# An initial attempt at a spatially structured stock assessment for the South African hake resource including explicit movement 

Rebecca A. Rademeyer and Doug S. Butterworth

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## INTRODUCTION

This document presents a first attempt at a spatially structured model for the South African hake (the model equations are set out in Appendix 1). It is currently assumed that $M$. paradoxus and M. capensis both comprise a single stock across the South African west and south coasts (with at least one of these species possibly extending to Namibia). From catch-at-length information, it is however clear that the fish are not distributed evenly in terms of age/length in different areas. M. capensis is usually found in depths of less than 400 m , with the largest biomass in the 100-200 m depth range, while the distribution of $M$. paradoxus is mainly between 150 m and 500 m . There is also a tendency for hake to move offshore into deeper water as they grow older. Age information from the surveys furthermore suggests that young (up to age 3) M. capensis are primarily restricted to the west coast. At intermediate ages, a large proportion of these fish move to the south coast. However, for the oldest fish (ages $6+$ ), there is some movement back to the west coast. Similarly for $M$. paradoxus, the smaller fish tend to be found more on the west coast.

The current assessment of the resource treats the each hake species as homogeneously distributed throughout the whole region, and reflects the different age/length structure on the west and south coasts by assuming different fishing selectivities-at-age for the commercial and survey fleets (so selectivity here combines both gear and availability effects). In the spatial model developed here however, the survey and commercial fishing selectivities-at-age are taken to be the same across all regions. The regional differences in the age/length distributions of the catches are therefore explained instead by the different proportions of each age class in each region.

The hake are modelled to move across regions through the use of movement matrices, which reflect the probability that a fish of a particular age in region $r^{\prime}$ at the start of the year, moves to region $r$ at the end of the year. These matrices are estimated for three age groups (rather than estimate a different matrix for each age class): a) ages 0-1, b) ages 2-4 and c) ages 5 and above, and are assumed to be constant over time (but variability in the form of random effects could also be included in the future if the data support their estimation).

Recruitment is a function of the total spawning biomass and is then distributed amongst regions on the basis of the movement matrix that redistributes the hake of ages 0 and 1 at the end of the year.

The model presented here comprises nine regions:

1. west coast, 0-100m;
2. south coast, 0-50m;
3. west coast, 101-200m;
4. south coast, $51-100 \mathrm{~m}$;
5. west coast, 201-300m;
6. south coast, 101-200m; and
7. west coast, $301-400 \mathrm{~m}$;
8. south coast, $200 \mathrm{~m}+$.
9. west coast, $401 \mathrm{~m}+$;

These nine regions follow the present survey area stratifications, so that all the survey data are readily available in this form. The commercial information is not currently available disaggregated into these nine regions, so that some crude assumptions have been made (not documented here as it is not the purpose of this document to present a description of every detail).

## Results

Very initial results are given below.

## REFERENCE

Rademeyer RA. 2011. Current methodology for assessing the South African hake resource: A gender-disaggregated assessment fitting directly to age-length keys. MARAM IWS/DEC11/H/MODEL/BG1.

Table 1: Model results

|  | M. paradoxus | M. capensis |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $K^{s p}$ | 2037.8 | 702.0 |  |  |
| $h$ (Ricker) | 1.4416 | 0.4511 |  |  |
| $B^{s p}{ }_{2011}$ | 466.8 | 432.2 |  |  |
| $B^{s p}{ }_{2011} / K^{s p}$ | 0.23 | 0.62 |  |  |
| $\mathrm{M}_{0}$ | 0.39 | 1.10 |  |  |
| $\mathrm{M}_{7+}$ | 0.23 | 0.50 |  |  |
|  | M. paradoxus |  | M. capensis |  |
| Survey $q^{\prime}$ 's | WC summer | WC winter | WC summer | WC winter |
| 0-100m | 0.000 | 0.000 | 1.827 | 2.245 |
| $101-200 \mathrm{~m}$ | 6.121 | 1.213 | 1.930 | 1.682 |
| $201-300 \mathrm{~m}$ | 3.440 | 1.383 | 1.002 | 1.728 |
| $301-400 \mathrm{~m}$ | 1.437 | 2.421 | 0.137 | 0.494 |
| 401m+ | 0.667 | 1.055 | 0.000 | 0.000 |
|  | SC autumn | SC spring | SC autumn | SC spring |
| 0-50m | - | - | 0.206 | 0.141 |
| $51-100 \mathrm{~m}$ | - | - | 2.239 | 1.407 |
| 101-200m | 0.180 | 0.029 | 4.033 | 4.108 |
| 201m+ | 1.197 | 0.950 | 0.358 | 1.599 |

Table 2: Average proportion, for three age groups, in each region, pre-exploitation and currently.


| Age |  | M. capensis |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Pre-exploitation |  |  | Current |  |  |
|  |  | 0-1 | 2-4 | 5+ | 0-1 | 2-4 | 5+ |
|  | 0-100m | 0.29 | 0.13 | 0.02 | 0.32 | 0.12 | 0.00 |
| N | $101-200 \mathrm{~m}$ | 0.14 | 0.20 | 0.03 | 0.18 | 0.36 | 0.01 |
| $\bigcirc$ | 201-300m | 0.01 | 0.17 | 0.20 | 0.01 | 0.12 | 0.04 |
| 3 | 301-400m | 0.00 | 0.01 | 0.18 | 0.00 | 0.00 | 0.06 |
|  | $401 \mathrm{~m}+$ | 0.01 | 0.01 | 0.29 | 0.00 | 0.01 | 0.78 |
| $\pm$ | 0-50m | 0.30 | 0.15 | 0.07 | 0.31 | 0.05 | 0.02 |
| 8 | 51-100m | 0.19 | 0.20 | 0.03 | 0.13 | 0.22 | 0.02 |
| $\stackrel{5}{ \pm}$ | $101-200 \mathrm{~m}$ | 0.06 | 0.12 | 0.12 | 0.03 | 0.12 | 0.03 |
| ¢ | 201m+ | 0.00 | 0.01 | 0.07 | 0.00 | 0.00 | 0.04 |

Table 3: Movement matrices estimated.

## M. paradoxus

| Ages 0-1 | into |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\left\lvert\, \begin{gathered} 0- \\ 100 \mathrm{~m} \end{gathered}\right.$ | $\begin{gathered} 101- \\ 200 \mathrm{~m} \end{gathered}$ | $\begin{aligned} & 201- \\ & 300 \mathrm{~m} \end{aligned}$ | $\begin{aligned} & 301- \\ & 400 \mathrm{~m} \end{aligned}$ | $401 \mathrm{~m}$ | 0-50m | $\begin{array}{r} 51- \\ 100 \mathrm{~m} \end{array}$ | $\begin{aligned} & 101- \\ & 200 \mathrm{~m} \end{aligned}$ | $201 \mathrm{~m}$ |
| 0-100m | 0.96 | 0.00 | 0.00 | 0.03 | 0.01 | - | - | 0.00 | 0.00 |
| 101-200m | 0.00 | 0.56 | 0.42 | 0.00 | 0.01 | - | - | 0.00 | 0.00 |
| 201-300m | 0.00 | 0.00 | 0.97 | 0.02 | 0.01 | - | - | 0.00 | 0.00 |
| $301-400 \mathrm{~m}$ | 0.00 | 0.00 | 0.00 | 0.79 | 0.01 | - | - | 0.00 | 0.20 |
| H $401 \mathrm{~m}+$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | - | - | 0.13 | 0.86 |
| 0-50m | - | - | - | - | - | - | - | - | - |
| 51-100m | - | - | - | - | - | - | - | - | - |
| 101-200m | 0.92 | 0.06 | 0.00 | 0.00 | 0.01 | - | - | 0.00 | 0.00 |
| $201 \mathrm{~m}+$ | 0.75 | 0.00 | 0.24 | 0.00 | 0.01 | - | - | 0.00 | 0.00 |

## M. capensis

| Ages 0-1 | into |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\left\lvert\, \begin{gathered} 0- \\ 100 \mathrm{~m} \end{gathered}\right.$ | $\begin{gathered} 101- \\ 200 \mathrm{~m} \end{gathered}$ | $\begin{aligned} & 201- \\ & 300 \mathrm{~m} \end{aligned}$ | $\begin{aligned} & 301- \\ & 400 \mathrm{~m} \end{aligned}$ | $401 \mathrm{~m}$ | 0-50m | $\begin{gathered} 51- \\ 100 \mathrm{~m} \end{gathered}$ | $\begin{gathered} 101- \\ 200 \mathrm{~m} \end{gathered}$ | $\underset{+}{201 \mathrm{~m}}$ |
| 0-100m | 0.34 | 0.30 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 | 0.00 | 0.00 |
| 101-200m | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.71 | 0.28 | 0.00 | 0.00 |
| 201-300m | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.49 | 0.49 | 0.01 | 0.00 |
| $301-400 \mathrm{~m}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.33 | 0.33 | 0.33 | 0.00 |
| $401 \mathrm{~m}+$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.48 | 0.04 | 0.00 |
| $0-50 \mathrm{~m}$ | 0.41 | 0.20 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 | 0.00 | 0.00 |
| $51-100 \mathrm{~m}$ | 0.50 | 0.31 | 0.02 | 0.00 | 0.01 | 0.00 | 0.16 | 0.00 | 0.00 |
| 101-200m | 0.52 | 0.34 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.12 | 0.00 |
| $201 \mathrm{~m}+$ | 0.87 | 0.08 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.03 |

## M. paradoxus



## M. capensis

| Ages 2-4 | into |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} 0- \\ 100 \mathrm{~m} \end{gathered}$ | $\begin{aligned} & 101- \\ & 200 \mathrm{~m} \end{aligned}$ | $\begin{aligned} & 201- \\ & 300 \mathrm{~m} \end{aligned}$ | $\begin{array}{r} 301- \\ 400 \mathrm{~m} \\ \hline \end{array}$ | $\begin{gathered} 401 \mathrm{~m} \\ + \end{gathered}$ | 0-50m | $\begin{gathered} 51- \\ 100 \mathrm{~m} \end{gathered}$ | $\begin{aligned} & 101- \\ & 200 \mathrm{~m} \end{aligned}$ | $\begin{gathered} \text { 201m } \\ + \end{gathered}$ |
| 0-100m | 0.34 | 0.30 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 | 0.00 | 0.00 |
| 101-200m | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.71 | 0.28 | 0.00 | 0.00 |
| 201-300m | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.49 | 0.49 | 0.01 | 0.00 |
| $301-400 \mathrm{~m}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.33 | 0.33 | 0.33 | 0.00 |
| $\rightleftarrows \quad 401 \mathrm{~m}+$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.48 | 0.04 | 0.00 |
| $0-50 \mathrm{~m}$ | 0.41 | 0.20 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 | 0.00 | 0.00 |
| 51-100m | 0.50 | 0.31 | 0.02 | 0.00 | 0.01 | 0.00 | 0.16 | 0.00 | 0.00 |
| 101-200m | 0.52 | 0.34 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.12 | 0.00 |
| $201 \mathrm{~m}+$ | 0.87 | 0.08 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.03 |

M. paradoxus

M. capensis

Ages 5+ into

|  | $\begin{gathered} 0- \\ 100 \mathrm{~m} \end{gathered}$ | $\begin{aligned} & 101- \\ & 200 \mathrm{~m} \end{aligned}$ | $\begin{aligned} & 201- \\ & 300 \mathrm{~m} \end{aligned}$ | $\begin{aligned} & 301- \\ & 400 \mathrm{~m} \end{aligned}$ | 401 m | 0-50m | $\begin{gathered} 51- \\ 100 \mathrm{~m} \end{gathered}$ | $\begin{aligned} & 101- \\ & 200 \mathrm{~m} \end{aligned}$ | 201m+ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0-100m | 0.00 | 0.00 | 0.99 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| 101-200m | 0.00 | 0.00 | 0.31 | 0.01 | 0.01 | 0.65 | 0.00 | 0.00 | 0.02 |
| 201-300m | 0.00 | 0.00 | 0.07 | 0.48 | 0.01 | 0.00 | 0.00 | 0.00 | 0.44 |
| 301-400m | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $401 \mathrm{~m}+$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.97 | 0.00 | 0.02 | 0.00 | 0.01 |
| 0-50m | 0.00 | 0.00 | 0.00 | 0.22 | 0.01 | 0.00 | 0.00 | 0.77 | 0.00 |
| 51-100m | 0.00 | 0.00 | 0.26 | 0.08 | 0.00 | 0.00 | 0.22 | 0.32 | 0.11 |
| 101-200m | 0.00 | 0.00 | 0.19 | 0.80 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| $201 \mathrm{~m}+$ | 0.14 | 0.27 | 0.00 | 0.00 | 0.59 | 0.00 | 0.00 | 0.0 | 0.00 |



Figure 1: Catches ('000t) assumed for each species, fleet and regions.


Figure 2: Estimated spawning biomass for each species and region.


Figure 3: Commercial and survey selectivity-at-length and at age estimated for each species.


Figure 4: Fit of the model to the CPUE data


Figure 5: Fit of the model to the survey biomass estmates, for each survey and region. M. capensis in red and M. paradoxus in black.


Figure 6: Fit to the commercial CAA data (left-hand plots) and CAL data (right-hand plots), as average over all the years with data available.
M. paradoxus


Figure 7: Fit to the west coast summer survey CAL data for the west coast summer.

## Appendix A - the model

## Population Dynamics

$r$ : an index for region, $r=1, \ldots, n_{\text {region }}$ (here $n_{\text {region }}=9$ )
$y$ : an index for year
$a$ : an index for age, $a=1, \ldots, m$ ( $m=15$, a plus group)
$f$ : an index for fleet, $f=1, \ldots n_{\text {fleet }}\left(n_{\text {fleet }}=4\right)$
The equations below apply to each hake species, with different parameter values by species. The species indices have been omitted to avoid clutter.

## Numbers-at-age:

We use Pope's approximation

$$
\begin{align*}
& N_{y+1,0}^{r}=\left(\frac{1}{n_{\text {region }}} \sum_{r^{\prime}}^{n_{\text {region }}} X_{y+1,0}^{\text {out } r^{\prime} \text { in } r}\right) R_{y+1}  \tag{1}\\
& N_{y+1, a+1}^{r}=\sum_{r^{\prime}}^{n_{\text {rater }}}\left(\left[\left\{N_{y, a}^{r^{\prime}} e^{-M_{a} / 2}-\sum_{f}^{n_{n a t a}} C_{f, y, a}^{r^{\prime}}\right\} e^{-M_{a} / 2}\right] X_{y, a}^{\text {out } r^{\prime} \text { in } r}\right) \quad \text { for } 0 \leq a \leq m-2  \tag{2}\\
& N_{y+1, m}^{r}=\sum_{r^{\prime}}^{n_{\text {maim }}}\left(\left[\left\{N_{y, m-1}^{r^{\prime}} e^{-M_{m-1} / 2}-\sum_{f}^{n_{n-m}} C_{f, y, m-1}^{r^{\prime}}\right\} e^{-M_{m-1} / 2}\right] X_{y, m-1}^{\mathrm{out}, \dot{\text { in }} r}\right) \\
& +\sum_{r^{\prime}}^{n_{\text {moin }}}\left(\left[\left\{N_{y, m}^{r^{\prime}, s p, s} e^{-M_{m} / 2}-\sum_{f}^{n_{\text {mam }}} C_{f, y, m}^{r^{\prime}}\right\} e^{-M_{m} / 2}\right] X_{y, m}^{\text {out } r^{\prime} \text { in } r}\right) \tag{3}
\end{align*}
$$

i.e. 1) recruit, 2) die of natural causes in first half of the year, 3) catch taken as pulse in the middle of the year, 4) second half of the natural mortality, 5 ) move.
$N_{y, a}^{r} \quad$ : the number of fish of age $a$ at the start of year $y$ in region $r$,
$m \quad$ : the maximum age considered (taken to be a plus-group),
$C_{f, y, a}^{r}$ : the number of fish of species $s p p$, gender $s$ and age $a$ caught in year $y$ and region $r$ by fleet $f$,
$X_{y, a}^{r^{\prime}, r} \quad$ : the probability that a fish of age $a$ in region $r^{\prime}$ at the start of year $y$ moves to region $r$ at the end of that year ( $X_{y, a}^{r, r}$ is the probability that a fish stays in region $r$ ).

For the moment, $X_{y, a}^{r, r^{\prime}}=X_{a}^{r, r^{\prime}}$, i.e the movement is the same across the years, but variability in the form of random effects could be included. Furthermore, at this stage, movement is estimated for three age groups: a) ages $0-1, b$ ) ages $2-4$ and c) ages 5 and above. Some form of relationship (with random effects), such as forcing older fish offshore, could be included. The movement is not density dependent for now.
$M_{a}$ : the natural mortality on fish of age $a$ (assumed to be region independent)

$$
M_{a}=\left\{\begin{array}{lll}
M_{2} & \text { for } & a \leq 1  \tag{4}\\
\alpha^{M}+\frac{\beta^{M}}{a+1} & \text { for } & 2 \leq a \leq 5 \\
M_{5} & \text { for } & a>5
\end{array}\right.
$$

Recruitment:
$R_{y}=f\left(S S B_{y}\right)$
the recruitment (number of 0 -year-old fish) at the start of year $y$, which is a function of the total spawning biomass ( $S S B_{y}$ ):

$$
\begin{equation*}
R_{y}=\frac{4 h R_{0} S S B_{y}}{K^{s p}(1-h)+(5 h-1) S S B_{y}} e^{\left(\varsigma_{y}-\sigma_{R}^{2} / 2\right)} \tag{6}
\end{equation*}
$$

for the Beverton-Holt stock-recruitment relationship and

$$
\begin{equation*}
R_{y}=\alpha S S B_{y} \exp \left(-\beta\left(S S B_{y}\right)^{\gamma}\right) e^{\left(\varsigma_{y}-\sigma_{R}^{2} / 2\right)} \tag{7}
\end{equation*}
$$

with
$\alpha=R_{0} \exp \left(\beta\left(S S B_{y}\right)^{\gamma}\right) \quad$ and $\quad \beta=\frac{\ln (5 h)}{\left(S S B_{y}\right)^{\gamma}\left(1-5^{-\gamma}\right)}$
for the modified Ricker relationship (for the true Ricker, $\gamma=1$ )
$R_{0}=S S B_{y} /\left[\sum_{a=1}^{m-1} m a t_{a} w_{a} e^{-\sum_{a^{\prime}=0}^{a-1} M_{a^{\prime}}}+\right.$ mat $\left._{m} w_{m} \frac{e^{-\sum^{m-1} M_{a^{\prime}}}}{1-e^{-M_{m}}}\right]$

## Spawning biomass:

$S S B_{y}=\sum_{r=1}^{n_{\text {region }}} \sum_{a=1}^{m} m a t_{a} w_{a} N_{y, a}^{r}$
$w_{a}$ : the begin-year mass of fish of age $a$
$m a t_{a}$ : the proportion of fish of age $a$ that are mature (converted from maturity-at-length).

## Initial distribution amongst regions

Start 20 years before 1917 to settle distribution by 1917.

## Catch:

The fleet-disaggregated catch by mass in year $y$ and region $r$ is given by:

$$
\begin{align*}
& C_{f, y}^{r}=\sum_{a} w_{a+1 / 2} C_{f, y, a}^{r}  \tag{14}\\
& C_{f, y, a}^{r}=N_{y, a}^{r} e^{-M_{a} / 2} \tilde{S}_{f, y, a} F_{f, y}^{r} \tag{15}
\end{align*}
$$

$\tilde{S}_{f, y, a}$ is the commercial selectivity (not region specific) at age $a$ for year $y$ and fleet $f$; when $V_{f, y, a}=1$, the age-class $a$ is said to be fully selected.
$F_{f, y}^{r}$ : the fished proportion of a fully selected age class for fleet $f$ in year $y$ and region $r$ and
$\tilde{S}_{f, y, a}=\tilde{w}_{f, y, a+1 / 2} / w_{a+1 / 2}$
$\tilde{S}_{f y a} \quad$ is the effective commercial selectivity at age $a$ for fleet $f$ and year $y$; with
$\tilde{w}_{f y, a+1 / 2}=\sum_{l} S_{f y l} w_{l} P_{a+1 / 2, l}$
$\widetilde{w}_{f y, a+1 / 2}$ is the selectivity-weighted mid-year weight-at-age $a$ for fleet $f$ and year $y$;
$w_{l} \quad$ is the weight of fish of length $/$;
$w_{a+1 / 2}$ is the mid-year weight of fish of age $a$, at median length for that age;
$S_{f y l}$ is the commercial selectivity of gender $g$ at length / for year $y$, and fleet $f$;
$P_{a+1 / 2, l}$ is the mid-year proportion of fish of age $a$ that fall in the length group $/$ (i.e., $\sum_{l} P_{a+1 / 2, l}=1$ for all ages $a$ ).

The matrix $P$ is calculated under the assumption that length-at-age is log-normally distributed about a mean given by the von Bertalanffy equation, i.e.:
$\ln l_{a} \sim N\left[\ln \left(l_{\infty}\left(1-e^{-\kappa\left(a-t_{0}\right)}\right)\right) ;\left(\frac{\theta_{a}}{l_{\infty}\left(1-e^{-\kappa\left(a-t_{0}\right)}\right)}\right)^{2}\right]$
where $\theta_{a}$ is the standard deviation of length-at-age $a$ :
$\theta_{a}=\beta l_{a}$
with $\beta$ estimated in the model fitting procedure.

## The likelihood function

The model inputs past catch estimates by species and is fit to

1) region- and species-specific GLM-CPUE
2) historical CPUE (species aggregated but disaggregated over some regions (change over time?))
3) survey abundance indices (region and species)
4) commercial catch-at-age data (species aggregated and aggregated over some regions)
5) commercial catch-at-age length (species aggregated and aggregated over some regions)
6) survey catch-at-length data (region and species specific)
7) stock-recruitment curve
8) (in the future to ALKs as well)

The contributions by each of these to the negative of the log-likelihood are as in the current Reference Case (see Rademeyer, 2011).

