contribution of the proportion at length data to the negative of the log-likelihood function when assuming an "adjusted" lognormal error distribution is given by:
$-\ell \mathrm{n} L^{\mathrm{length}}=0.1 \sum_{y} \sum_{l}\left\lfloor\ln \left(\sigma_{\text {len }}^{i} / \sqrt{p_{y l}^{i}}\right)+p_{y l}^{i}\left(\ln p_{y l}^{i}-\ln \hat{p}_{y l}^{i}\right)^{2} / 2\left(\sigma_{l e n}^{i}\right)^{2}\right\rfloor$
where
the superscript ' $\imath$ refers to a particular series of proportions at length data which reflect a specified fleet, and species (or combination thereof); and
$\sigma_{l e n}^{i} \quad$ is the standard deviation associated with the proportion at length data, which is estimated in the fitting procedure by:
$\hat{\sigma}_{l e n}^{i}=\sqrt{\sum_{y} \sum_{l} p_{y l}^{i}\left(\ln p_{y l}^{i}-\ln \hat{p}_{y l}^{i}\right)^{2} / \sum_{y} \sum_{l} 1}$
The initial 0.1 multiplicative factor is a somewhat arbitrary downweighting to allow for correlation between proportions in adjacent length groups. The coarse basis for this adjustment is the ratio of effective number of age-classes present to the number of length groups in the minimisation, under the argument that independence in variability is likely to be more closely related to the former.

Commercial proportions at length are incorporated in the likelihood function using equation App.II.37, for which the summation over length $l$ is taken from length $l_{\text {minus }}$ (considered as a minus group) to lpus (a plus group). The length for the minus- and plus-groups are fleet specific and are chosen so that typically a few percent, but no more, of the fish sampled fall into these two groups.

## II.3.4 Survey proportions at length

The survey proportions at length are incorporated into the negative of the log-likelihood in an analogous manner to the commercial catches-at-age, assuming an adjusted log-normal error distribution (equation App.II.36). In this case however, data are disaggregated by species, and for some surveys further disaggregated by gender:
$p_{s y l}^{g, s u r v}=\frac{C_{\text {syl }}^{g, s u r v}}{\sum_{l^{\prime}} C_{s y l^{\prime}}^{g, \text { surv }}}$
is the observed proportion of fish of species $s$, gender $g$ and length $l$ from survey surv in
year $y$; and
$\hat{p}_{s y l}^{g, s u r v}$ is the expected proportion of fish of species $s$, gender $g$ and length $l$ in year $y$ in the survey surv, given by:
$\hat{p}_{s y l}^{g, s u r v}=\frac{\sum_{a} S_{s l}^{g, s u m} P_{\text {sal }}^{g} N_{s y a}^{g}}{\sum_{l^{\prime}} \sum_{a} S_{s l^{\prime}}^{g, s u m} P_{\text {sal }}^{g} N_{\text {sya }}^{g}}$
(App.II.39)
for begin-year (summer) surveys, or

$$
\begin{equation*}
\hat{p}_{s y l}^{g, s u r v}=\frac{\sum_{a} S_{s l}^{g, \text { win }} P_{s, a+1 / 2, l}^{g} N_{s y a}^{g} e^{-M_{s a}^{g} / 2}\left(1-\sum_{f} S_{s f y a}^{g} F_{s f y} / 2\right)}{\sum_{l^{\prime}} \sum_{a} S_{s l^{\prime}}^{g, w i n} P_{s, a+1 / 2, l}^{g} N_{s y a}^{g} e^{-M_{s a}^{g} / 2}\left(1-\sum_{f} S_{s f y a}^{g} F_{s f y} / 2\right)} \tag{App.II.40}
\end{equation*}
$$

for mid-year (autumn, winter or spring) surveys.

## II.3.5 Age-length keys

Under the assumption that fish are sampled randomly with respect to age within each length-class, the contribution to the negative log-likelihood for the ALK data (ignoring constants) is:
$-\ln L^{A L K}=-w \sum_{i} \sum_{l} \sum_{a}\left[A_{i, l, a}^{o b s} \ln \left(\hat{A}_{i, l, a}\right)-A_{i, l, a}^{o b s} \ln \left(A_{i, l, a}^{o b s}\right)\right]$
where
$w \quad$ is a downweighting factor to allow for overdispersion in these data compared to the expectation for a multinomial distribution with independent data; this weight factor is set to 0.01 ;
$A_{i, a, l}^{\text {obs }} \quad$ is the observed number of fish of age $a$ that fall in the length class $l$, for ALK $i$ (a specific combination of survey, year, species and gender);
$\hat{A}_{i, a, l} \quad$ is the model estimate of $A_{i, a, l}^{\text {obs }}$, computed as:

$$
\begin{equation*}
\hat{A}_{i, a, l}=W_{i, l} \frac{C_{i, l} A_{a, l}}{\sum_{a^{\prime}} C_{i, l} A_{a^{\prime}, l}} \tag{App.II.42}
\end{equation*}
$$

where
$W_{i, l} \quad$ is the number of fish in length class $l$ that were aged for ALK $i$,
$A_{a^{\prime}, l}=\sum_{a} P\left(a^{\prime} \mid a\right) A_{a, l}$ is the ALK for age $a$ and length $l$ after accounting for age-reading error,
with $P\left(a^{\prime} \mid a\right)$, the age-reading error matrix, representing the probability of an animal of true age $a$ being aged to be that age or some other age $a$.

Age-reading error matrices have been computed for each reader and for each species as reported in Appendix III.

When multiple readers age the same fish, these data are considered to be independent information in the model fitting.

## II.3.6 Stock-recruitment function residuals

The stock-recruitment residuals are assumed to be log-normally distributed. Thus, the contribution of the recruitment residuals to the negative of the log-likelihood function is given by:
$-\ell n L^{S R}=\sum_{s}\left[\sum_{y=y 1}^{y 2} \zeta_{s y}^{2} / 2 \sigma_{R}^{2}+\left(\sum_{y=y 1}^{y 2} \zeta_{s y}\right)^{2} / 0.01^{2}\right]$
where
$\varsigma_{s y} \quad$ is the recruitment residual for species $s$, and year $y$, which is assumed to be log-normally distributed with standard deviation $\sigma_{R}$ and which is estimated for year $y 1$ to $y 2$ (see equation App.II.4) (estimating the stock-recruitment residuals is made possible by the availability of catch-at-age data, which give some indication of the age-structure of the population); and
$\sigma_{R} \quad$ is the standard deviation of the log-residuals, which is input.
The stock-recruitment residuals are estimated for years 1985 to 2006, with recruitment for other years being set deterministically (i.e. exactly as given by the estimated stock-recruitment curve) as there is insufficient catch-at-age information to allow reliable residual estimation for earlier years. A limit on the recent recruitment fluctuations is set by having the $\sigma_{\mathrm{R}}$ (which measures the extent of variability in recruitment - see equation -

App.II.43) decreasing linearly from 0.45 in 2004 to 0.1 in 2009, effectively forcing recruitment over the last years to lie closer to the stock-recruitment relationship curve.

The second term on the right hand side is introduced to force the average of the residuals estimated over the period from $y 1$ to $y 2$ to be close to zero, for reasons elaborated in the main text.

## II. 4 Model parameters

## II.4.1 Estimable parameters

The primary parameters estimated are the species-specific female virgin spawning biomass $\left(K_{s}^{q_{s p}}\right)$ and "steepness" of the stock-recruitment relationship $\left(h_{s}\right)$. The standard deviations $\sigma^{i}$ for the CPUE series residuals (the species-combined as well as the GLM-standardised series) as well as the additional variance $\left(\sigma_{A}^{i}\right)^{2}$ for each survey abundance series are treated as estimable parameters in the minimisation process. Similarly, in the case of the species-combined CPUE, $q_{C}^{W C, z 1}, q_{C}^{W C, z 2}, q_{P}^{W C}, r$ and $\gamma$ are directly estimated in the fitting procedure.

The species- and gender-specific von Bertalanffy growth curve parameters ( $l_{\infty}, \boldsymbol{\kappa}$ and $t_{0}$ ) are estimated directly in the model fitting process, as well as $B_{0}, \alpha$ and $\beta$, values used to compute the standard deviation of the length-at-age $a$.

The following parameters are also estimated in the model fits undertaken (if not specifically indicated as fixed):

## App.II.4.1.1 Natural mortality:

Natural mortality $\left(M_{s a}^{g}\right)$ is assumed to be age-specific and is calculated using the following functional form (the selection of the specific form here is based on convenience and is somewhat arbitrary):
$M_{s a}^{\circ}=\left\{\begin{array}{lll}M_{s 2}^{\circ} & \text { for } & a \leq 1 \\ \alpha_{s}^{M}+\frac{\beta_{s}^{M}}{a+1} & \text { for } & 2 \leq a \leq 5 \\ M_{s 5}^{\circ} & \text { for } & a>5\end{array}\right.$
and
$M_{s a}^{\text {males }}=v^{s} M_{s a}^{\text {females }}$
$M_{s 0}$ and $M_{s 1}$ are set equal to $M_{s 2}\left(=\alpha_{s}^{M}+\beta_{s}^{M} / 3\right)$ as there are no data (hake of ages younger than 2 are rare in catch and survey data) which would allow independent estimation of $M_{s 0}$ and $M_{s 1}$.

When $M$ values are estimated in the fit, a penalty is added to the total $-\operatorname{lnL}$ so that $M_{s 2} \geq M_{s 5}$ :

$$
\begin{equation*}
\operatorname{pen}^{M}=\sum_{s}\left(M_{s 5}-M_{s 2}\right)^{2} / 0.01^{2} \quad \text { if } M_{s 2}<M_{s 5} \tag{App.II.46}
\end{equation*}
$$

## II.4.1.2 Stock-recruitment residuals:

Stock-recruitment residuals $\varsigma_{s y}$ are estimable parameters in the model fitting process. They are estimated separately for each species from 1985 to the present, and set to zero pre-1985 because there are no catch-at-length data for that period to provide the information necessary to inform estimation.

Table App.II. 1 summarises the estimable parameters, excluding the selectivity parameters.

## II.4.1.3 Survey fishing selectivity-at-length:

The survey selectivities are estimated directly for seven pre-determined lengths for $M$. paradoxus and $M$. capensis. When the model was fitted to proportion-at-age rather than proportion-at-length, survey selectivities were estimated directly for each age (i.e. seven age classes). The lengths at which selectivity is estimated directly are survey specific (at constant intervals between the minus and plus groups) and are given in Table App.II.2. Between these lengths, selectivity is assumed to change linearly. The slope from lengths $l_{\text {minus }}$ to $l_{\text {minus }}+1$ is assumed to continue exponentially to lower lengths down to length 1 , and similarly the slope from lengths $l_{\text {plus }}-1$ to $l_{\text {plus }}$ for $M$. paradoxus and $M$. capensis to continue for greater lengths.

For the south coast spring and autumn surveys, gender-specific selectivities are estimated for $M$. paradoxus. Furthermore, the female selectivities are scaled down by a parameter estimated for each of these two surveys to allow for the male predominance in the survey catch.

A penalty is added to the total $-\ln \mathrm{L}$ to smooth the selectivities:
pen $^{\text {survS }}=\sum_{i} \sum_{L=L_{1}+1}^{L_{7}-1} 3\left(S_{L-1}^{i}-2 S_{L}^{i}+S_{L+1}^{i}\right)^{2}$
where $i$ is a combination of survey, species and gender.

## II.4.1.4 Commercial fishing selectivity-at-length:

The fishing selectivity-at-length (gender independent) for each species and fleet, $S_{s f l}$, is estimated in terms of a logistic curve given by:
$S_{s f l}=\left[1+\exp \left(-\left(l-l_{s f}^{c}\right) / \boldsymbol{\delta}_{s f}^{c}\right)\right]^{-1}$
where
$l_{s f}^{c} \mathrm{~cm}$ is the length-at-50\% selectivity,
$\delta_{s f}^{c} \mathrm{~cm}^{-1}$ defines the steepness of the ascending limb of the selectivity curve.
The selectivity is sometimes modified to include a decrease in selectivity at larger lengths, as follows:
$S_{s f l}=S_{s, f, l-1} e^{-s_{s f l}} \quad$ for $l>l_{s l o p e,}$,
where
$s_{s f l}$ measures the rate of decrease in selectivity with length for fish longer than $l_{\text {slope }}$ for the fleet concerned, and is referred to as the "selectivity slope"; and
$l_{\text {slope }}$ is fixed externally from the model, values for each fleet and species are given in Table App.II.2.
Periods of fixed and changing selectivity have been assumed for the offshore trawl fleet to take account of the change in the selectivity at low ages over time in the commercial catches, likely due to the phasing out of the (illegal) use of net liners to enhance catch rates.

On the south coast, for $M$. paradoxus, the female offshore trawl selectivity (only the trawl fleet is assumed to catch M. paradoxus on the south coast) is scaled down by a factor taken as the average of those estimated for the south coast spring and autumn surveys. Although there is no gender information for the commercial catches, the south coast spring and autumn surveys catch a much higher proportion of male $M$. paradoxus than female (ratios of about $7: 1$ and $3.5: 1$ for spring and autumn respectively). This is assumed to reflect a difference in distribution of the two genders which would therefore affect the commercial fleet similarly.

Details of the fishing selectivities (including the number of parameters estimated) that are used in the assessment are shown in Table App.II.4.

## II.4.2 Input parameters and other choice for application to hake

## II.4.2.1 Age-at-maturity:

The proportion of fish of species $s$, gender $g$ and length $l$ that are mature is assumed to follow a logistic curve with the parameter values given below (from Fairweather and Leslie, 2008, "stage 2, $>40 \mathrm{~cm}$ " for females and Fairweather, pers. commn for males):

|  | $l_{50}(\mathrm{~cm})$ | N |
| ---: | :---: | :---: |
| M. paradoxus: |  |  |
| Males | 28.63 | 5.07 |
| Females | 42.24 | 4.46 |
| M. capensis: |  |  |
| Males | 34.35 | 7.38 |
| Females | 40.80 | 7.51 |

Maturity-at-length is then converted to maturity-at-age as follows:
$f_{s a}^{s}=\sum_{l} f_{s l}^{s} P_{a, l}^{s}$

## II.4.2.2 Weight-at-length:

The weight-at-length for each species and gender is calculated from the mass-at-length function, with values of the parameters for this function listed below (from Fairweather, 2008, taking the average of the west and south coasts):

|  | $\perp\left(\mathrm{gm} / \mathrm{cm}^{\perp}\right)$ | $\perp$ |
| ---: | :---: | :---: |
| M. paradoxus: |  |  |
| Males | 0.007541 | 2.988 |
| Females | 0.005836 | 3.065 |
| M. capensis: |  |  |
| Males | 0.006307 | 3.061 |
| Females | 0.005786 | 3.085 |

## II.4.2.3 Minus- and plus-groups

Because of a combination of gear selectivity and mortality, a relatively small number of fish in the smallest and largest length classes are caught. In consequence, there can be relatively larger errors (in terms of variance) associated with these data. To reduce this effect, the assessment is conducted with minus- and plusgroups obtained by summing the data over the lengths below and above $l_{\text {minus }}$ and $l_{\text {pus }}$ respectively. The minusand plus-group used are given in Table App.II. 5 (and plotted in Figures.I. 2 and 3). Furthermore, the proportions at length data (both commercial and survey) are summed into 2 cm length classes for the model fitting.

Table App.II.1: Parameters estimated in the model fitting procedure, excluding selectivity parameters.

* if not fixed on input

|  | No of parameters | Parameters estimated |
| :---: | :---: | :---: |
| $K^{9}$ | 2 | $\ln \left(K_{\text {cap }}\right)$ and $\ln \left(K^{\text {Para }}\right.$ ) |
| $h$ | 2 | $h_{\text {cap }}$ and $h_{\text {para }}$ |
| $M_{a}$ | $4(6)^{*}$ | For each species: $M_{2}, M_{5}$ (and $\cup$ ) |
| Additional variance | 2 | $\sigma_{A, c a p}$ and $\sigma_{\text {Apara }}$ |
| Recruitment residuals | 50 | $\zeta_{\text {cap,1985-2009 }}$ and $\zeta_{\text {para, 1985-2009 }}$ |
| $\sigma_{\text {CPUE }}$ | 6 | 1 for each series (lower bounds imposed) |
| ICSEAF CPUE | 5 | $q_{\mathrm{C}}{ }^{W C, z I}, q_{\mathrm{C}}{ }^{\mathrm{WC}, z 2}, q_{\mathrm{P}}{ }^{\mathrm{WC}}, r \text { and } \gamma$ |
| $\theta_{a}$ | 12 | For each species and gender: $\theta_{0}, \theta_{1}$ and $\theta_{14}$ |
| Growth | 12 | For each species and gender: $L_{5}, \kappa$ and $t_{0}$ |

Table App.II.2: Lengths (in cm ) at which survey selectivity is estimated directly.

| $\begin{gathered} \text { n} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 8 \end{gathered}$ | West coast summer | 13 | 18 | 23 | 28 | 32 | 37 | 42 | 47 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | West coast winter | 13 | 18 | 24 | 29 | 35 | 40 | 46 | 51 |
|  | South coast spring | 21 | 26 | 30 | 35 | 39 | 44 | 48 | 53 |
|  | South coast autumn | 21 | 26 | 31 | 36 | 42 | 47 | 52 | 65 |
| $\begin{aligned} & \text { n } \\ & \text { む̃ } \\ & \text { S } \\ & \text { si } \end{aligned}$ | West coast summer | 13 | 20 | 26 | 33 | 39 | 46 | 52 | 59 |
|  | West coast winter | 13 | 17 | 21 | 30 | 40 | 47 | 54 | 61 |
|  | South coast spring | 13 | 19 | 28 | 38 | 46 | 54 | 63 | 71 |
|  | South coast autumn | 13 | 19 | 28 | 36 | 44 | 52 | 61 | 69 |

Table App.II.3: Length (cm) at which selectivity starts to decrease ( $l_{\text {slope }}$ ) for each species and fleet.

|  | M. paradoxus | M. capensis |
| :--- | :---: | :---: |
| WC offshore trawl | 40 | 70 |
| SC offshore trawl | 70 | 70 |
| SC inshore trawl | - | 55 |
| WC longline | 85 | 85 |
| SC longline | - | 85 |
| SC handline | - | 70 |

Table App.II.4: Details for the commercial selectivity-at-length for each fleet and species combination, as well as indications of what data are available.

|  | No of est. parameters | M. paradoxus <br> Comments | No of est. parameters | M. capensis <br> Comments | data available |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. West coast offshore $\begin{aligned} & 1917-1976 \\ & 1977-1984 \\ & 1985-1992 \\ & 1993-2009 \end{aligned}$ | 0 <br> 3 <br> 0 <br> 2 | set equal to 1989 <br> two logistic parameters estimated (same slope as 1993+) <br> linear change between 1984 and 1993 selectivity <br> two logistic + slope parameters estimated | 0 <br> 0 <br> 0 <br> 0 | set equal to 1989 <br> differential shift compared to 1993+ as for paradoxus, slope $1 / 3$ of inshore linear change between 1984 and 1993 selectivity same as SC inshore but shifted to the right by 5 cm , slope $1 / 3$ of inshore | species combined species combined species combined |
| 2. South coast offshore 1917-1976 1977-1984 1985-1992 <br> 1993-2009 | 0 <br> 3 <br> 0 <br> 3 <br> 0 | set equal to 1989 <br> differential shift compared to 1993+ as for WC (same slope as 1993+) <br> linear change between 1984 and 1993 selectivity <br> two logistic + slope parameters estimated <br> female downscaling factor (av. of SC spring and autumn surveys's factors) | 0 <br> 0 <br> 0 <br> 0 | set equal to 1989 <br> differential shift compared to 1993+ as for paradoxus, slope $1 / 3$ of inshore linear change between 1984 and 1993 selectivity same as SC inshore but shifted to the right by 10 cm , slope $1 / 3$ of inshore | species combined <br> species combined <br> species combined |
| 3. South coast inshore | - | - | 3 | two logistic + slope parameters estimated | M. capensis |
| 4. West coast longline | 3 | two logistic + slope parameters estimated | 0 | same as South Coast longline | species combined |
| 5. South coast longline | - | - | 3 | two logistic + slope parameters estimated | M. capensis |
| 6. South coast handline | - | - | 0 | average of South Coast longline and inshore |  |
| West coast summer survey <br> Africana old <br> Africana new <br> West coast winter survey <br> Africana old <br> South coast spring survey <br> Africana old Africana new <br> South coast autumn survey <br> Africana old Africana new | 7 <br> 5 <br> 7 <br> 7 <br> 5 <br> 1 <br> 7 <br> 5 <br> 1 | estimated for 7 specified lengths same slope as old estimated for 7 specified lengths estimated for 7 specified lengths same slope as old female downscaling factor estimated for 7 specified lengths same slope as old female downscaling factor | 7 <br> 5 <br> 7 <br> 7 <br> 5 <br> 7 <br> 5 | estimated for 7 specified lengths same slope as old estimated for 7 specified lengths estimated for 7 specified lengths same slope as old estimated for 7 specified lengths same slope as old | species disaggregated species disaggregated <br> species disaggregated <br> species disaggregated species disaggregated <br> species disaggregated species disaggregated |
| Total | 59 |  | 49 |  |  |

Table App.II.5: Minus- and plus-groups taken for the surveys and commercial proportion at length data.
SUR VEY DATA

|  | M. paradoxus |  | M. capensis |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Minus | Plus | Minus | Plus |
| West coast summer | 13 | 47 | 13 | 59 |
| West coast winter | 13 | 51 | 13 | 61 |
| South coast spring | 21 | 53 | 13 | 71 |
| South coast autumn | 21 | 65 | 13 | 69 |

COMMERCIAL DATA

|  | Minus | Plus |
| :--- | :---: | :---: |
| West coast offshore, species combined | 23 | 65 |
| South coast offshore, species combined | 27 | 75 |
| South coast inshore, $M$. capensis | 27 | 65 |
| West coast longline, species combined | 51 | 91 |
| South coast longline, $M$. capensis | 51 | 91 |
| Both coasts offshore, species combined | 25 | 65 |

## APPENDIX III - Age-reading error matrices for Merluccius paradoxus and M. capensis

## III. 1 Introduction

The current stock assessment models used for the South African M. paradoxus and M. capensis resources are age structured models, making use of ageing data, either in the form of catch-at-age data or more recently directly as age-length keys. Until this assessment the assumption had been made that age classes are determined without error, when in fact some level of misclassification is often to be expected. Age-reading error occurs when estimates of age based on reading hard structures such as otoliths differ from the true age of the animal concerned. There are two sources of uncertainty in the relationship between the ages obtained from reading otoliths and the true age of the animal: bias and imprecision. Ageing bias occurs when there is a systematic difference between the true age of an animal and the age assigned to it, whereas ageing imprecision occurs when age-reading errors occur at random (Punt et al., 2008).

Errors in ageing can be taken into account by supplying an ageing-error matrix (Fournier and Archibald, 1982; Richards et al., 1992; Punt et al., 2008), which defines the probability of assigning a particular age to a fish with a given true age. The method described in Punt et al. (2008) is used here to construct such matrices for the two hake species for use in these assessments.

## III. 2 Data and Method

Punt et al. (2008) model the probability of reader $i$ (of $I$ readers) assigning an animal of true age $a$ an age of $a^{\prime}, P^{i}\left(a^{\prime} \mid a\right)$, by assuming that both the ageing bias and the age-reading error standard deviation depend on the reader and the true age of the animal, and that age-reading error is normally distributed about the expected age (i.e., the expected age given any bias in age reading):
$P^{i}\left(a^{\prime} \mid a, \varphi\right) \propto \exp \left[\frac{-\left(a^{\prime}-b_{a}^{i}(\varphi)\right)^{2}}{2\left(\sigma_{a}^{i}(\varphi)\right)^{2}}\right]$
(App.III.1)
where
$b_{a}^{i} \quad$ is the expected age when reader $i$ determines the age of an animal of true age $a$;
$\sigma_{a}^{i} \quad$ is the standard deviation for reader $i$ of the age-reading error for animals of true age $a$; and
$\varphi \quad$ is the vector of parameters that determines the age-reading error matrix.
The ageing bias is modelled by:
$b_{a}=\left\{\begin{array}{cc}b_{L}+\left(b_{H}-b_{L}\right) \frac{1-e^{-\lambda(a-L)}}{1-e^{-\lambda(H-L)}} & \text { if } \lambda \neq 0 \\ b_{L}+\left(b_{H}-b_{L}\right) \frac{a-L}{H-L} & \text { if } \lambda=0\end{array}\right.$
where
$b_{L} \quad$ is the expected age of animal of prespecified minimum age $L$;
$b_{H} \quad$ is the expected age of animal of prespecified maximum age $H$; and
$\lambda \quad$ determines the extent of nonlinearity between the true age and the expected age (note that $\lambda=0$ reflects the special case of linear dependence).

The age-reading error standard deviation is modelled by:
$\sigma_{a}=\left\{\begin{array}{cc}\sigma_{L}+\left(\sigma_{H}-\sigma_{L}\right) \frac{1-e^{-\alpha(a-L)}}{1-e^{-\alpha(H-L)}} & \text { if } \alpha \neq 0 \\ \sigma_{L}+\left(\sigma_{H}-\sigma_{L}\right) \frac{a-L}{H-L} & \text { if } \alpha=0\end{array}\right.$
where
$\sigma_{L} \quad$ is the age-reading error standard deviation for a prespecified minimum age $L$;
$\sigma_{H} \quad$ is the age-reading error standard deviation for a prespecified maximum age $H$; and
$\alpha \quad$ determines the extent of nonlinearity between age and the age-reading error standard deviation (note that $\alpha=0 \quad$ reflects the special case of linear dependence).

The values for the parameters that determine the age-reading error matrix for each reader are estimated by maximizing the following likelihood function:
$L(A \mid \beta, \varphi)=\prod_{j=1}^{J} \sum_{a=L}^{H} \beta_{a} \prod_{i=1}^{I} P^{i}\left(a_{i, j} \mid a, \varphi\right)$
where
$a_{i, j} \quad$ is the age assigned by reader $i$ to the $j$ th otolith;
$A \quad$ is the entire data set of otolith readings; and
$\beta_{a} \quad$ are nuisance parameters that can be interpreted as the relative frequency of animals of (true) age $a$ in the sample (rather than in the population from which the sample was taken).

In general, not all otoliths are read by all readers. Therefore, the likelihood function is more generally the product of eqn (App.III.4) over sets of otoliths that were all read by the same group of readers, and a separate set of $\beta$ 's is estimated for each such set of otoliths.

For this hake case the ageing error matrices were computed for each species separately. The data were aggregated over sex and over all sources of data (survey, commercial offshore and commercial longline). For each species, the data were divided into three groups of three readers:
a) Alexia Daniels (AD), Luke Bester (LB) and 'Unknown Reader' (UR);
b) Kevin Gradie (KG), John Prinsloo (JP) and Andy Payne/Dave Japp (AP/DJ) (these two readers have been aggregated as they read otoliths only when KG and JP did not agree, so that relatively very few data are involved); and
c) Phoeby Mullins (PM), Teressa Akkers (TA) and Kashif Booley (KB).

Table App.III. 1 give details on the data available for each group.

## III. 3 Results and Discussion

In each group of three readers, one reader at least was assumed to be unbiased, as the age-reading errors would be confounded otherwise (Punt et al., 2008). For each group of three readers, Akaike's information criterion (AIC) was used to select among alternative models (including which reader should be assumed to be unbiased).

The final models for age-reading error are summarised in Table App.III.2, while Figures App.III. 1 and App.III. 2 show plots of the age-reading error matrices for each reader and species. Figure App.III. 3 plots the
estimated ageing bias for each reader within each set of three readers. The fact that in some instances there is a bias for the true age zero is related to the use of a minus group.

Table App.III.1: Number of aged hake by species for each reader.

|  |  | M. capensis |  |  | M. paradoxus |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Year | UR | AD | LB | UR | AD | LB |
|  |  |  |  |  | 324 |  | 324 |
| WC summer | 1999 | 314 | 351 | 358 | 263 | 299 | 299 |
| WC summer | 2006 |  |  |  |  | 465 | 465 |
| WC summer | 2007 |  | 369 | 369 |  | 554 | 554 |
| WC summer | 2008 |  | 451 | 451 |  | 409 | 409 |
| WC winter | 2004 |  | 808 | 808 |  |  |  |
| SC spring | 2006 |  |  |  |  | 243 | 243 |
| SC autumn | 1999 |  | 265 | 264 |  | 139 | 139 |
| SC autumn | 2005 |  |  |  |  | 192 | 192 |
| SC autumn | 2007 |  | 626 | 626 |  | 358 | 358 |
| SC autumn | 2008 |  | 638 | 638 |  | 214 | 214 |


|  |  | M. capensis |  |  | M. paradoxus |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Year | KG | JP | AP/DJ | KG | JP | AP/DJ |
| WC summer | 1992 | 389 | 389 | 33 | 310 | 310 | 44 |
| WC summer | 1993 | 351 | 351 | 62 | 311 | 311 | 49 |
| WC summer | 1994 | 282 | 282 | 6 | 290 | 290 | 4 |
| WC summer | 1995 | 0 | 368 |  | 0 | 303 | 0 |
| SC autumn | 1992 | 329 | 329 | 91 | 40 | 40 | 5 |
| SC autumn | 1993 | 407 | 407 | 40 | 95 | 95 | 23 |
| SC autumn | 1994 | 390 | 391 | 83 | 72 | 69 | 27 |
| Comm Offshore | 1992 | 260 | 260 | 28 | 521 | 521 | 46 |
| Comm Offshore | 1993 | 115 | 115 | 17 | 645 | 645 | 75 |
| Comm Offshore | 1994 | 126 | 126 | 5 | 330 | 330 | 38 |
| Comm Longline | 1994 |  |  |  | 314 | 314 | 9 |


|  |  | M. capensis |  |  | M. paradoxus |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Year | PM | TA | KB | PM | TA | KB |
| WC autumn | 1999 | 408 | 406 | 400 | 140 | 140 | 140 |

Table App.III.2: Selected model for age-reading error for each reader and species.

|  | M. paradoxus <br> bias |  | M. capensis |  |
| :---: | :---: | :---: | :---: | :---: |
|  | precision | bias | precision |  |
| AD | Eqn App.8.III.2 | Eqn App.8.III.3 | Eqn App.8.III.2 | Eqn App.8.III.3 |
| LB | Eqn App.8.III.2 | Eqn App.8.III.3 | Eqn App.8.III.2 | Eqn App.8.III.3 |
| UR | Unbiased | Eqn App.8.III.3 | Unbiased | Eqn App.8.III.3 |
| KG | Eqn App.8.III.2 | Eqn App.8.III.3 | Eqn App.8.III.2 | Eqn App.8.III.3 |
| JP | Eqn App.8.III.2 | Eqn App.8.III.3 | Eqn App.8.III.2 | Eqn App.8.III.3 |
| AP/DJ | Unbiased | Eqn App.8.III.3 | Unbiased | Eqn App.8.III.3 |
| PM | Linear | Eqn App.8.III.3 | Eqn App.8.III.2 | Eqn App.8.III.3 |
| TA | Unbiased | Eqn App.8.III.3* | Unbiased | Eqn App.8.III.3 |
| KB | As PM | As PM | As PM | Eqn App.8.III.3* |

* Eqn App.III. 3 pertains to the coefficient of variation rather than the standard deviation.


Figure App.III.1: Plots of the ageing-error matrices ('true' vs. expected age - the area of the bubble represents the proportion expected at each age) for $M$. paradoxus and M. capensis, for the two current readers (AD and LB) and the 'unknown reader'.


Figure App.III.2: Plots of the ageing-error matrices ('true' vs. expected age - the area of the bubble represents the proportion expected at each age) for $M$. paradoxus and $M$. capensis for past readers (KG, JP, AP/DJ, PM and $\mathrm{KB})$.


Figure App.III.3: Plots of the true vs. mean expected age across readers for M. paradoxus and M. capensis.


[^0]:    ${ }^{1}$ In the interests of less cumbersome notation, subscripts have been separated by commas only when this is necessary for clarity.

[^1]:    ${ }^{2}$ Strictly it is a penalised log-likelihood which is maximised in the fitting process, as some contributions that would correspond to priors in a Bayesian estimation process are added.

[^2]:    ${ }^{3}$ There are insufficient data in any series to enable this to be tested with meaningful power.

