# Assessment of the South African anchovy resource using data from 1984 - 2010: posterior distributions for the two base case hypotheses 

C.L. de Moor* and D.S. Butterworth<br>Correspondence email: carryn.demoor@uct.ac.za


#### Abstract

The operating model (OM) for the South African anchovy resource has been updated from that used to develop OMP-08 given four more years of data and a revised time series of commercial catch. The OM with results at the posterior mode has already been presented (de Moor and Butterworth 2011). The posterior distributions for the two base case hypotheses are similar in many respects except for parameters relating to the stock recruitment relationship.


## Introduction

The operating model of the South African anchovy resource has recently been updated to be used in developing and simulation testing OMP-12. The full model description is given in Appendix A of de Moor and Butterworth (2011). Two base case hypotheses have been chosen, one assuming a constant adult natural mortality over time, $\mathrm{A}_{\text {cstM }}$, and the other allowing for random effects about annual adult natural mortality, $\mathrm{A}_{\mathrm{HS}}$. de Moor and Butterworth (2011) present results at the posterior mode for these two base case hypotheses as well as for a range of robustness tests. In this document the posterior distributions for these two base case hypotheses are presented.

## Bayesian Estimation

The objective function consisting of the negative log likelihood (equation (A.7) of de Moor and Butterworth 2011) added to the negative of the log prior distributions was minimised using AD Model Builder (Otter Research Ltd. 2000) to fit the model to the observed data and estimate the parameters at the posterior mode. A glossary defining all parameters is given in Appendix C of de Moor and Butterworth (2011) and the prior distributions utilised are listed in Table 1. The posterior probability distributions were estimated using Markov Chain Monte Carlo (Gelman et al. 1995) in AD Model Builder. The length of the chain, the thinning and burnin applied are given in Table 1. Results presented in this document are based on a random sample of 5000 from the remaining chain. Convergence of the chains was tested using the BOA (Bayesian Output Analysis) package (Smith 2003).

## Results and Discussion

The posterior medians, means and CVs of key model parameters and outputs for $\mathrm{A}_{\mathrm{HS}}$ and $\mathrm{A}_{\text {cstM }}$ are given in Table 2. The posterior distributions of key model parameters and outputs are plotted in Figure 1 and the annual biomass posterior distributions are plotted in Figure 2.

[^0]The posterior distribution of the maximum median recruitment, and consequently that of carrying capacity, under $\mathrm{A}_{\text {cstM }}$ is centered on a lower number than under $\mathrm{A}_{\mathrm{HS}}$. The variance about the stock recruitment curve is estimated to be slightly larger under $\mathrm{A}_{\mathrm{cstM}}$ compared to $\mathrm{A}_{\mathrm{HS}}$.

The main difference between the biomass distributions for the two base case hypotheses are during the peak years of 2001 to 2003. This can also be seen in Figure 3 which plots the median and $95 \%$ posterior interval over time. This 2001 - 2003 period corresponds with a dramatic change from low to high adult mortality under $A_{H S}$, compared to a constant value over all years for $A_{c s t M}$ (Figure 4). A difference between $A_{H S}$ and $A_{c s t M}$ is also evident in the numbers at age 0 in November each year, with a much higher peak in the late 1990s to early 2000 s in $\mathrm{A}_{\mathrm{HS}}$ compared to $\mathrm{A}_{\text {cstM }}$ (Figure 5). As was evident at the posterior mode, the loss to predation during the past decade as estimated by $\mathrm{A}_{\mathrm{HS}}$ is much higher than that estimated by $\mathrm{A}_{\mathrm{cst}}$ (Figure 6).

## References

de Moor, C.L., and Butterworth, D.S. 2011. Assessment of the South African anchovy resource using data from 1984-2010: results at the posterior mode. Department of Agriculture, Forestry and Fisheries Document FISHERIES/2011/SWG-PEL/66/ 29pp.
Gelman, A., Carlin, J.B., Stern, H.S. \& Rubin, D.B. 1995. Bayesian Data Analysis. Chapman \& Hall. 552pp.
Otter Research Ltd. 2000. An Introduction to AD Model Builder Version 4: For Use in Nonlinear Modeling and Statistics. Otter Research Ltd. (http://www.otter-rsch.com/)

Smith, B.J. 2003. Bayesian Output Analysis Program (BOA) Version 1.0 User's Manual.

Table 1. A list of the model parameters and their prior distributions (see Appendix A of de Moor and Butterworth (2011) for further details). Where population numbers are concerned, the units are billions.

| Parameter | Description |
| :--- | :--- |
| $\ln \left(k_{N}^{A}\right) \sim U[-100,0.7]$ | Log of the constant of proportionality associated with the acoustic survey <br> estimate of adult anchovy biomass from the November survey |
| $\ln \left(k_{r}^{A}\right) \sim U[-100,0.7]$ | Log of the constant of proportionality associated with the acoustic survey <br> estimate of recruit survey numbers from the recruit survey |
| $\ln \left(k_{p}^{A}\right) \sim U[-100,0.7]$ | Log of the constant of proportionality associated with the proportion of 1- <br> year-olds in the November survey |
| $\left(\lambda_{r}^{A}\right)^{2} \sim U[0,100]$ | Additional variance associated with the recruit survey |
| $\varepsilon_{y}^{A} \sim N\left(0,\left(\sigma_{r}^{A}\right)^{2}\right), y=1984, \ldots, y_{n-1}$ | Annual lognormal deviation of anchovy recruitment |
| $\left(\sigma_{r}^{A}\right)^{2} \sim U[0.4,10]$ | Standard deviation in the residuals about the stock recruitment curve |
| $N_{1983, a}^{A} \sim U[0,500], a=0,1$ | Numbers at age in the initial year |
| $N_{1983, a}^{A} \sim U[0,0.01], a=2,3$ | Numbers at age in the initial year. These are set effectively at zero because <br> there are insufficient data to determine these values with reasonable <br> precision. |
| $\eta_{y}^{a d} \sim N\left(0, \sigma_{a d}^{2}\right)$ | Normally distributed error used in calculating the annual residuals about <br> adult natural mortality |
| $\sigma_{a d} \sim U[0.20,0.50]$ | Standard deviation in the annual residuals about adult natural mortality |
| $p \sim U[0,1]$ | Annual autocorrelation coefficient in annual residuals about adult natural <br> mortality |
| $\sigma_{p}^{A} \sim U[0.09,10]$ | Standard deviation associated with the estimated proportion of 1-year-olds in <br> the November survey |
| $\ln \left(a^{A}\right) \sim U[0,7.2]$ | Log of the maximum median recruitment in the hockey stick stock <br> recruitment curve |
| $\frac{b^{A}}{K^{A} \sim U[0,1]}$ | The biomass above which median recruitment is not impaired in the hockey <br> stick stock recruitment curve as a proportion of carrying capacity |

Table 2. The MCMC chain length, thinning and burn-in used to get a sample from the posterior distribution for the robustness tests. The posterior means and CVs of key model parameters and outputs are also shown. Biomasses are given in thousands of tons and numbers in billions. Parameters fixed for MCMC runs are given in bold (initial testing showed very little movement in the chain from the posterior mode)

|  | $\mathrm{A}_{\mathrm{HS}}$ |  |  | $\mathrm{A}_{\text {cstM }}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total chain length | 120000000 |  |  | 240000000 |  |  |
| Thinning | 3000 |  |  | 3000 |  |  |
| Chain excluded (eg for burn-in) | 10000000 |  |  | 30000000 |  |  |
| Length of chain used for posterior | 30000 |  |  | 50000 |  |  |
| Parameter | Median | Mean | CV | Median | Mean | CV |
| $k_{N}^{A}$ | 1.07 | 1.08 | 0.12 | 1.07 | 1.08 | 0.13 |
| $k_{R}^{A}$ | 0.78 | 0.79 | 0.14 | 1.02 | 1.03 | 0.14 |
| $k_{p}^{A}$ | 0.86 | 0.87 | 0.05 | 0.95 | 0.94 | 0.06 |
| $\left(\lambda_{r}^{A}\right)^{2}$ | 0.11 | 0.12 | 0.49 | 0.20 | 0.22 | 0.43 |
| $\left(\sigma_{p}^{A}\right)^{2}$ | 0.09 | 0.09 | N/A | 0.85 | 0.91 | 0.35 |
| $N_{1983,0}^{A}$ | 174 | 178 | 0.18 | 180 | 189 | 0.32 |
| $N_{1983,1}^{A}$ | 129 | 133 | 0.28 | 146 | 149 | 0.38 |
| $N_{1983,2}^{A}$ | 0.005 | 0.005 | 0.58 | 0.005 | 0.005 | 0.55 |
| $N_{1983,3}$ | 0.005 | 0.005 | 0.57 | 0.005 | 0.005 | 0.58 |
| $\sigma_{a d}$ | 0.45 | 0.44 | 0.10 | N/A | N/A | N/A |
| $\rho$ | 0.59 | 0.57 | 0.31 | N/A | N/A | N/A |
| $\eta_{2010}^{a d}$ | 0.42 | 0.40 | 0.65 | N/A | N/A | N/A |
| $a^{\text {A }}$ | 903 | 906 | 0.26 | 474 | 551 | 0.45 |
| $b^{A}$ | 4318 | 4380 | 0.30 | 2600 | 3146 | 0.55 |
| $\sigma_{r}^{A}$ | 0.66 | 0.67 | 0.18 | 0.75 | 0.76 | 0.17 |
| $K^{\text {A }}$ | 7239 | 7259 | 0.26 | 3798 | 4418 | 0.45 |
| $\eta_{2009}^{A}$ | -0.56 | -0.57 | 0.63 | -0.52 | -0.53 | 0.79 |
| $s_{c o r}^{\text {a }}$ | 0.16 | 0.17 | 0.59 | 0.17 | 0.17 | 0.61 |
| $\bar{B}_{\text {Nov }}^{A}$ | 1268 | 1274 | 0.11 | 1251 | 1260 | 0.12 |
| $N_{2010,1}^{A}$ | 159 | 162 | 0.22 | 89 | 93 | 0.34 |
| $N_{2010,2}^{A}$ | 40 | 42 | 0.34 | 57 | 58 | 0.27 |
| $N_{2010,3}^{A}$ | 8.84 | 9.57 | 0.55 | 40 | 41 | 0.23 |
| $N_{\text {2010,4+ }}^{\text {A }}$ | 1.04 | 1.39 | 0.86 | 16 | 16 | 0.21 |
| $B_{2010, N}^{A}$ | 2183 | 2217 | 0.18 | 2451 | 2495 | 0.18 |



Figure 1. Posterior distributions for key model parameters and outputs for the two base case hypotheses $\mathrm{A}_{\mathrm{HS}}$ (solid lines) and $\mathrm{A}_{\text {cstM }}$ (dotted lines). The prior distributions for model parameters estimated are shown by the thin dashed lines. The prior distribution for $\eta_{2010}^{a d}$ is not plotted as it depends on the distribution of $\sigma_{a d}^{2}$, as $\eta_{y}^{a d} \sim N\left(0, \sigma_{a d}^{2}\right)$.


Figure 2. The posterior pdfs of annual November biomass from the two base case hypotheses $\mathrm{A}_{\mathrm{HS}}$ (solid lines) and $\mathrm{A}_{\text {cstM }}$ (dotted lines) - note that in some cases where these are virtually identical, the latter is not visible as it is covered by the former.








Figure 2 (continued).


Figure 3. The posterior median and $95 \%$ probability intervals of the annual anchovy November $1+$ biomass. The posterior medians for $\mathrm{A}_{\text {HS }}$ (solid) and $\mathrm{A}_{\text {cssm }}$ (doted) are plotted together in c ).


Figure 4. The posterior median and $95 \%$ probability intervals for adult natural mortality for $\mathrm{A}_{\mathrm{HS}}$.


Figure 5. The posterior median and $95 \%$ probability intervals of the annual numbers of anchovy at age 0 in November for a) $A_{H S}$ and $b$ ) $A_{c s t M}$. The posterior medians for $A_{H S}$ (solid) and $\mathrm{A}_{\text {cstM }}$ (dotted) are plotted together in c ).




Figure 6. The posterior median and $95 \%$ probability intervals in anchovy loss to predation for a) $A_{H S}$ and $b$ ) $A_{\text {cstM }}$. The posterior medians for $A_{H S}$ (solid) and $A_{\text {cstM }}$ (dotted) are plotted together in c).


[^0]:    * MARAM (Marine Resource Assessment and Management Group), Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch, 7701, South Africa.

